

Identification of subsystems in a tilt-rotor

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Abstract

An application of subsystem selection in a tilt-rotor prior to an ATPA experimental test and a SEA study is presented. In [1] the use of the transmission paths as defined in the GTDT method [2] is proposed. Specifically, the direct transfer matrix is used for this purpose. The existence or not of weakly coupled subsystems is related to the formation of a block diagonal matrix as the powers of the direct transfer matrix increase. As the \mathbf{T}^D matrix is composed by summing the powers of the \mathbf{T}^D , i.e. summing the power of any order, when subsystems are not weakly coupled, the direct transfer matrix becomes more uniform and no contrast can be found. In the present work, these methods are applied to the identification of the possible subsystems in a real tilt-rotor cabin case.

1 Introduction

Within the PIANO project, in the framework of Horizon 2020 - Clean Sky 2 research and innovation program, a vibro-acoustic study of a tilt-rotor developed by the company Leonardo Helicopter Division (LHD) is carried out.

A general objective of this project is to improve acoustic noise comfort in the tilt-rotor by applying active attenuation which will use the information corresponding to the noise transmission paths propagating through different parts of the aircraft. Quantification of noise transmission paths is intended to be done experimentally through the application of Advanced Transfer Path Analysis (ATPA), which needs a previous subdivision of the aircraft in sub-parts (called subsystems).

In the framework of PIANO project, the set of identified subsystems are used for a double objective.

- Obtaining a model of the energy coupling between the subsystems identified through the application of a specific SEA methodology [3]; and, therefore, a numerical model.
- Applying the ATPA experimental methodology [2], with the purpose of establishing the same model that relates the identified subsystems at experimental level.

A comparison of the coupling factors obtained with both methodologies will be done in order to assess to what extent the numerical models are able to capture the physics of the problem.

The work presented in this paper only includes the automatic identification of the subsystems into which the aircraft model under study can be virtually subdivided and the intensity of the connection between them.

In the framework of SEA, subsystem identification approaches are usually based on modal density calculation and inevitably require modal analysis [4]. In the ATPA context, subsystems are mostly chosen by experience in similar problem-cases [5]. The approach presented here differs from those. The identification of the subsystems and their coupling strength is done by means of the direct transfer matrix (\mathbf{T}^D) and its powers, according to the method developed in [1].

First, some concepts related to GTDT theory – which underlies ATPA – are exposed (chapter 2). Then, a description of the method used, the associated language and its physical justification are presented in chapter 3. Then, a brief description of the tilt-rotor numerical model is shown. Finally, the results obtained are evaluated and conclusions of the work are outlined.

2 GTDT framework

The method for identifying subsystems in GTDT framework is based on the path concept. Thus, some concepts linked to the path analysis, which are the basis of ATPA method as well, are presented in this section.

Assuming that a mechanical system is described by a linear system of equations

$$\mathbf{Ax} = \mathbf{b} \quad x_i, \mathbf{b}_i, A_{ij} \in \mathbb{C} \tag{1}$$

with \mathbf{A} being a generalized stiffness (dynamic) matrix and \mathbf{b} a vector of external excitations.

The system matrix \mathbf{A} can be split into a diagonal matrix \mathbf{D} , an upper triangular \mathbf{U} and a lower triangular \mathbf{L} :

$$\mathbf{A} = \mathbf{\Lambda} + \mathbf{L} + \mathbf{U} \tag{2}$$

Then, the linear system can be rewritten as:

$$(\mathbf{\Lambda} + \mathbf{L} + \mathbf{U})\mathbf{x} = \mathbf{b} \tag{3}$$

$$\mathbf{\Lambda}(\mathbf{I} + \mathbf{\Lambda}^{-1}(\mathbf{L} + \mathbf{U}))\mathbf{x} = \mathbf{b} \tag{4}$$

$$(\mathbf{I} + \mathbf{\Lambda}^{-1}(\mathbf{L} + \mathbf{U}))\mathbf{x} = \mathbf{\Lambda}^{-1}\mathbf{b} \tag{5}$$

$$(\mathbf{I} - \mathbf{T}^D)\mathbf{x} = \mathbf{\Lambda}^{-1}\mathbf{b} \tag{6}$$

$$\mathbf{x} = \mathbf{\Lambda}^{-1}\mathbf{b} + \mathbf{T}^D\mathbf{x} \tag{7}$$

where \mathbf{T}^D is the direct transfer matrix (the transposed of the direct transfer matrix defined in [2] with zeros in the diagonal) and can be expressed as

$$\mathbf{T}^D = -\mathbf{\Lambda}^{-1}(\mathbf{L} + \mathbf{U}) \tag{8}$$

The meaning of equation (7) is shown in Figure 1 with an example, a plate with 3 nodes with an external punctual force F applied at node 1. Displacement at node 1 (x_1) can be decomposed in 3 terms: the first one is the effect of the force at node 1 (the displacement at node 1 only due to the force applied, expressed as x_F) and the other two represent the effect of the displacement of the rest of the nodes at node 1. Going back to equation (7), the first term represents, for each node of the system, the effect of external forces when the other nodes are blocked and the second term represents the effect of the displacement (or acceleration or any other magnitude) of the rest of nodes when the rest of nodes are blocked and there is no external force.

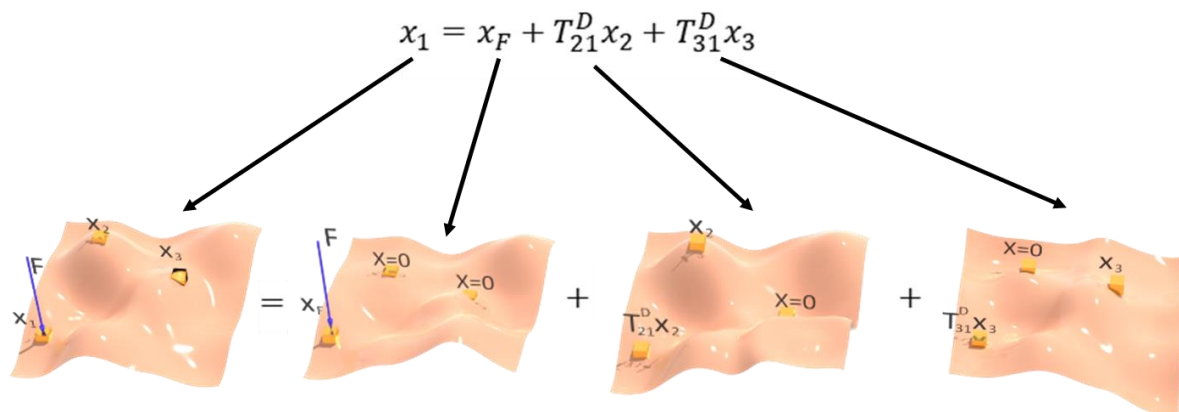


Figure 1: Decomposition of displacement at node 1 (x_1)

The way in which the \mathbf{T}^D matrix is obtained depends on the context: at experimental level, it is determined from the measured global transfer functions [2], while in a computational model it can be computed from the system matrix \mathbf{A} .

The \mathbf{T}^D matrix quantifies how the signal (acceleration, velocity or displacement) in each of the system's degrees of freedom (DOF) depends on the signal at the rest of DOFs. This corresponds to the concept that the signal at one degree of freedom can be decomposed into a part due to the motion of the other degrees of freedom and a part coming from external forces. This is made mathematically explicit in equation (8).

The commonly used approach that the motion of all the points of a system is a consequence of the application of external forces to all the points of the system is now translated into the idea that each DOF motion can be attributed to the motion of the other points of the system plus an external force contribution only into that point.

This concept is in line with Green's integral formulation, which states that the signal at any point can be obtained from the knowledge of the signal in a closed contour which contains it independently of the force that has produced that signal in the contour.

As a final remark on the \mathbf{T}^D matrix, it should be noted that it corresponds exactly to the Jacobi matrix used in the iterative method that bears its name.

When the variable used is the energy of each subsystem, as for example in SEA, the signal at the degrees of freedom and their values are replaced by the energy of each subsystem, and therefore \mathbf{T}^D is the contribution of the energy of each subsystem to other subsystems.

The \mathbf{T}^D matrix can be obtained from the fundamental dynamic equation of a mechanical system:

$$([\mathbf{K}] - \omega^2[\mathbf{M}]) \cdot \{x\} = f \quad (9)$$

$$[\mathbf{H}(\omega)] \cdot \{x\} = f \quad (10)$$

$$[\mathbf{H}(\omega)] = [\mathbf{K}] - \omega^2[\mathbf{M}] \quad (11)$$

$$[\mathbf{T}^D] = [\mathit{diag}(\mathbf{H}(\omega))]^{-1} \cdot ([\mathbf{H}(\omega)] - \mathit{diag}([\mathbf{H}(\omega)])) \quad (12)$$

3 Selection of subsystems and coupling strength

3.1 General outline

Subsystems are often defined as coinciding with the differential elements that constitute the object of study. In the context of an aircraft or other kinds of vehicle, they can be windows, doors, parts of the structure that are connected to each other, ribs, etc.

From the point of view of the paths, what seems reasonable is to group as subsystems those sets of degrees of freedom whose vibratory motion is essentially defined by themselves with a minor dependence on the degrees of freedom of the other subsystems.

The mathematical method used here to distinguish when this is true and therefore to define subsystems is the clustering of the direct transfer matrix.

As a second step, for quantifying the degree of coupling between subsystems a method based on the powers of the direct transfer matrix will be used [1].

As already mentioned, the direct transfer function from subsystem i to subsystem j indicates how the movement at i influences the movement at j along the path linking i and j , since it is the influence of i on j when the rest of subsystems are blocked.

The power of order two of this matrix quantifies all the paths of order two, that is to say, element T_{ij}^D is the sum of all the paths of order 2 which link i and j . Second order paths are defined as those that pass through

another degree of freedom before reaching j , i.e. paths of the type $T_{ik}^D T_{kj}^D$. Generalising, the n^{th} power of the direct transfer matrix is formed by the n^{th} -order paths.

The idea underlying this method is that a subsystem will be the more weakly coupled the more dependent it is on itself. If that happens, the “connection” paths are less important than the “internal paths” and, in consequence, the behaviour of the subsystem is mainly independent of the rest of the system.

After explaining how the powers of the \mathbf{T}^D matrix represent higher order paths, let us see how the solution of equation (2) depends on these paths. From equation (5), it can be immediately obtained:

$$(\mathbf{I} - \mathbf{T}^D)\mathbf{x} = \mathbf{\Lambda}^{-1}\mathbf{b} \quad (13)$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{T}^D)^{-1}\mathbf{\Lambda}^{-1}\mathbf{b} \quad (14)$$

The term $(\mathbf{I} - \mathbf{T}^D)^{-1}\mathbf{\Lambda}^{-1}$ is the global transfer matrix which corresponds in this case to a receptance and the matrix $\mathbf{\Lambda}^{-1}$ is a diagonal matrix corresponding to the displacement at each node when a unit force is applied to it.

In [6] it is shown that, for any case, it is satisfied that

$$(\mathbf{I} - \mathbf{T}^D)^{-1} = \mathbf{T}^D + \mathbf{T}^{D^2} + \dots + \mathbf{T}^{D^m} + \sum_{k=m+1}^n \gamma_k \mathbf{T}^{D^k} \quad (15)$$

for a suitably chosen parameter γ_k .

Thus, the general solution to equation (1) can be obtained by summing the powers of \mathbf{T}^D i.e., by summing the paths of any order, and the structure of the powers of the matrix \mathbf{T}^D will be reflected in the structure of the general solution.

The structure of the power matrices coincides with the structure of the eigenvectors of the system under study if it is homogeneous, otherwise they still retain a close relationship with them. This is important because it indicates that the requirement for the paths to be essentially internal to the subsystem in order to have high contrasts corresponds to the requirement that for the subsystems to be considered weakly coupled, the resonances must be local. This is a criterion normally used to identify weakly coupled subsystems.

3.2 Method

Previous works on clustering ([1],[6]) showed that, for weakly coupled subsystems, the performance of subsystem identification algorithm increased when using powers of the \mathbf{T}^D matrix.

Exponentiation of \mathbf{T}^D matrix has the underlying physical meaning of establishing higher order connections between nodes. If \mathbf{T}^{D^1} has the meaning of the direct energy connection between nodes, \mathbf{T}^{D^2} can be interpreted as the energy transfer between nodes, after the first transfer of energy has taken place, so it represents 2^{nd} order paths.

Under the hypothesis that SEA vibro-acoustic subsystems are defined by the fact that most of the energy flows inside the same subsystem, so connectivities inside the subsystem are much stronger than connectivities to DOFs out of this subsystem, exponentiation tries to underline this behaviour. It tries to do so by numerically strengthen the connectivity between elements of the same subsystems and dilute weak connectivities with DOFs belonging to other subsystems. Exponentiation of \mathbf{T}^D is performed by sequential products of matrices in a 2^n scheme, to have better control of results in each computation step. This translates to the fact that only powers of two are evaluated.

The clustering algorithm proposed relies on the fact that, if there is a weakly coupled subsystem, the magnitude of the elements of the \mathbf{T}^D matrix raised to the n^{th} power must be greater for the DOFs of the subsystem itself. Therefore, in this case a square matrix will appear centered on the diagonal with values much greater than those of the rest of rows and columns. An example with synthetic signals consisting of two totally uncoupled subsystems is shown in Figure 2.

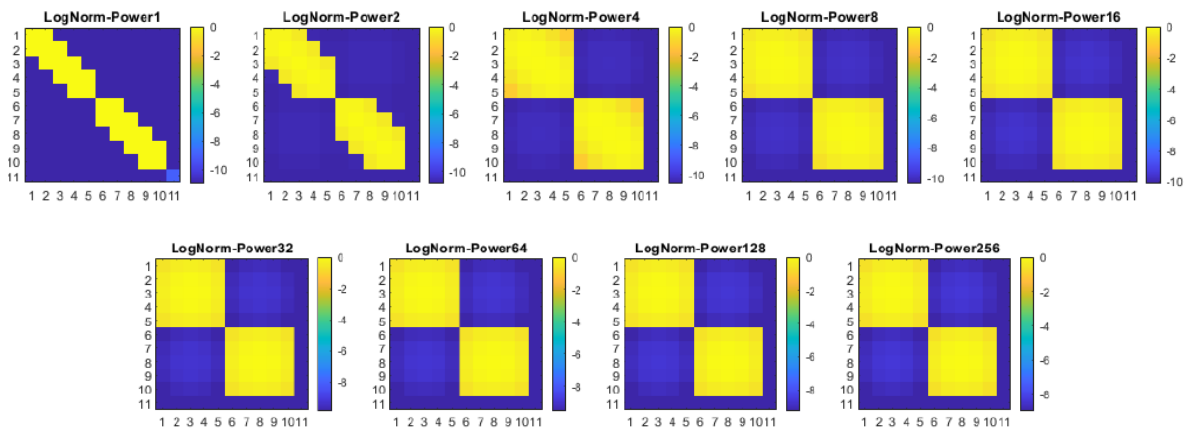


Figure 2: Behaviour of powers of synthetic \mathbf{T}^D matrix composed of two subsystems

So, the proposed method is based on assessing whether the matrix powers form visible square matrices on the diagonal or not, and quantifies the relation of their norm with their coupling matrices, which is known as contrast.

The contrast matrix norm, shows the degree of strength of the union between two subsystems, that is, it gives a continuous scale of the strength or weakness of its union to the rest of the subsystems. Moreover, its inverse value establishes an upper limit to the error assumed if the behavior of the subsystem is calculated independently from the other subsystem [1].

The contrast, as the power of the \mathbf{T}^D matrix increases, converges to a limit which depends at each frequency on its largest eigenvalue. This limit can therefore be known even without calculating the successive powers.

In a finite-element model, the \mathbf{T}^D is a sparse matrix, as it is the system matrix \mathbf{A} . As the power of the \mathbf{T}^D matrix increases, the matrix gradually fills up as the order of the paths increases and more distant paths are connected. Once the matrix is fully filled, the influence between DOFs starts to become visible. This process for a matrix of large dimensions is a costly process which can be simplified by selecting a certain number of DOFs, generating a smaller matrix than the original one. In this way, there are paths between any degree of freedom however far away it is, the matrix is filled and already takes into account the interactions between degrees of freedom.

As a final underlining, it is crucial to bear in mind that this is not a method for defining subsystems based on certain hypothesis, but a method for locating them if they exist (with data from experimental or numerical origin). The existence of subsystems does not depend on the presented method itself, but it is an underlying characteristic of the structure under study and the connectivity among its defining parts.

4 Results

4.1 Direct Transfer Matrix

4.1.1 Model

The provided FE model of the tilt-rotor was large in terms of number of elements and therefore difficult to manage regarding memory and computational resources. For this reason, two main simplifications were carried out:

- It was first simplified by using super-elements which replaced some parts of the structure, including the wings, the nose and the back cavity. The final model used for subsystem subdivision is shown in Figure 3.

- A second simplification was performed by doing a selection of elements in the model to be considered in the \mathbf{T}^D calculation. This can be seen in Figure 4.

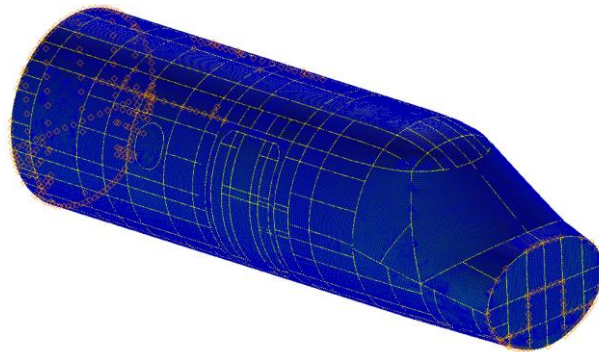


Figure 3: Numerical model used for subsystem subdivision

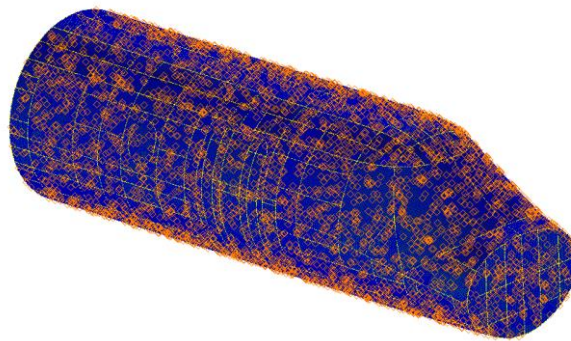


Figure 4: Selected nodes for the \mathbf{T}^D calculation

4.1.2 Results for the Direct Transfer Matrix

Results of subsystem subdivision process through the clustering of the \mathbf{T}^D matrix are shown in Figure 5.

The \mathbf{T}^D matrix unsorted (depicted on Figure 5a) does not seem to have any information about subsystems distinction but after proper clustering several blocks appear on the diagonal (Figure 5b). The correspondence between the elements of the matrix and the tilt-rotor cabin elements is shown in different colors. All the elements found within the same cluster form a subsystem.

The number of blocks selected is an input parameter of the clustering methodology and, as such, is arbitrary. However, the general structure of the system can be observed in the sorted matrix in any case, independently of the number of selected blocks.

In Figure 6 two of the found clusters have been identified as 1 and 2. Their coupling matrix, i.e. the matrix which indicates how the degrees of freedom of matrix 1 depend on those of matrix 2 is identified as 3. It can be seen that matrix 3 is represented with a very similar color to 1 and 2, which means that although 1 and 2 are identifiable as distinct subsystems their coupling is not weak, i.e. their contrast is small.

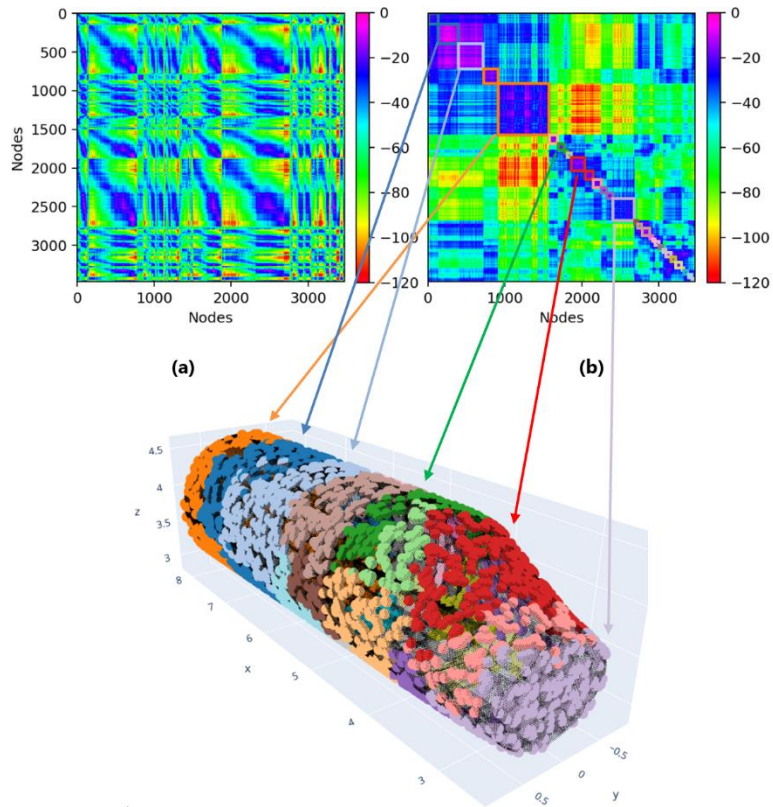


Figure 5: (a) Unsorted \mathbf{T}^D and (b) clustered (sorted) \mathbf{T}^D , with the corresponding position of the clustered elements in the tilt-rotor model.

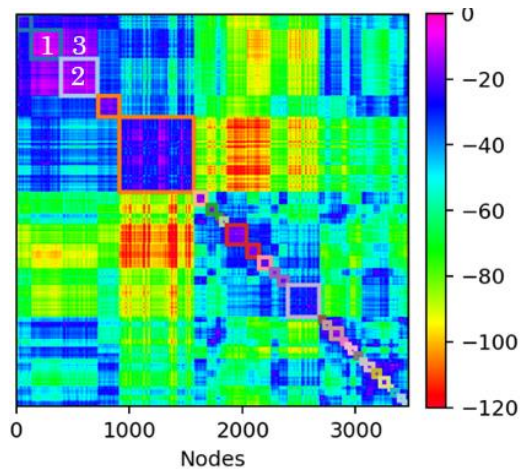


Figure 6: Sorted \mathbf{T}^D : Blocks numbered as 1 (boxed in dark green) and 2 (boxed in light grey) represent elements of subsystems 1 and 2. Block number 3 represents the coupling matrix between 1 and 2.

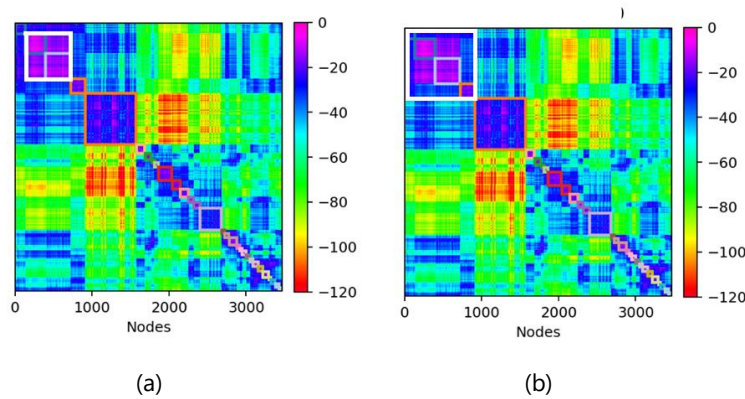


Figure 7: Alternative subsystems options

If, instead of selecting subsystems 1 and 2 as different subsystems, the union of 1 and 2 was selected as a single subsystem, as shown with the white square in Figure 7a, the resulting coupling matrix would have a higher contrast, i.e. the two resulting subsystems would be more weakly coupled. Subsystem selection in Figure 7b, shows an even more weakly coupling between subsystems when taking a wider white square.

So far, the T^D matrix has been useful in the identification of subsystems as long as it informs about groups of degrees of freedom whose movement is more dependent on themselves than on other groups of degrees of freedom. The extent to which they are dependent on other subsystems can be seen from their coupling matrix, but will be more evidenced by higher powers as it will be shown in the next section.

4.1.3 Powers of the T^D matrix

Figure 7 and Figure 8 show the evolution of the clustering of the T^D matrix raised to higher powers. It can be observed how, as the length of the paths increases, the contrast between subsystems disappears, making the matrix more uniform, unlike to what should happen in a weak coupling situation. In [1] it is shown how, for a weak-coupled system, the contrast between blocks would increase as the power of the matrix increases.

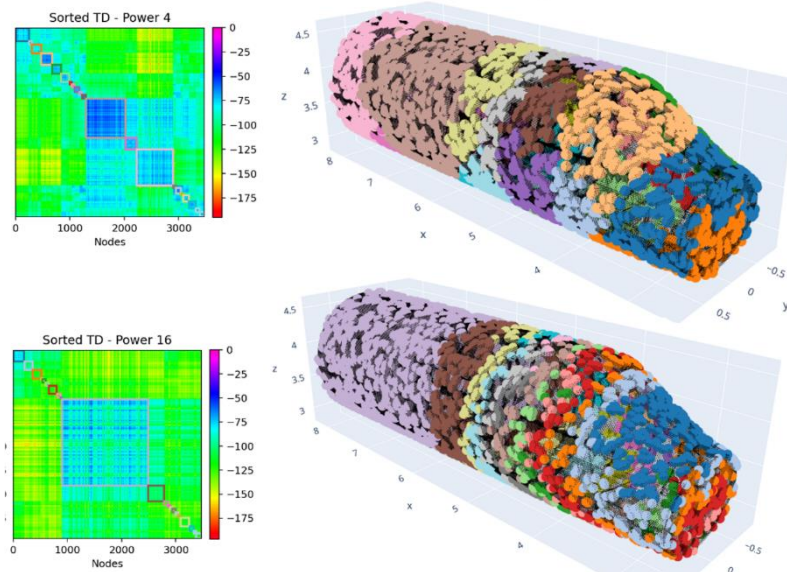


Figure 8: T^D raised to the 4th and 16th power

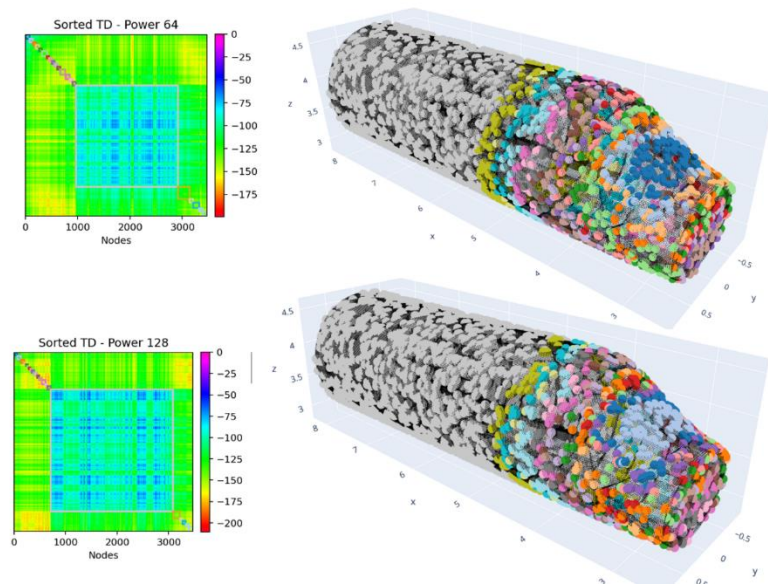


Figure 9: T^D raised to the 64th and 128th power

In Figure 9 it can be seen how, for higher order paths, subsystems become more dependent on each other, degrading subsystems identification process. It can be therefore stated that, even if subsystems are identifiable with the initial T^D matrix, they are not weakly coupled, as their coupling matrices do not have high contrast (Figure 6) because the higher order paths link them in an important way, as seen here.

Finally, subsystems in the tilt-rotor cabin have been selected based on the T^D . This selection is shown in Figure 10.

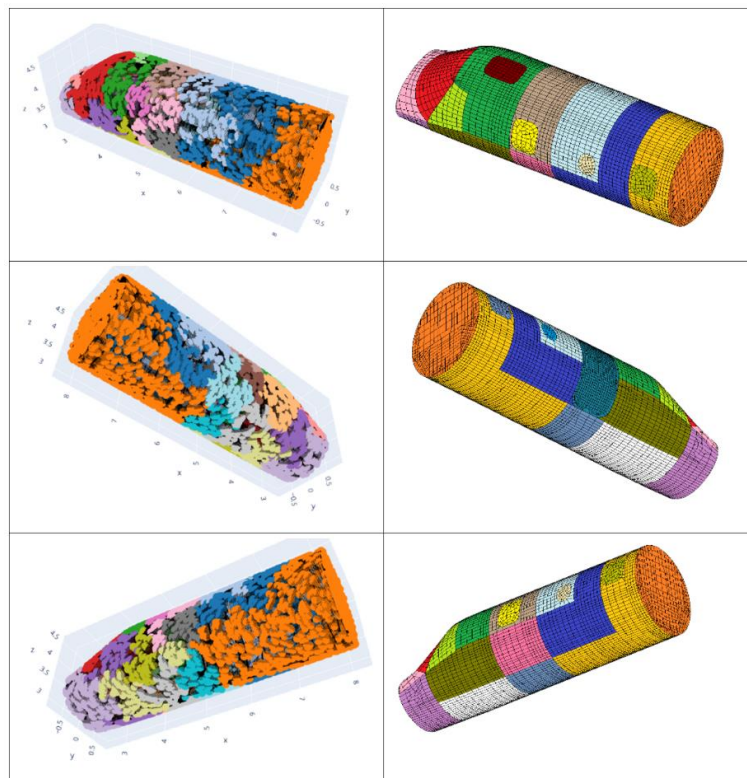


Figure 10: Final subsystem selection in the tilt-rotor cabin according to the applied methodology

5 Conclusions

In the present work, a methodology based on the direct transfer matrix (\mathbf{T}^D) has been used for identifying subsystems in a tilt-rotor cabin. The powers of the \mathbf{T}^D have been used to analyze the degree of coupling between DOFs. In the current analysis, the degree of coupling between subsystems has been identified as quite strong. As a result, SEA requirement of weakly connected subsystems is not satisfied. The powers of the \mathbf{T}^D were originally proposed in [1] as a tool to highlight weak connections in subsystems. Therefore, in a system with weakly coupled subsystems, this procedure reinforces contrast in the obtained matrix, and thus, eases up identification. However, if subsystem identification can be made through the \mathbf{T}^D matrix (meaning direct energy connection between nodes), the powers of the \mathbf{T}^D matrix result in a useful tool to analyze the degree of connection between already identified sub-blocks, corresponding to identified subsystems, which might be weakly or strongly coupled. In this way, in the framework of this investigation, the exponentiation process can be best categorized as a method for analyzing the strength of connection between already identified subsystems.

Acknowledgements

The present work has been carried out in the framework of the PIANO project (Path Identification for Active Noise Control), funded by EU Horizon 2020 and Clean Sky JU under the Grant Agreement no. 885976.

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