# Estimation of the transfer function in rotating machinery with high instantaneous speed fluctuations

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## Abstract

The transfer function between the rotating component and the sensor complicates the diagnosis process. In this study transfer function estimation is considered, based on spectrum background estimation and minimum phase. The ability to suppress peaks that are smeared as result of high instantaneous speed fluctuations is limited, and thus we extend former work for mitigating this challenge. The synchronous properties of the gear signal are used in the cycle domain for separating them from the wideband noise using liftering in the cepstrum domain. The signal is divided to consecutive segments for enabling original phase restoration for converting back the low quefrencies of the noises that correspond to the original spectrum background to the time domain.

## 1 Introduction

A vibration signal is composed from the vibration of the rotating components that have propagated through a transfer function. The transfer function distorts the shape of the vibration signal, complicating its analysis. The estimation of the transfer function could be an important stage in the analysis of the measured signal for several purposes, including: (1) validation of realistic models [1], [2], (2) suppression of the transfer function for an accurate diagnosis of the fault state [3], [4] (3) or for transfer across different machines for machine learning procedures that included domain adaptation [5], [6].

Although the estimation of the transfer function based only on the measured vibration signal is a challenge in prognostic health maintenance (PHM), the current knowledge and the available techniques in the field are limited. The current techniques can be divided to two groups: (1) using the spectrum background of the signal and specific assumptions on the phase of the transfer function [7]–[10] and (2) estimation of the transfer function based on properties of the original signal [11], [12] like minimum entropy deconvolution (MED) [3], [13].

However, when the signal has high instantaneous speed fluctuations the ability to estimate its background is limited, especially for rotating components that their peaks in the backgrounds are very dense. In this study we improve and combine former techniques for estimating the transfer function by background estimation under high instantaneous speed fluctuations and minimum phase estimation of the transfer function using the cepstrum domain.

In Section 2 the technique for estimating the background under high instantaneous speed fluctuations is presented and in Section 3 minimum phase transfer function estimation is discussed. In Section 4 the two stages are articulated together and Section 5 summarizes this study and proposed new ideas for future research.

## 2 Background estimation under high instantaneous speed fluctuations

Some rotating components have a wideband noise that propagates through the same transfer function as the signals of the components. For example, pair of gears in a gearbox generates wide band random fluctuation caused by unideal surface shape [14] and transmission errors [3], [15]. Thus, for such cases, estimation of the background enables to approximate the transfer function magnitude.

Basically, as can be seen in Figure 1 (a), the background is the slow variations of the PSD without the sharp peaks. When the peaks are sharp the background can be estimated using several techniques including ACS [16] and Cepstrum liftering [3], [4], [17], [18]; these techniques are basically based on implementing a "low pass filter" on the "PSD signal" as explained in [8].

However, when the machinery has high instantaneous speed fluctuations the peaks are smeared and hence cannot be filtered out using the mentioned-above "low pass filter" [7]. As can be seen in Figure 1 (b), the peaks are smeared and seem as part of the background.

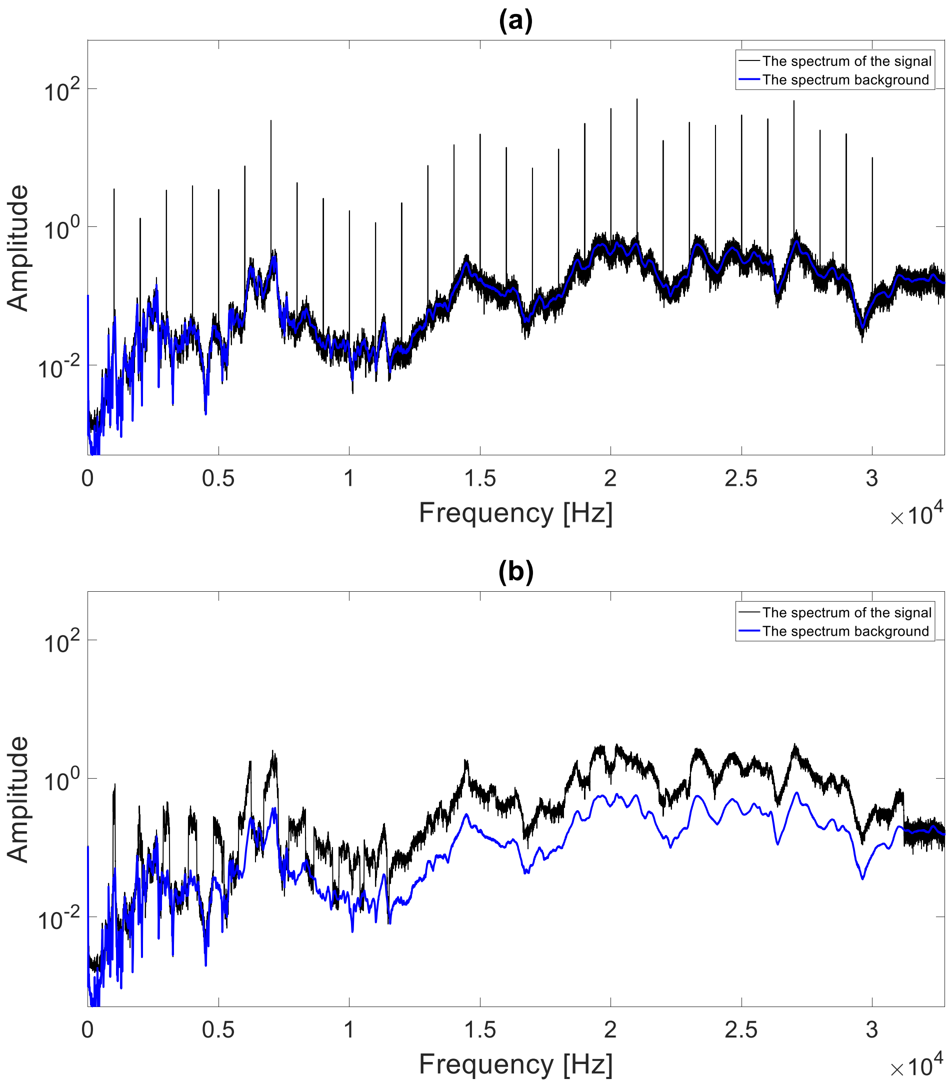


Figure 1: The spectrum of the signal with its spectrum background for (a) stationary signal and (b) non-stationary signal.

The speed of the rotating machinery can be sampled or estimated [19], [20] and the vibration signal can be resampled accordingly using angular resampling [21], [22]. Angular resampling can be seen as a base transformation where the representation of the signal is converted from time representation to cycle representation. Under high instantaneous speed fluctuations, the signal is non-periodic in the time domain and thus its peaks are smeared; in the cycle domain, after angular resampling, the signal is periodic and hence its peaks are clearly seen in the PSD. However, the wideband noise associated with the background is stationary in the time domain and non-stationary in the cycle domain, due to an artificial compression and decompression during the angular resampling process. Thus, the background in the order domain (the frequency domain of the cycle) is different from the background in the original frequency domain. The background in the order domain can be easily estimated using ACS and Cepstrum liftering, but it is not relevant for transfer function estimation which is defined in the frequency domain.

In this study we propose to filter out the peaks in the cycle domain by separating the signal into consecutive segments and filtering out the peaks in each segment using cepstrum liftering. In the liftering process the high quefrencies in each segment corresponding to the peaks are liftered out and the low quefrencies that correspond to the background remain. The original phase [16], [23] of the signal before the liftering is saved and added to the signals after the liftering process for converting the signal back to the cycle domain. Although the quefrencies of the background are different for each segment due to the non-stationary nature of them in the cycle domain, they are all mainly composed from low quefrencies and thus are not affected by the liftering process. In the last step the cycle signal is converted back to time domain and the spectrum background is estimated in the frequency domain by cepstrum liftering on the PSD. The liftering operation in the last step lifter out high variations of the PSD that can be attributed to randomness of the noise, and hence "smooths" the estimated background.

The idea for this process can be inspired from decomposition of the signal to the noises of the background and the peaks of the rotating component. When the noise is converted to the cycle domain, low-liftered in consecutive segments and then converted to the time domain is not distorted. When the peaks are converted to the cycle domain and then are low-liftered they are suppressed. In Figure 2 the estimated backgrounds of a non-stationary signal are compared with the real background. As can be seen, the background that is filtered in the cycle domain is the most accurate one.

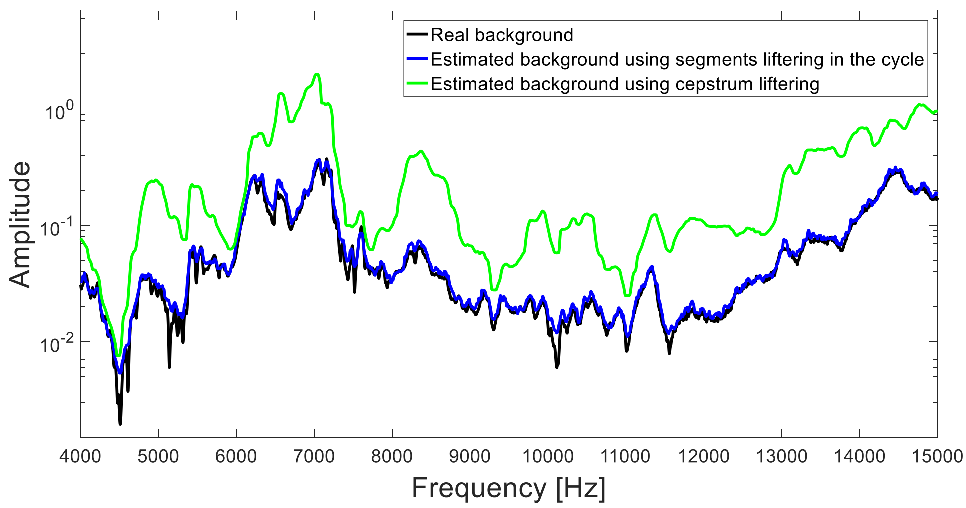


Figure 2: Estimation of the background directly in the frequency domain (green) and using segments liftering in the cycle domain (blue).

## 3 Minimum phase estimation

The background is associated with the magnitude of the transfer function for rotating components which generate wideband. However, the phase of the transfer function cannot be extracted directly from the spectrum background because in the frequency domain the noise that is used for estimating the magnitude has random phase, and it remains random after the multiplexing with the phase of the transfer function. For mechanical systems the phase of the transfer function can be approximated by estimating the corresponding minimum phase of the magnitude of the transfer function.

Transfer functions can be approximated by finite number of poles and zeros. For mechanical systems the poles are inside the unit circle. If we assume that the inverse transfer function is also stable the zeros must be also inside the unit circle because they correspond to the poles of the inverse transfer function. In such cases the transfer function holds the minimum phase assumption.

Under the minimum phase assumption, the phase can be directly extracted from the background. Oppenheim suggested to approximate the minimum phase using the cepstrum domain [9] by setting the negative quefrencies to zero and double the positive quefrencies of the background in the cepstrum domain. This technique was also used in other studies [3]. Figure 3 demonstrates this process on a simulated transfer function.

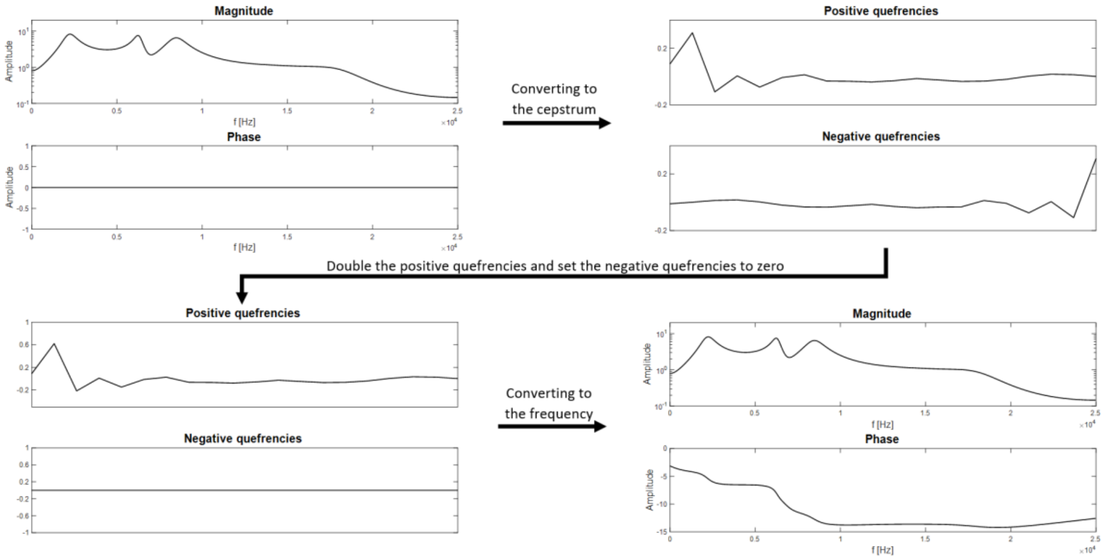


Figure 3: Example of minimum phase estimation using the cepstrum domain.

## 4 Transfer function estimation for non-stationary signals

The transfer function between the rotating component and the sensor can be estimated by combining the two mentioned-above techniques, namely estimation of the background by segments liftering in the cycle domain and minimum phase estimation. In Figure 4 an example of estimated transfer function is depicted. The magnitude was estimated using segments liftering in the cycle domain and the phase was restored using minimum phase estimation.

The estimated background has random "fluctuations" as results of high quefrencies. These fluctuations can be liftered out but however on the expense of fine estimation of sharp region of the background.

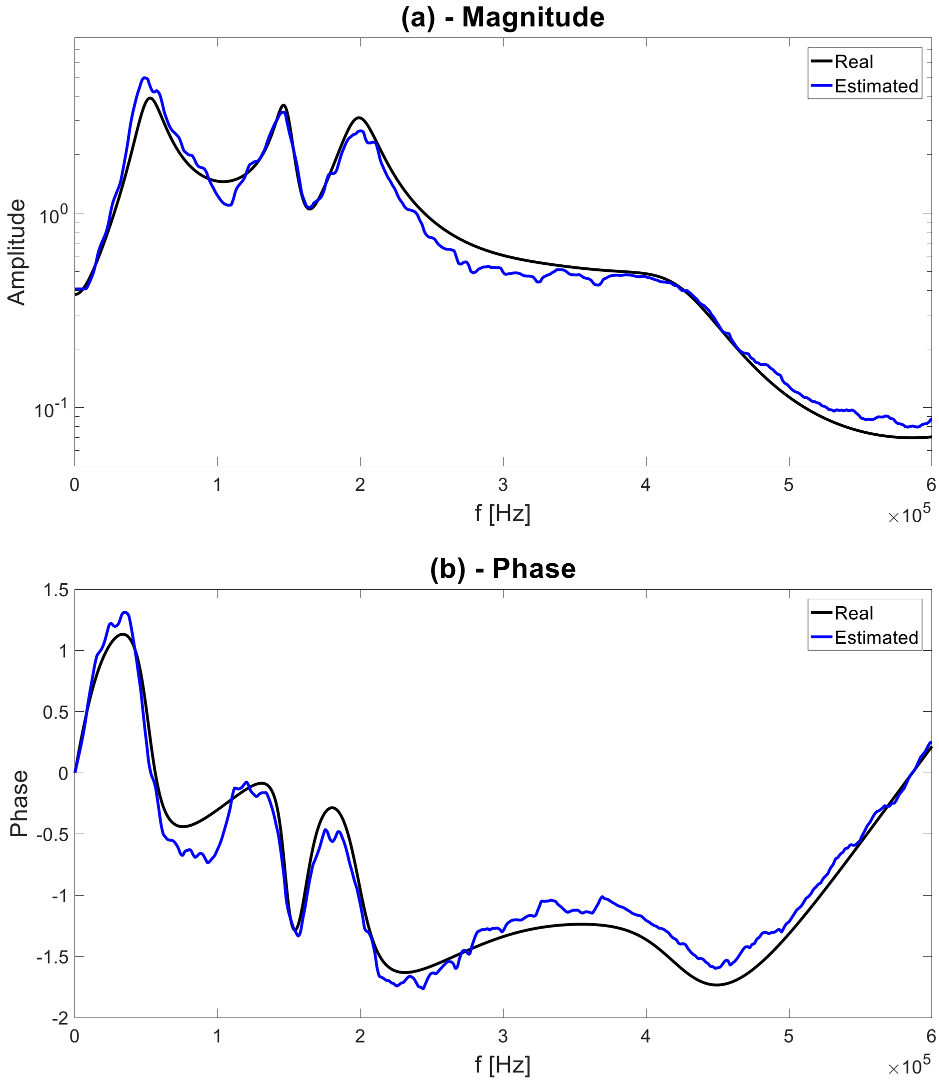


Figure 4: The estimated transfer function, magnitude (a) and phase (b) using segments liftering in the cycle domain and minimum phase estimation.

## 5 Summary and discussion

Transfer functions are greatly affecting the vibration signals of rotating components. In real systems the variations of the instantaneous speed are high and thus background-based transfer function estimation directly in the frequency domain is inaccurate. In this study we presented how to use segments liftering in the cycle domain for mitigating these problems and we showed how the estimation of the transfer function using also minimum phase enables transfer function estimation - magnitude and phase.

One of the main effects of the transfer function is the ability to transfer knowledge across machines in what is known as transfer across different machines (TDM) [6]. The transfer function disrupts important feature that facilitating fault diagnosis, and thus its estimation is important for TDM when few faulty examples are available from the target domain. This process that was also explained in [5] is important issue and need a further research.

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