Experimental evaluation of modal parameter based system identification procedure

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Abstract
Correct modelling of the foundation of a rotor bearing foundation system (RBFS) is an invaluable asset for the balancing and efficient running of turbomachinery. Numerical experiments have shown that a modal parameter based identification approach could be feasible for this purpose but experimental verification of the suitability of the approach for even the simplest systems is lacking. In this paper the approach is tested on a simple experimental rig comprising a clamped horizontal bar with lumped masses. It is shown that apart from damping, the proposed approach can identify reasonably accurately the relevant modal parameters of the rig; and that the resulting equivalent system can predict reasonably well the frequency response of the rig. Hence, the proposed approach shows promise. However, further testing is required, since application to identifying the foundation of an RBFS involves the additional problem of obtaining the force excitation from motion measurements, giving rise to additional error in the required input data.

1 Introduction

Correct modelling of a rotor bearing foundation system (RBFS) is an invaluable asset for the balancing and efficient running of turbomachinery but modelling of the foundation is still problematic, particularly for existing installations [1–4]. A promising approach for such modelling uses motion measurements of the rotor and foundation at the bearing supports and at select points on the foundation to identify the relevant modal parameters for an equivalent foundation, defined as a foundation which, when substituted for the actual foundation, reproduces the vibration behaviour of the RBFS over the operating speed range of interest. If successful, such an identification technique would be applicable to supporting structures of existing turbomachinery installations using readily available monitoring instrumentation.

In earlier work, an approach was developed which successfully identified, via numerical experiments, the modal parameters of an equivalent foundation for a relatively simple RBFS consisting of an unbalanced rotor supported by two hydrodynamic bearing pedestals fixed to a flexibly supported flexible foundation block, using as input data the numerically generated motion of the foundation and forces transmitted to the foundation at the bearing supports [5].

Ref. [5] did not fully consider problems associated with experimental measurement data generation and processing; nor did it fully consider practical instrumentation limitations. Hence, experimental evaluation of the modal parameter identification technique using experimentally generated input measurement data is in order. While recognising that an actual RBFS is more complex and has more sources of input data error, this paper, as a first step, evaluates the ability to successfully determine, in a laboratory environment, the modal parameters needed to define an equivalent system for a flexible flat bar with lumped masses, estimated to have five degrees of freedom (DOF) in the horizontal plane over this frequency range. Extension of this simple system into an RBFS is left for future work.
2 The identification equations

As shown in the appendix, the equations of motion of a stable \( n \) DOF linear system subjected to harmonic excitation forces can be written as:

\[
(-\Omega^2 I + i\Omega\zeta + \lambda)A^T \ddot{X} = m^{-1}\Phi^T \ddot{F}
\]

Equation (1) comprises the \( n \) identification equations \((k = 1, \ldots, n)\):

\[
(-\Omega^2 + i\Omega\zeta_k + \lambda_k) \sum_{j=1}^{n} a_{jk} \dot{X}_j - \sum_{j=1}^{n} \Phi_{jk} F_j / m_k = 0
\]

Knowledge of the measured values of \( F_j \) and \( \dot{X}_j \) at a sufficient number of excitation frequencies \( \Omega \) suffices to identify the elements of \( \zeta, m, \lambda \) and \( \Phi \) [5]. These parameters define the desired equivalent system.

3 Experimental procedure

3.1 Experimental rig

Figures 1 and 2 show the experimental rig which consists of a 3 mm by 20 mm rectangular steel bar with five masses clamped along its length. The ends of the bar are clamped in vices using pairs of ‘L’ keys and each vice is fixed to the laboratory floor via two bolts with a nominal clamping torque of 41 N \( \cdot \) m. The end fixities ensure that the bar is in tension when clamped in the vices and that there is no axial motion of the bar ends during the experiments. Thus, a piece of steel is welded to the left end of the bar and bears against the end face of the left vice; and wedges bear against the end face of the right vice and against a steel pin located through the right end of the bar.

Figure 1: The general view of experimental rig

(a) Vice at the left end (b) Vice at the right end

Figure 2: Bar end fixities
Figure 3 and table 1 show the mass weights and their locations. These were selected to obtain 5 natural frequencies with horizontal mode shapes within the planned excitation frequency range. Since the steel bar is flat and thin, the natural frequencies of the vibration modes in the horizontal direction are lower than those in the vertical direction. Accordingly, all force excitations and displacement measurements were to be in the horizontal plane; and the modal parameters defining the equivalent system were to be identified using the measured horizontal excitation forces applied at masses $M_2$ and $M_4$ and the measured horizontal accelerations of the five masses.

### 3.2 Experimental setup

Figure 4 is a schematic of the experimental setup. The system is to be excited by two horizontal shakers connected to masses $M_2$ and $M_4$ respectively. A signal generator is used to generate stable sinusoidal voltage signals. These signals are fed to two PA25E-CE amplifiers, the output voltages from which actuate two V200 Series shakers (each shaker having a lower frequency limitation of 3 Hz for effective operation) so that each shaker output force will have the same frequency but may differ in amplitude [6]. The stingers of the two shakers are attached to masses $M_2$ and $M_4$ respectively to supply excitation forces in the transverse horizontal directions; and force transducers are used to measure these excitation forces.
Accelerometers are attached to a flat surface of each cylindrical mass to measure the accelerations of the masses in the transverse horizontal directions. The measured force and acceleration signals are amplified by charge amplifiers, digitised by a 16 channel PC-30D data acquisition board and stored in a computer for subsequent data processing using in house data processing software.

### 3.3 Determination of actual natural frequencies

The actual natural frequencies of the system were determined by applying hammer tests at each mass in the transverse horizontal direction. Figure 5 shows the resulting frequency spectra of the horizontal accelerations of the five masses over the frequency range from 0 to 150 Hz with a frequency resolution of 0.125 Hz. The spectra for all five masses display, to varying degrees, the five natural frequencies shown in table 2. As expected, only five dominant natural frequencies appear over this frequency range. Two system identification experiments were carried out consecutively. Apart from providing the evaluation yardstick, the hammer tests were applied before and after the identification experiments to provide assurance that the system had not changed significantly during the experiments. The natural frequencies as determined by the hammer tests before and after each experiment are shown in table 2. It can be seen that there was minimal change in the properties of the rig and one can assume there was negligible change in the system during the experiments.

![Figure 5: Frequency spectra of the horizontal accelerations of each mass](image)

<table>
<thead>
<tr>
<th>Modes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Expt1</td>
<td>4.250</td>
<td>11.875</td>
<td>23.250</td>
<td>40.500</td>
<td>65.750</td>
</tr>
<tr>
<td>After Expt1; Before Expt2</td>
<td>4.250</td>
<td>11.750</td>
<td>23.250</td>
<td>40.500</td>
<td>65.625</td>
</tr>
<tr>
<td>After Expt2</td>
<td>4.125</td>
<td>11.750</td>
<td>23.125</td>
<td>40.375</td>
<td>65.625</td>
</tr>
</tbody>
</table>

Table 2: Natural frequencies as determined by hammer test (Hz)

### 3.4 Finite element estimation of modal parameters

It was possible to estimate its modal properties using ANSYS finite element modelling (FEM) using block elements. For this computation, a Young’s modulus of 210 GPa, a density of 7850 kg/m³ and a shear modulus of 80 GPa were assumed. Table 3 shows the change in the natural frequencies as the number of elements used to model the flat bar is increased to 9600. Single precision was used for these computations. There were found to be seven natural frequencies in the vicinity of the excitation range of interest (3 Hz to 80 Hz) and, as can be seen in table 3, all natural frequencies except the fourth and seventh had converged. Figure 6 shows the first seven natural frequencies and their corresponding mode shapes. It can be seen that the 4th and the 7th modes are vertical vibration modes. These would not be picked up in the hammer test, as all hammer excitations were in the horizontal direction as were all accelerometer measurements. For the
same reason, one would not identify these vertical mode shapes in the identification procedure. Thus, the
fact that their values had not yet converged (see Table 3) was of little consequence.

<table>
<thead>
<tr>
<th>No. of Elements</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
<th>$\omega_6$</th>
<th>$\omega_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>240</td>
<td>4.33</td>
<td>12.83</td>
<td>25.69</td>
<td>52.87</td>
<td>45.75</td>
<td>74.92</td>
<td>123.3</td>
</tr>
<tr>
<td>400</td>
<td>4.28</td>
<td>12.67</td>
<td>25.26</td>
<td>53.08</td>
<td>44.56</td>
<td>73.25</td>
<td>123.6</td>
</tr>
<tr>
<td>1280</td>
<td>4.28</td>
<td>12.67</td>
<td>25.20</td>
<td>53.37</td>
<td>44.52</td>
<td>72.94</td>
<td>123.9</td>
</tr>
<tr>
<td>2400</td>
<td>4.27</td>
<td>12.66</td>
<td>25.16</td>
<td>51.48</td>
<td>44.43</td>
<td>72.82</td>
<td>120.3</td>
</tr>
<tr>
<td>6000</td>
<td>4.27</td>
<td>12.66</td>
<td>25.15</td>
<td>46.12</td>
<td>44.42</td>
<td>72.77</td>
<td>110.5</td>
</tr>
<tr>
<td>9600</td>
<td>4.27</td>
<td>12.65</td>
<td>25.14</td>
<td>44.24</td>
<td>44.41</td>
<td>72.77</td>
<td>107.0</td>
</tr>
</tbody>
</table>

Table 3: FEM natural frequencies (Hz)

<table>
<thead>
<tr>
<th>Modes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hammer test</td>
<td>4.250</td>
<td>11.875</td>
<td>23.250</td>
<td>N/A</td>
<td>40.500</td>
<td>65.750</td>
<td>N/A</td>
</tr>
<tr>
<td>FEM</td>
<td>4.27</td>
<td>12.65</td>
<td>25.14</td>
<td>44.41</td>
<td>44.24</td>
<td>72.77</td>
<td>107.0</td>
</tr>
</tbody>
</table>

Table 4: Hammer test and FEM natural frequencies (Hz)

Figure 6: Natural frequencies and mode shapes using FEM
Table 4 compares the FEM and hammer test natural frequencies. There was reasonable qualitative agreement between the FEM and hammer test results, particularly at the lower natural frequencies. This was as expected, as the FEM modelling is only approximate. Also, the FEM results are for undamped natural frequencies whereas the hammer tests yield damped natural frequencies, and so are expected to be slightly lower, depending on the actual system damping. Thus, the FEM results give a reasonable qualitative description, and can be used as a yardstick for evaluating the yet to be identified mode shapes.

3.5 System identification experiments

Two identification experiments, viz. experiment 1 and experiment 2, with different levels of force excitation were carried out. The measured amplitudes of the excitation forces applied to masses M2 and M4 are shown in figure 7, wherein the solid lines correspond to experiment 1. The measurement data from each experiment was used to identify the system modal parameters, thereby yielding two identified equivalent systems, viz. System 1 and System 2; the main difference being that the measurements used to identify System 2 involved smaller excitation force amplitudes so that these measurements would presumably have smaller signal to noise ratios. In each experiment, measurement data was collected at every Hertz over the frequency range of 3 to 80 Hz, resulting in 78 different measurement frequencies. Since the force and displacement amplitudes are not necessarily in phase, one has 156 real and imaginary measurement data sets available for the identification. The identification procedure detailed in ref. [5] was then used to identify the modal parameters of System 1 and System 2, with each equivalent system assumed to have five DOF.

![Excitation forces](image)

(a) Excitation force amplitudes on mass M2  
(b) Excitation force amplitudes on mass M4

Figure 7: Excitation forces

4 Results and discussion

4.1 Identified parameters: System 1

The results are first discussed for System 1. Initially the equivalent system was identified using measurement data at every 5 Hz, with the first measurement at 3 Hz. In all cases, data at 50 Hz was ignored. Table 5 shows that all the five horizontal mode natural frequencies could be identified.

<table>
<thead>
<tr>
<th>Step Intervals</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Hz</td>
<td>4.27</td>
<td>11.62</td>
<td>23.09</td>
<td>38.09</td>
<td>66.05</td>
</tr>
<tr>
<td>4 Hz</td>
<td>4.26</td>
<td>11.62</td>
<td>22.72</td>
<td>36.01</td>
<td>65.98</td>
</tr>
<tr>
<td>3 Hz</td>
<td>4.27</td>
<td>11.71</td>
<td>22.47</td>
<td>38.31</td>
<td>66.03</td>
</tr>
<tr>
<td>2 Hz</td>
<td>4.27</td>
<td>11.75</td>
<td>22.35</td>
<td>37.28</td>
<td>66.00</td>
</tr>
<tr>
<td>1 Hz</td>
<td>4.29</td>
<td>11.77</td>
<td>22.14</td>
<td>37.93</td>
<td>65.94</td>
</tr>
<tr>
<td>Hammer Test</td>
<td>4.125</td>
<td>11.750</td>
<td>23.125</td>
<td>40.375</td>
<td>65.625</td>
</tr>
</tbody>
</table>

Table 5: Identified natural frequencies (Hz) using increased number of data sets. System 1
There are obvious deviations from the yardstick hammer test results, especially in the 4\textsuperscript{th} natural frequency. Increasing the number of data sets by using measurement data at every 4 Hz, 3 Hz, 2 Hz and 1 Hz respectively did not seem to improve the identification. Indeed, occasionally it appeared to be worse.

In theory, if the identification is accurate, the number of data sets used should not affect the results. However, in practice it is impossible to avoid measurement error as a result of the accuracy limitations of the instrumentation and noise. Hence, a weighting procedure was applied to improve the identification result. Again, data at 50 Hz was ignored. In this weighting procedure, measurement data with 5 Hz step size was first used to identify the equivalent system so that there were 16 measurement speeds within the range of 3 to 80 Hz. Following this, additional measurement data in the vicinity of the identified natural frequencies was added, viz. data corresponding to three consecutive measurement speeds, differing by 1 Hz, before and after an initially identified frequency; i.e. 12 extra data sets were added to obtain the identification results. If the new identified frequency was different, it replaced the original one. This was continued until there was minimal change in that frequency.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st}</td>
<td>4.27</td>
<td>11.61</td>
<td>23.18</td>
<td>39.65</td>
<td>66.14</td>
</tr>
<tr>
<td>2\textsuperscript{nd}</td>
<td>4.27</td>
<td>11.66</td>
<td>23.22</td>
<td>41.61</td>
<td>66.15</td>
</tr>
<tr>
<td>3\textsuperscript{rd}</td>
<td>4.27</td>
<td>11.66</td>
<td>23.22</td>
<td>41.82</td>
<td>66.15</td>
</tr>
<tr>
<td>Hammer Test</td>
<td>4.125</td>
<td>11.750</td>
<td>23.125</td>
<td>40.375</td>
<td>65.625</td>
</tr>
</tbody>
</table>

Table 6: Identified natural frequencies (Hz) using iterative weighting procedure. System 1

The corresponding results are given in table 6. Adding additional data sets did little to change $\omega_1$, $\omega_2$, $\omega_3$ or $\omega_5$. Hence, they are assumed to be accurate. However there were significant changes in the identified $\omega_4$, which changed from 39.65 Hz to 41.61 Hz, thereby predicting further iteration with weighting data now centred near 41 Hz. The subsequent change in $\omega_4$ from 41.61 Hz to 41.82 Hz is now less than 1 Hz so that no further improvement can be expected with this weighting procedure.

The final identified modal parameters defining System 1 are given in table 7. There is good agreement between the actual and identified natural frequencies except for a significant discrepancy of 1.3 Hz (3.3\%) in $\omega_4$. All other errors are less than 1\%. The identified damping ratios $\xi_k$ are small (of order $10^{-5}$). Negative $\xi_k$ are identified in the 1\textsuperscript{st} and 4\textsuperscript{th} modes. This is inconsistent with theory and suggests excessive error build up due to instrumentation accuracy limitations, noise, data processing inaccuracy and/or that the assumption of a diagonalisable damping matrix is inappropriate.

<table>
<thead>
<tr>
<th>Hammer Test (Hz)</th>
<th>$\omega_k$ (Hz)</th>
<th>$\Phi_{1k}$</th>
<th>$\Phi_{2k}$</th>
<th>$\Phi_{3k}$</th>
<th>$\Phi_{4k}$</th>
<th>$\Phi_{5k}$</th>
<th>$m_k$ (kg)</th>
<th>$\xi_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.250</td>
<td>4.27</td>
<td>0.12</td>
<td>0.30</td>
<td>0.39</td>
<td>0.44</td>
<td>0.25</td>
<td>0.59</td>
<td>-0.013</td>
</tr>
<tr>
<td>11.750</td>
<td>11.66</td>
<td>-0.21</td>
<td>-0.35</td>
<td>-0.15</td>
<td>0.23</td>
<td>0.35</td>
<td>0.38</td>
<td>0.013</td>
</tr>
<tr>
<td>23.250</td>
<td>23.21</td>
<td>0.44</td>
<td>0.25</td>
<td>-0.31</td>
<td>-0.12</td>
<td>0.43</td>
<td>0.64</td>
<td>0.031</td>
</tr>
<tr>
<td>40.500</td>
<td>41.82</td>
<td>0.48</td>
<td>-0.10</td>
<td>-0.27</td>
<td>0.34</td>
<td>-0.10</td>
<td>0.25</td>
<td>-0.023</td>
</tr>
<tr>
<td>65.625</td>
<td>66.15</td>
<td>-0.32</td>
<td>0.40</td>
<td>-0.21</td>
<td>0.19</td>
<td>-0.08</td>
<td>0.37</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 7: Final identified modal parameters. System 1

To further investigate the effect of damping, the system was re-identified by assuming zero damping. The re-identified modal parameters are shown in table 8 which shows that the identified natural frequencies are generally closer to the hammer test values; indeed significantly so for the 4\textsuperscript{th} horizontal mode. There is significant difference in the modal masses corresponding to the 3\textsuperscript{rd} and 4\textsuperscript{th} modes. The mode shape values are much the same with and without damping.

Figure 8 compares the identified mode shapes with the FEM predictions. Except for some discrepancies in the 4\textsuperscript{th} horizontal mode shape, there is generally good agreement between the identified foundation mode shape predictions and the FEM mode shape predictions.
Table 8: Identified modal parameters assuming zero damping ratios. System 1

<table>
<thead>
<tr>
<th>Hammer Test (Hz)</th>
<th>ω_k (Hz)</th>
<th>Φ_{1k}</th>
<th>Φ_{2k}</th>
<th>Φ_{3k}</th>
<th>Φ_{4k}</th>
<th>Φ_{5k}</th>
<th>m_k (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.250</td>
<td>4.26</td>
<td>0.13</td>
<td>0.30</td>
<td>0.39</td>
<td>0.44</td>
<td>0.25</td>
<td>0.59</td>
</tr>
<tr>
<td>11.750</td>
<td>11.68</td>
<td>-0.22</td>
<td>-0.36</td>
<td>-0.14</td>
<td>0.23</td>
<td>0.35</td>
<td>0.39</td>
</tr>
<tr>
<td>23.250</td>
<td>23.19</td>
<td>0.45</td>
<td>0.27</td>
<td>-0.32</td>
<td>-0.14</td>
<td>0.46</td>
<td>1.04</td>
</tr>
<tr>
<td>40.500</td>
<td>40.58</td>
<td>0.48</td>
<td>-0.04</td>
<td>-0.31</td>
<td>0.34</td>
<td>-0.07</td>
<td>0.41</td>
</tr>
<tr>
<td>65.625</td>
<td>66.10</td>
<td>-0.33</td>
<td>0.41</td>
<td>-0.22</td>
<td>0.20</td>
<td>-0.08</td>
<td>0.42</td>
</tr>
</tbody>
</table>

4.2 Responses to experiment 1 excitation

Figure 9(a) compares the actual to the predicted responses obtained using the identified System 1, using for excitation input the excitation forces corresponding to experiment 1 in figure 7. Note that the excitation forces are of variable amplitude, giving rise to response peaks and dips at speeds other than the natural frequencies. There is good, though not perfect, agreement between the predicted and actual frequency responses at each mass. The major discrepancies occur around 20 Hz and around 40 Hz. Error around 40 Hz was expected, considering the errors in identifying the natural frequency around 41 Hz and in the corresponding mode shape. The error around 20 Hz is more mystifying, could be because the damping matrix in equation (A1) is assumed to be diagonalisable. It is beyond the scope of this present investigation to ascertain the extent to which this assumption is valid. The more accurate assumption of a symmetric though not necessarily diagonalisable damping matrix adds considerable complexity to the analysis [7]. Alternatively, the assumption of structural rather than viscous damping may in this instance have been more appropriate. Such software enhancements are left for future work.

Figure 9(b) is the same as figure 9(a) except that the damping is assumed to be zero. Apart from the expected discrepancies in the vicinity of some of the resonances, the agreement between actual and predicted responses is inferior even between resonances. Hence, damping effects are worth including even though some if not all of the identified damping ratios are erroneous.

4.3 Identified parameters: System 2

System 2 was obtained using the same identification procedure as was used for System 1, the only difference being an attenuation in the force excitation (and hence response measurement) amplitudes. Table 9 shows the final identified modal parameters. Again, there is generally good agreement between the actual and identified natural frequencies except for a discrepancy of 1.5 Hz (3.6%) in \( \omega_4 \) and a
discrepancy of 0.46 Hz (2.0%) in $\omega_3$. Except for $\xi_1$, which has a surprisingly high value of 0.1, the identified damping ratios are again quite small and bear little resemblance to those of System 1. Again, some of the small damping ratios are negative, suggesting excessive error build up due to a combination of instrumentation accuracy limitations, noise, data processing inaccuracy and/or the assumption of a diagonalisable damping matrix. For this reason it is suspected that a significant part of the identified $\xi_k$ could derive from such accumulative error rather than from the actual damping. Comparing the identified modal masses in System 1 and System 2, there are significant differences in the modal masses, particularly in those corresponding to the 4th and 5th modes.

The identified mode shapes for both System 1 and System 2 are compared with the FEM predictions in figure 10. There is good agreement between the identified foundation mode shapes and the FEM mode shape prediction except for huge discrepancies in the 4th horizontal mode shape in the case of System 2.

Figure 9: Actual and identified responses at the five masses with and without damping. Excitation as per experiment 1
Table 9: Final identified modal parameters. System 2.

<table>
<thead>
<tr>
<th>Hammer Test (Hz)</th>
<th>$\omega_k$ (Hz)</th>
<th>$\phi_{1k}$</th>
<th>$\phi_{2k}$</th>
<th>$\phi_{3k}$</th>
<th>$\phi_{4k}$</th>
<th>$\phi_{5k}$</th>
<th>$m_k$ (kg)</th>
<th>$\xi_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.125</td>
<td>4.18</td>
<td>0.13</td>
<td>0.29</td>
<td>0.39</td>
<td>0.43</td>
<td>0.26</td>
<td>0.53</td>
<td>0.107</td>
</tr>
<tr>
<td>11.750</td>
<td>11.78</td>
<td>-0.17</td>
<td>-0.30</td>
<td>-0.17</td>
<td>0.25</td>
<td>0.38</td>
<td>0.26</td>
<td>0.019</td>
</tr>
<tr>
<td>23.125</td>
<td>23.59</td>
<td>0.44</td>
<td>0.27</td>
<td>-0.34</td>
<td>-0.08</td>
<td>0.41</td>
<td>0.98</td>
<td>0.017</td>
</tr>
<tr>
<td>40.375</td>
<td>38.92</td>
<td>0.10</td>
<td>0.46</td>
<td>-0.61</td>
<td>0.58</td>
<td>-0.20</td>
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<td>-0.034</td>
</tr>
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<td>65.45</td>
<td>-0.46</td>
<td>0.58</td>
<td>-0.34</td>
<td>0.29</td>
<td>-0.12</td>
<td>1.78</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

4.4 Responses to experiment 2 excitation

Figure 11(a) compares the actual to the predicted responses obtained using the identified System 1, using for excitation input the excitation forces corresponding to experiment 2 in figure 7. Again note that the excitation forces are of variable amplitude. The agreement between the actual and predicted responses is as good as that obtained in figure 9(a) even though the excitation forces are different from those used to obtain System 1. This shows that the identified System 1 is not biased by the measurement data used to obtain it.

Figure 11(b) compares the actual to the predicted responses obtained using the identified System 2, using this time for excitation input the excitation forces corresponding to experiment 2 in figure 7. It is clear that the response predictions using System 1 (in figure 11(a)) are superior to those using System 2 (in figure 11(b)). This is not surprising since the measurement data used to identify System 1 had larger amplitudes than those used to identify System 2 (as may be clearly seen when comparing figures 12 and 13, which show the amplitudes of the corresponding measured accelerations) and hence were less likely to be affected by noise and instrumentation accuracy limitations.

Figure 14 shows the frequency spectra of the horizontal accelerations of the five masses with the bar stationary, prior to running the experiments. Since noise levels are random and could be different when the bar is excited, these amplitudes are only indicative of what the noise level may have been during the experiments. Figures 15 and 16 show the corresponding noise to signal ratios of the acceleration amplitudes pertaining to experiments 1 and 2 respectively. As expected, these ratios are much larger for measurements pertaining to experiment 2, where the ratio occasionally exceeds 0.5 and in one instance is as high as 0.8. (Note that 50 Hz measurements were never used for the identifications). However, it can be seen that the noise to signal ratios are quite small for measurements in the vicinity of the identified natural frequencies; and hence are unlikely to be the main reason for the discrepancies between the real and identified responses around 23 Hz and 41 Hz as seen in figures 9(a) and 11(a). Since the actual measurement amplitudes in the vicinity of these frequencies are quite small, it is felt more likely that these discrepancies are due to instrumentation accuracy limitations and/or computational round off errors.
The reasonably good predictions using both System 1 and System 2 suggest that the excitation forces were sufficiently large to ensure that they and the measured accelerometer signals did not suffer from excessive noise and did not suffer from excessive instrumentation accuracy limitations. Thus, this experimental evaluation demonstrated that the proposed system identification approach is practically feasible, provided one can overcome the problems of excessive noise, instrumentation accuracy limitations and computational round off errors, which errors are felt to be the likely cause for inaccurate damping ratios.

Note that in an actual RBFS, the damping due to hydrodynamic bearings will dominate the system damping and accurate identification of the relatively small foundation damping is not so important. However, since force excitations are due to rotor unbalance, the error in the input measurement data will be larger than in these experiments for several reasons. Firstly, the magnitude of the force excitations is unbalance dependent so that accelerations will be small at the lower rotor speeds. Secondly, the force...
excitations cannot be measured directly but are derived from rotor and foundation motion measurements.
Thirdly, since the unbalance is unknown, signal subtractions are needed to furnish the measurement data.
Hence, further experimentation is still needed to properly evaluate the application of the proposed identification procedure to turbomachinery foundations.

Figure 12: Acceleration amplitudes. Experiment 1

Figure 13: Acceleration amplitudes. Experiment 2

Figure 14: Frequency spectra of the accelerations of each mass in absence of excitation
5 Summary of conclusions

The system to be identified had five horizontal mode natural frequencies over the excitation range.

FEM natural frequency predictions agree qualitatively with hammer test results; and FEM mode shape predictions can be used as a yardstick for evaluating the identified mode shapes.

Apart from damping, the proposed identification approach could identify reasonably accurately the relevant modal parameters. In particular, all five horizontal mode natural frequencies could be identified and there was generally good agreement between the actual and identified natural frequencies. There was generally good agreement between the identified mode shapes and the FEM predictions; and the equivalent system could predict reasonably well the frequency response of the experimental rig.

The identified system is not biased by the measurement data used to obtain it.

Possible causes of the discrepancies between the real and identified responses were the assumption of a diagonalisable damping matrix, instrumentation accuracy limitations, computational round-off errors, and the presence of noise (least likely).

The identified damping ratios are of order $10^{-2}$ and some, if not all, are incorrect. Nevertheless, inclusion of damping via an assumed diagonalisable damping matrix significantly improved the predicted responses.

This experimental evaluation demonstrated that the proposed system identification approach is practically feasible, provided one can overcome the problems of excessive noise, instrumentation accuracy limitations and computational round-off errors, which errors are the likely cause for inaccurate damping ratios.

Further experimentation is needed to properly evaluate the applicability of the proposed identification procedure to turbomachinery foundations.
Nomenclature

\[ A = \text{modified modal matrix} = (\Phi^{-1})^T; \ a_{jk} = \text{elements of } A \]
\[ C = \text{diagonalisable system damping matrix} \]
\[ c = \text{modal damping matrix} \]
\[ \vec{f}; \vec{F} = \text{excitation force vector; amplitude thereof} \]
\[ I = \text{identity matrix} \]
\[ K = \text{symmetric system stiffness matrix} \]
\[ k = \text{modal stiffness matrix} \]
\[ M = \text{symmetric system mass matrix} \]
\[ m = \text{modal mass matrix}; \ m_k = \text{modal mass of the } k^{th} \text{ mode} \]
\[ \vec{x}; \vec{X} = \text{displacement vector; amplitude thereof} \]
\[ \lambda = \text{eigenvalue matrix} = m^{-1}k; \ \lambda_k = k^{th} \text{ eigenvalue} = \omega_k^2 \]
\[ \zeta = m^{-1}c; \ \zeta_k = k^{th} \text{ element of } \zeta \]
\[ \xi = \text{damping ratio matrix} = c/(2\sqrt{k \cdot m}); \ \xi_k = k^{th} \text{ element of } \xi \]
\[ \Phi = \text{system modal matrix}; \ \Phi_{jk} = \text{elements of } \Phi; \ \Phi_k = k^{th} \text{ mode shape vector} \]
\[ \Omega = \text{excitation frequency} \]
\[ \omega = \text{natural frequency matrix}; \ \omega_k = k^{th} \text{ natural frequency} \]

References

Appendix: The identification equations

A brief summary of the derivation of equations (1) and (2) is given below. Full derivation details and the numerical procedure adopted to solve for the required modal parameters are given in ref. [5].

Assuming the equivalent system can be represented by \( n \) DOF, its equations of motion are given by:

\[
M \dddot{x} + C \ddot{x} + K x = \ddotel
\]  
(A1)

The elements of the vector \( x \) are the \( n \) independent displacements chosen to coincide with convenient measurement locations and include the excitation force application points and directions. The elements of the vector \( \ddotel \) are the excitation forces acting on the system. Assuming that the excitation is harmonic with excitation frequency \( \Omega \), and that the system response is periodic with fundamental frequency \( \Omega \), one can obtain the equation:

\[
-\Omega^2 M \ddot{R} + i \Omega C \dot{R} + K R = \ddotel
\]  
(A2)

Letting \( R = \Phi \ddotel \) and premultiplying both sides of the equation by \( \Phi^T \), one obtains:

\[
(-\Omega^2 \Phi^T M \Phi + i \Omega \Phi^T C \Phi + \Phi^T K \Phi) \ddotel = \Phi^T \ddotel
\]  
(A3)

or

\[
(-\Omega^2 m + i \Omega c + k) \Phi^{-1} \ddotel = \Phi^T \ddotel
\]  
(A4)

Defining \( A = (\Phi^{-1})^T \) and substituting into equation (A4) gives:

\[
(-\Omega^2 I + i \Omega \zeta + \lambda) A^T \ddotel = m^{-1} \Phi^T \ddotel.
\]  
(A5)

Equation (A5) corresponds to equation (1). It comprises the \( n \) identification equations \((k = 1, \ldots, n)\):

\[
(-\Omega^2 + i \Omega \zeta_k + \lambda_k) \sum_{j=1}^{n} a_{jk} x_j - \sum_{j=1}^{n} \Phi_{jk} f_j / m_k = 0.
\]  
(A6)

Equations (A6) correspond to equations (2).

As explained in ref. [5], knowledge of the measured values of \( F_j \) and \( X_j \) at a sufficient number of excitation frequencies \( \Omega \) suffices to identify the elements of \( \zeta, m, \lambda \) and \( \Phi \), which parameters define the desired equivalent system. Thus one can recover the mass, damping and stiffness matrices according to:

\[
M = (\Phi^T)^{-1} m \Phi^{-1}
\]  
(A7)

\[
K = (\Phi^T)^{-1} k \Phi^{-1} = (\Phi^T)^{-1} m \lambda \Phi^{-1} = M \lambda \Phi^{-1}
\]  
(A8)

\[
C = (\Phi^T)^{-1} c \Phi^{-1} = (\Phi^T)^{-1} m \zeta \Phi^{-1} = M \zeta \Phi^{-1}
\]  
(A9)

and thereby calculate the frequency response of the equivalent system. Alternatively, one could obtain this response using the modal parameters directly according to:

\[
\ddotel = \Phi [m(-\Omega^2 I + 2i \Omega \omega \xi + \lambda)]^{-1} \Phi^T \ddotel
\]  
(A10)