A simple model for the damping properties of additively manufactured particle damper

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Abstract
We propose modeling the damping behavior of a structure containing embedded particle dampers using a friction-based model, where a lumped mass, representing the trapped particles, slides within the structure and dissipates its kinetic energy due to a frictional interface. The model was studied numerically and its dynamics were analyzed using a Hilbert-transfer-based identification method. To validate the model, beam-like structures were fabricated using additive manufacturing and their dynamic behavior was studied experimentally. The experiment results were analyzed using the same Hilbert-transfer-based methods. The Hilbert-transfer-based method was used to identify the parameters of an equivalent, general single-degree-of-freedom, nonlinear model. Using these tools, the instantaneous (amplitude-dependent) modal properties of the numerical model and experimental system were identified. The identification results showed that the model and the experimental system exhibit similar behavior, which corroborates the model.

1 Introduction

Particle dampers are a form of passive vibration mitigation method, in which particle-filled enclosures are attached to a vibrating structure to dissipate its kinetic energy. These dampers have proven to be efficient, durable, and simple to implement, albeit modeling their behavior proves to be a challenge, as no agreed-upon model has been obtained yet [1].

Recent advancements in additive-manufacturing technologies, such as selective laser sintering (SLS), allow for the fabrication of structures containing embedded particle dampers [2]. In SLS, metallic powder layers are applied gradually to a build surface and selectively sintered using an energetic beam to form a solid structure. The powder that is used as a feedstock to the SLS process can be trapped in cavities within the solid structure, while the particles are free to move within the cavity. These powder-filled cavities act as embedded particle dampers: exciting the structure causes the particles to collide and rub with one another and with the cavities' walls, interchanging and dissipating the kinetic energy of the structure and reducing its overall response.

We propose modeling the dynamics of the structure containing embedded particle dampers as a two-mass system with a frictional interface between them, similar to the model for vibratin shear cell proposed in [3]. The model consists of a lumped mass, representing the trapped particles, that slides within a larger mass, representing the structure (structure mass). The friction force which opposes the motion of the masses dissipates their kinetic energy. The model is studied numerically and its dynamics are characterized using Hilbert-transfer-based identification tools.

To validate the model, beam-like structures are fabricated using SLS out of Ti_6Al_4V (Titanium) and $AlSi_{10}Mg$ (aluminum), and their dynamic behavior is studied experimentally. In most works where particle dampers were studied experimentally, linear analysis tools such as identification of a frequency response function were used. In this work, we use Hilbert-transfer-based analysis tools to identify the parameters of a general nonlinear model for embedded particle dampers and characterize the rich dynamic behavior they exhibit. These tools allow for the estimation of the instantaneous (amplitude-dependent) natural frequency and damping ratio of the system and derive physical insights from the identified parameters.

This paper is organized as follows: in this chapter, the background and motivation were laid out. In chapter 2 the theoretical model is presented. In chapter 3 the analysis methods and numerical investigation of the model are brought. In chapter 4 the experimental system and test specimen used in for validating the model are brought followed by the experimental results. In chapter 5 the experimental results are discussed and compared to the theoretical model and chapter 6 summarizes the findings and contributions of this paper

2 Friction based damping model

We propose modeling the dynamics of structures containing embedded particle dampers as a two-mass system with a Coulomb friction interface, similar to a friction damper [4]. By considering the structure's response near a single mode of vibration, the structure is modeled as a spring-mass-dashpot system, while the powder is modeled as a lumped mass, sliding within the structure mass (the mass representing the structure). When the structure mass is excited at low amplitudes, the lumped mass moves with it due to friction, in a regime called "stick". When the excitation amplitude increases, causing the inertia of the lumped mass to exceed the friction force, the lumped mass starts sliding within the structure mass, performing "slip" motion. The friction force produced by the relative motion between the masses opposes their motion and dissipates their kinetic energy. We assume the lumped mass does not travel significantly, hence it does not reach a wall and no impact occurs during vibration. A schematic description of the friction damper is brought in Figure 1. The model's equation of motion take the form:

$$M\ddot{q} + C\dot{q} + Kq = Q(t) + f \tag{1}$$

Where q is a vector of the system's degrees of freedom, M, C, K are the mass, damping and stiffness matrices respectively, $Q(t)$ is the external excitation force and f is the Coulomb friction force acting between the masses. The friction force is calcualted by

$$f = \begin{cases} \lambda & \textit{during stick} \\ \mu m_2 g & \textit{during slip} \end{cases} \tag{2}$$

Where λ is a Lagrange multiplier associated with a constraint enforcing rigid body motion of the two masses, i.e. $\dot{x}_1 = \dot{x}_2$. The derivation of the model's equations of motion is brought in the appendix.

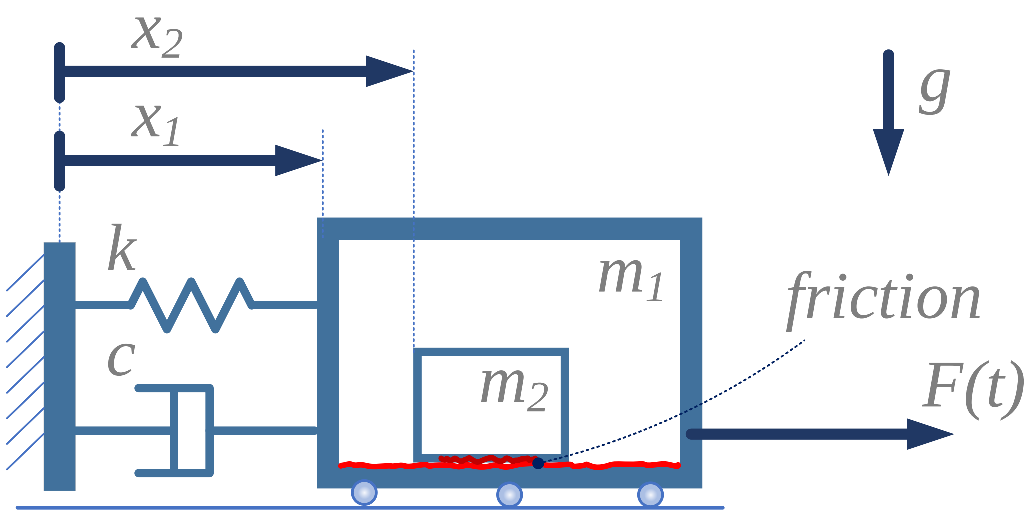


Figure 1: Schematic description of the two-mass model for a structure with an embedded particle damper

3 Theoretical model investigation

We use numerical integration to simulate and study the behavior of the two-mass system. Due to the nonlinear nature of the system, we seek to find a comprehensive description of the system using analytic signal theory.

3.1 Hilbert transform and the FORCEVIB method

Analytic signal theory [5] states that many processes, including the response of vibrating systems, can be represented as a combination of slowly varying functions, envelope $A(t)$ and phase $\psi(t)$, as:

$$y(t) = A(t)\cos(\psi(t)). \tag{3}$$

The instantaneous frequency of the signal is defined as the time derivative of the phase:

$$\omega(t) = \dot{\psi}(t) \tag{4}$$

In the FORCEVIB method [6], Hilbert transform and analytic signal representation are used to identify the modal parmeters of a system whose forced response is modeled as a single degree of freedom (SDOF) system with the following equation of motion:

$$\ddot{y}(t) + 2\omega_n(A)\zeta(A)\dot{y}(t) + \omega_n^2(A)y(t) = \frac{x(t)}{m} \tag{5}$$

Where $\zeta(A)$ and $\omega_n(A)$ are the instantaneous damping characteristic and natural frequency of the equivalent SDOF model, x is the excitation signal and m is the modal mass of the equivalent SDOF model. It can be shown that the instantaneous parameters can be expressed in terms of the components of the analytic representation of the input $X(t)$ and the response $Y(t)$ as:

$$\begin{aligned} \omega_n^2(t) &= \omega^2 + \frac{\mathrm{Re}\left[X(t)/Y(t)\right]}{m} - \frac{\mathrm{Im}\left[X(t)/Y(t)\right]\dot{A}}{A\omega m} - \frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} + \frac{\dot{A}\dot{\omega}}{A\omega}, \\ h_0(t) &= \frac{\mathrm{Im}\left[X(t)/Y(t)\right]}{2\omega m} - \frac{\dot{A}}{A} - \frac{\dot{\omega}}{2\omega} \end{aligned} \tag{6}$$

The analytic signals, the instantaneous envelope and frequency, and their derivatives can be obtained directly from experimental measurements of the excitation force and the response of a test structure. The modal mass, however, needs to be estimated in some other way. By assuming the instantaneous natural frequency varies only slightly during a short period of time $\hat{t}$, it is possible to utilize an optimization method to obtain the mass value for which the estimated values of ω_n vary the least during all time periods $\hat{t}$. This method requires that the instantaneous frequency of the excitation signal would vary in time, so a swept sine is usually a favorable excitation signal. After estimating the mass value and the instantaneous natural frequency and damping ratio, the instantaneous spring and dashpot coefficients can be calculated as well and be used for estimating the internal elastic and damping forces of the system.

3.2 Numerical investigation

We use a constant-frequency and amplitude modulated (AM) signal to excite the structure mass and by using the FORCEVIB method we investigate how the instantaneous natural frequency and damping ratio of the system vary with the response amplitude. Different excitation frequencies are used in the investigation to study the effects of both excitation frequency and amplitude on the system's instantaneous parameters. Since for low amplitudes the two masses move as a rigid body the system behaves linearly, we chose excitation frequencies in the vicinity of the natural frequency of the system's linear regime. The investigation results are brought in Figure 2, where each line represents the analysis of the response to a single-frequency AM signal, and the transition of the lines' colors from blue to red corresponds to increasing excitation frequency. In graphs (a) and (b), the instantaneous natural frequency and damping ratio (respectively) are shown as a function of the acceleration response amplitude. Graphs (c) and (d) show the characteristic curves of the equivalent spring and damper, respectively. As can be seen, the system's behavior is indeed linear up to a certain acceleration amplitude. This acceleration amplitude corresponds to the inertial force exceeding the friction force and is equal to

$$\ddot{x}_2 = \mu g \, . \tag{7}$$

Above this acceleration amplitude, slipping starts to take place and the system's behavior varies. We can see in Figure 2 (a) that the natural frequency of the system increases and approaches a limiting value. The increase in the natural frequency is attributed to the lumped mass momentarily detaching from the structure mass, causing the system's average mass in a period to decrease. With the increase in excitation amplitude, the lumped mass spends more time slipping rather than sticking to the structure mass. This continues until barely any rigid body motion is exhibited, causing the system to reach a limiting natural frequency value. The increase in the instantaneous damping ratio, seen in Figure 2 (b) is also attributed to the onset of slip – when the masses undergo relative motion, the friction force dissipates their kinetic energy. With the increase in excitation amplitude, the relative duration of slip increases, hence more energy is dissipated on average in a period. However, we can see that with the increase in response amplitude, the damping ratio reaches a maximum and then decreases back to its original value. Since the magnitude of the friction force is constant, while the damping force of the dashpot-like component is proportional to the velocity amplitude, for very large response amplitudes the influence of the friction force on the overall dissipation becomes negligible.

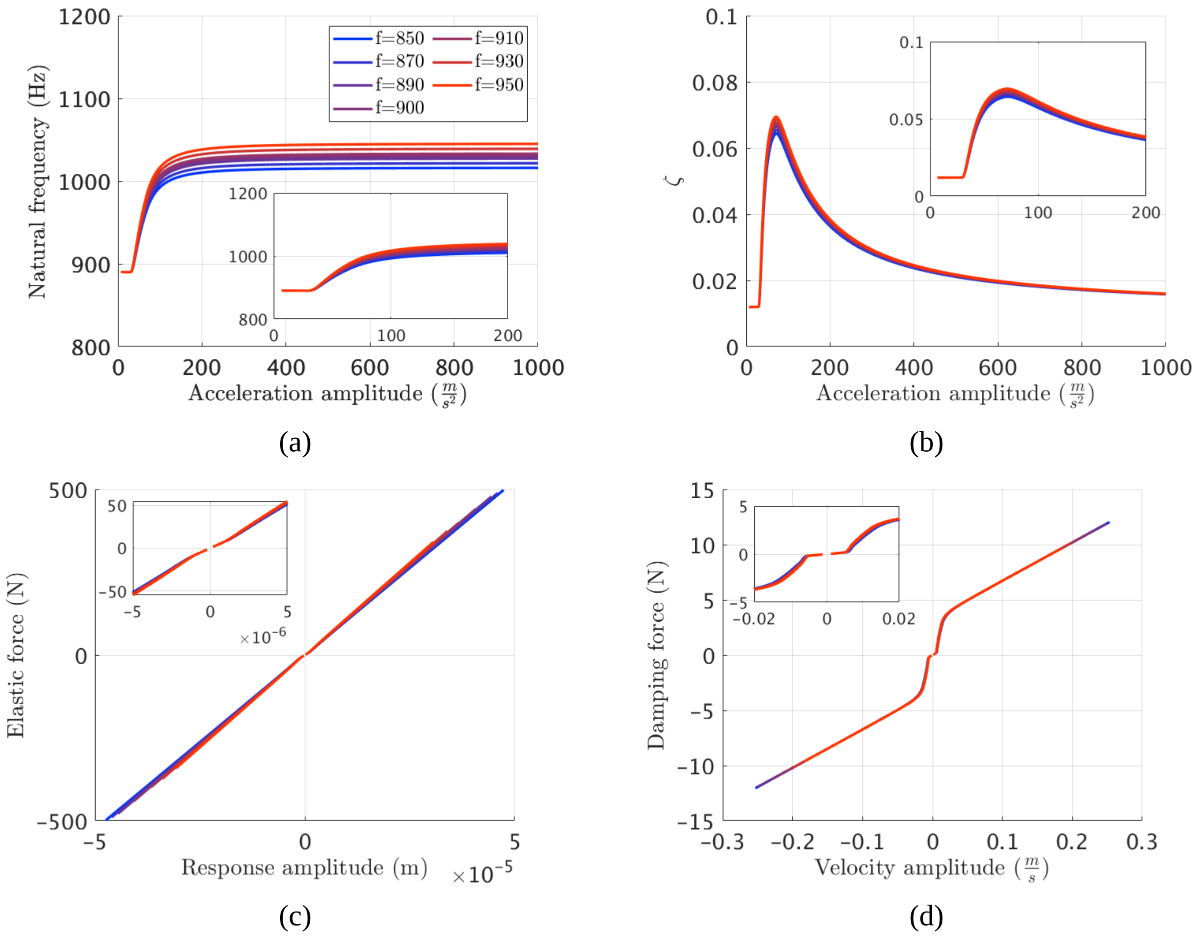


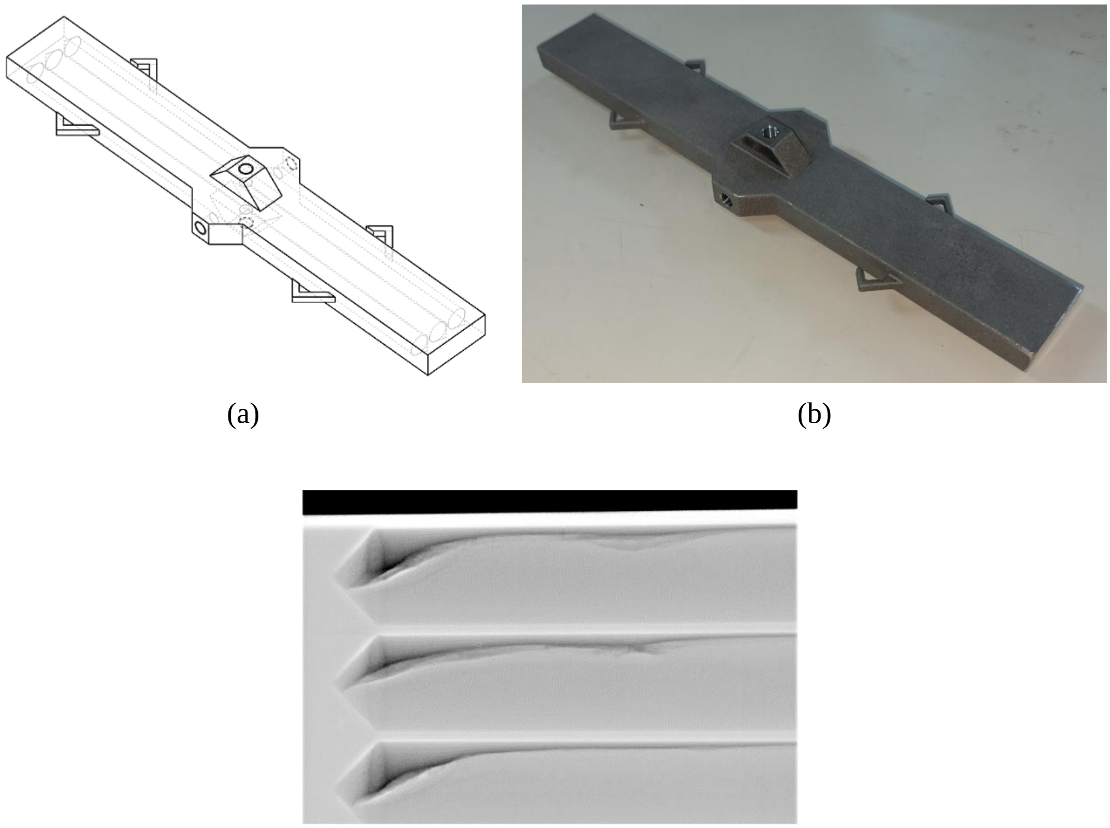
Figure 2: Numerical investigation results of the two-mass model: (a) instantaneous frequency, (b) instantaneous damping ratio, (c) elastic force curve, and (d) damping force curve. Each line is obtained from the response to AM excitation. Lines' colors transition from blue to red, corresponding to increasing excitation frequency.

4 Experimental model validation

Previous works studying particle dampers have reported similar phenomena to those exhibited by the theoretical model – an increase in natural frequency and a similar qualitative variation in the damping ratio [7]–[9]. To validate our model, we fabricated a test structure with embedded particle dampers and used the same excitation signals and analysis tools to identify the dependency of the structure's instantaneous parameters on its response amplitude.

4.1 Experimental setup

We used selective laser sintering to fabricate a beam with 3 longitudinal cavities filled with powder, inspired by [10]. Two geometrically identical beams were fabricated, out of Titanium (Ti_6Al_4V) and Aluminum ($AlSi_{10}Mg$), where the cavities take up about 50% of the structure's volume. In Figure 3 (a) the CAD model of the investigated structure is shown, along with a picture of the Titanium beam (b) and an X-ray scan (c), showing the loose powder within the cavities (dark areas correspond to lower density).



(a) (b)

(c)

Figure 3: (a) CAD model of a beam with embedded particle dampers (b) picture of Titanium beam (c) X-ray scan of Titanium beam.

The structures containing particle-filled enclosures described above were excited and their vibrations were measured to study their damping properties. The freely suspended (on elastomer supports) beams were excited using an electrodynamic shaker (Labworks' MT-163-2 Modal Thruster) via a stinger connected to the shaker on one side, and a force sensor on the other. The force sensor (Endevco model 2311 Isotron force sensor), which measures the force input into the beam, is connected between the stinger and the beam. The shaker's input voltage is produced by a signal generator (Agilent 33522a) and amplified using a power amplifier (Labworks PA-151 Linear power amplifier). The signal generator is controlled using a PC. The beam's response is measured using Polytec™ single-point laser vibrometer model HSV-100. The signal generator's output, the force sensor, and the laser vibrometer's measurements were all sampled using an analog-to-digital converter (data translation DT9873C) and recorded to the PC through a MATLAB™ program. A schematic drawing of the system is shown in Figure 4 and a photo of the setup is shown in Figure 5.

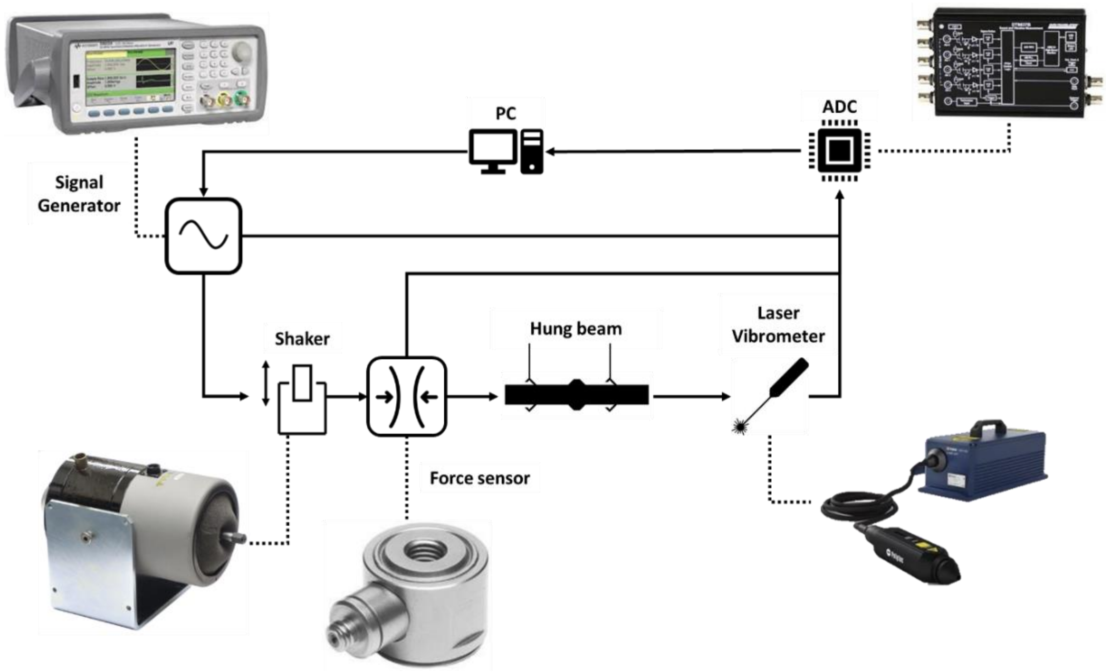


Figure 4: Schematic drawing of the experimental setup

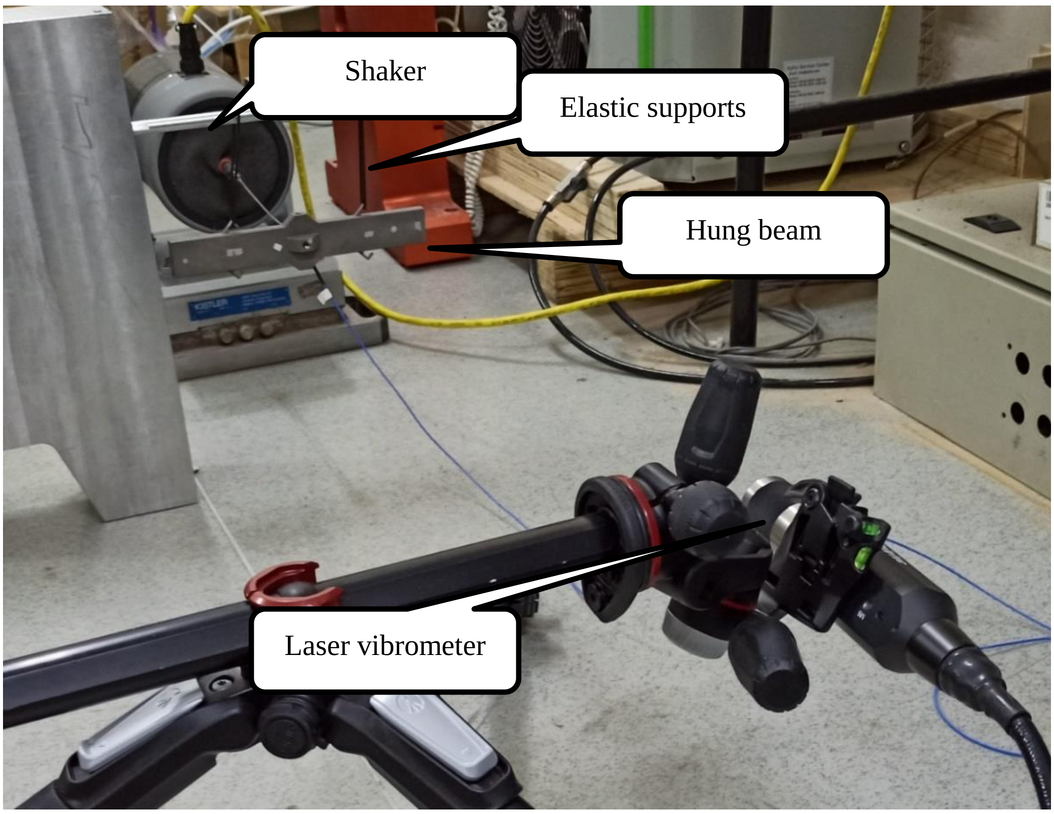


Figure 5: Picture of the experimental system

4.2 Analysis method

In practical situations, the control signal, provided to the exciter, and the excitation force, exciting the test structure, are not identical. Usually, the excitation amplitude will depend on the excitation frequency due to the test structure's dynamics affecting the exciter's performance. To separate the effects of the excitation frequency from the effects of the excitation amplitude on the test structure's dynamics without employing a control loop, we used a constant frequency signal with a slowly modulated amplitude. However, as mentioned before, excitation frequency variation is essential for modal mass estimation in the FORCEVIB method, so the following methodology was used for the complete investigation of a test structure:

a. Excite the test structure using a frequency varying signal (such as swept sine) near the natural frequency of interest.
b. Use the FORCEVIB method and the optimization scheme to estimate the structure's modal mass.
c. Excite the test structure using amplitude-modulated signals of varying frequencies.
d. Use the FORCEVIB method and the modal mass value from step b to estimate the system's instantaneous damping characteristic and natural frequency
e. Plot the estimation results – instantaneous parameters as functions of the response amplitude and excitation frequency.

4.3 Experimental results

The estimated instantaneous characteristics are shown in Figure 6 for the Titanium (left) and aluminum (right) beams. In each experiment, an amplitude modulated signal with constant frequency was fed into the exciter. The dependence between the instantaneous parameters and the acceleration response amplitude was obtained using the FORCEVIB method and is shown for various experiments at various frequencies. Each line color represents a single experiment, and the variation in the line's colors from blue to red corresponds to increasing excitation frequency (blue – low frequency, red – high frequency).

For low acceleration amplitudes, the beams exhibit linear behavior: the instantaneous parameters are constant and do not vary with the response amplitude. However, above a certain acceleration threshold, the instantaneous parameters start to vary: the natural frequency increases with the increase in amplitude and tends to a slightly higher value. The damping ratio increases with the increasing response amplitude, then it reaches a peak value and then decreases back, approaching its original value. The instantaneous parameters are shown against the acceleration response amplitude to emphasize that for all excitation frequencies, the transition from the linear to the nonlinear behavior occurs at the same acceleration level, indicating that an inertial threshold is exceeded, causing the variation in the system's behavior.

In Figure 7, the internal elastic and damping forces are shown against the response amplitude and the velocity amplitude respectively. These graphs can be thought of as the characteristic graphs of the nonlinear spring and damper in the SDOF model. The same linear behavior is observed for low response amplitudes, and a variation in the spring and damper characteristics that corresponds to the variation in the instantaneous parameters is seen. All of these variaition patterns are very similar to those exhibited by the modal parameters of the two-mass model depicted in Figure 2.

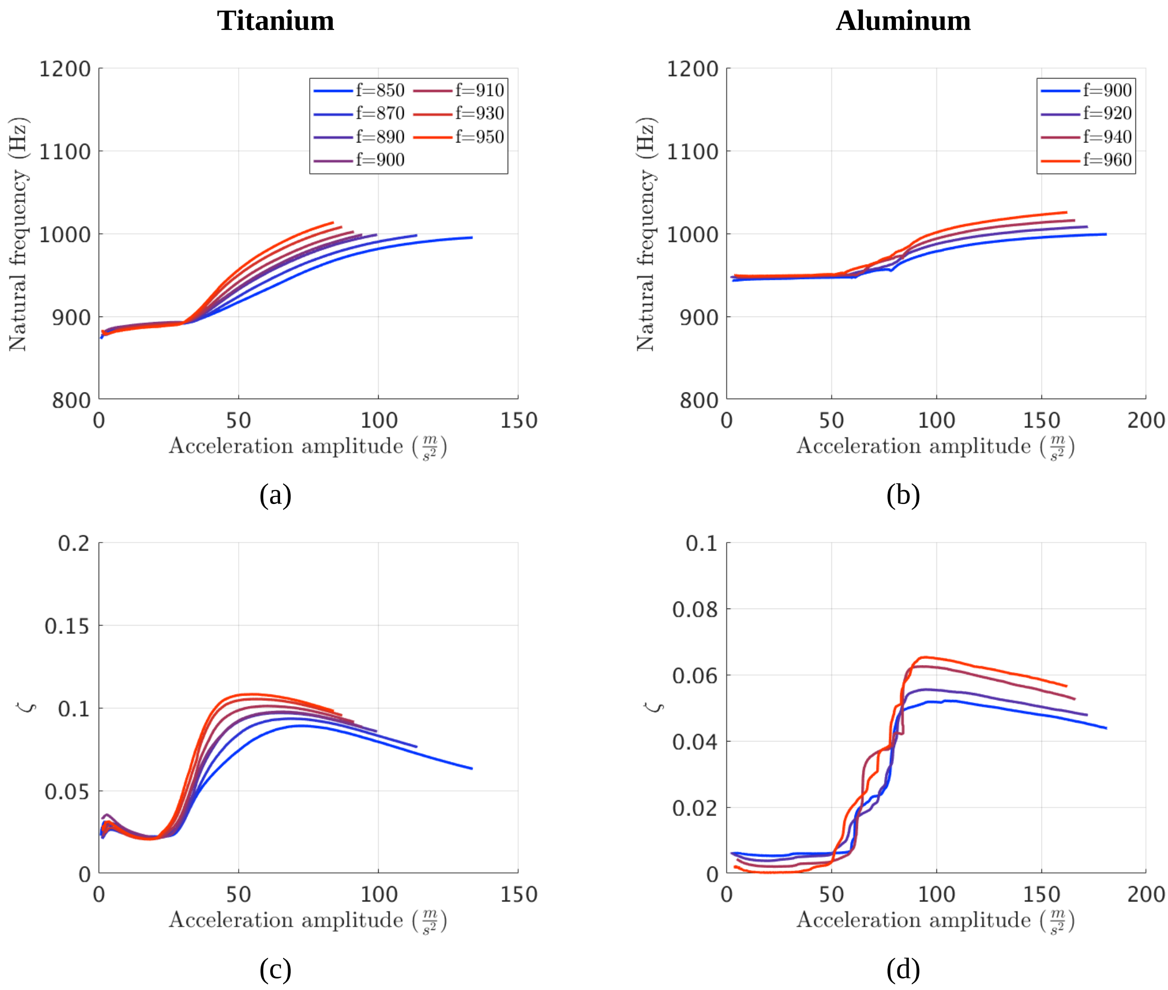


Figure 6: Experimental estimation of instantaneous natural frequency (a+b) and damping ratio (c+d) for Titanium (left) and aluminum (right) beams. The lines' colors transition from blue to red, corresponding to increasing excitation frequency.

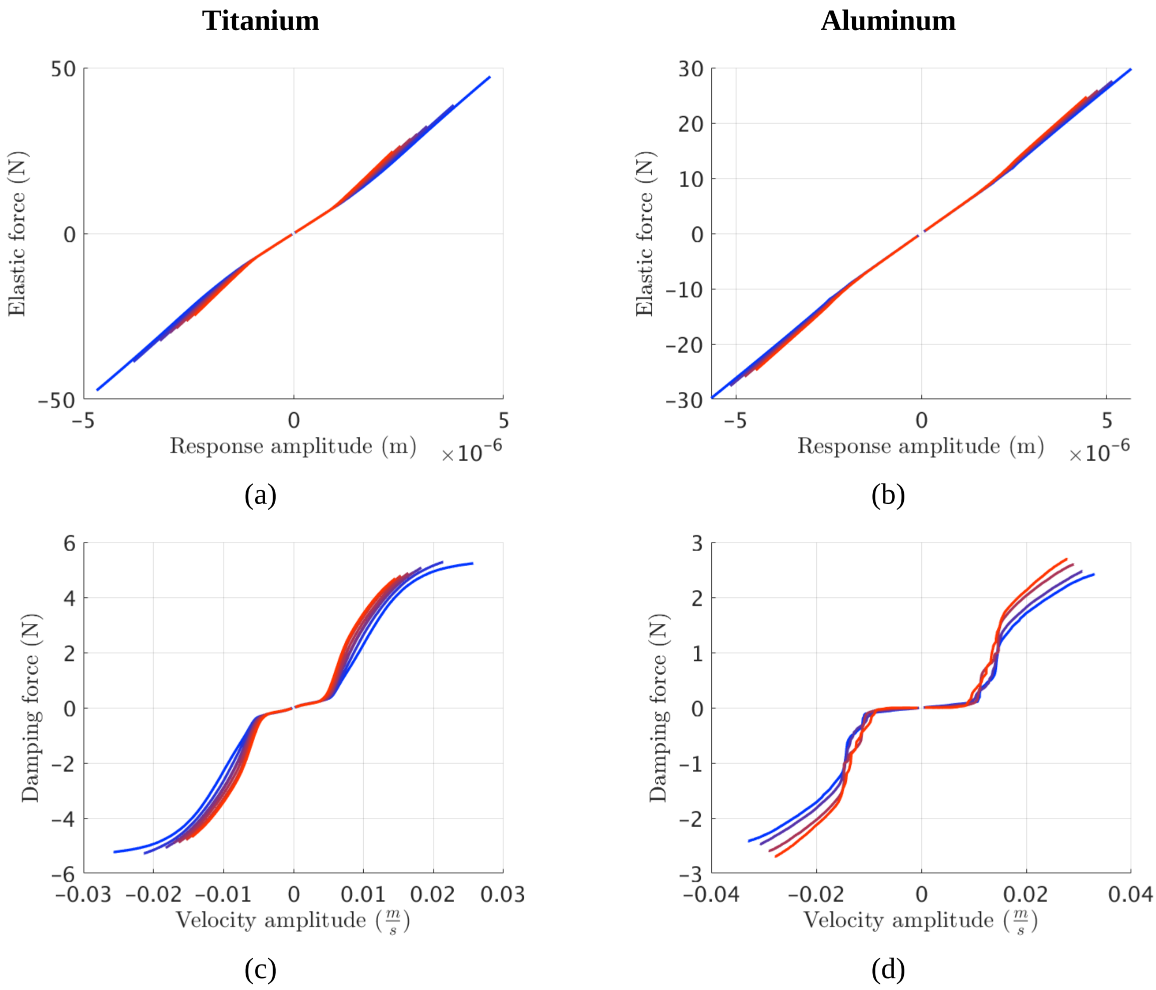


Figure 7: Experimental estimation of internal elastic force (a+b) and internal damping force (c+d) for Titanium (left) and aluminum (right) beams. The lines' colors transition from blue to red, corresponding to increasing excitation frequency.

5 Discussion

It was shown that the particle dampers' behavior transitions from a linear regime to a non-linear regime when an inertial threshold is exceeded. The nonlinear regime is characterized by an increase in the natural frequency and a variation of the damping characteristic with the response amplitude that obtains a maximal value. This behavior can be explained by considering the inertial forces acting on the powder. Below a certain vibration amplitude, the internal friction between the particles is dominant and the particles move mostly as a rigid body attached to the main structure. When the inertial forces exceed a certain energy barrier, the particles start moving more freely in their cavity. This transition from a "solid" phase to a "liquid", or, "gaseous" state is well documented in various granular systems [11]. During the particles' free motion, the effective mass of the test structure is reduced, due to the particles' motion being independent of the solid structures' motion. This decrease in effective mass corresponds to the increase in natural frequency, and the vigorous motion of the particles accounts for the increase (and decrease) in the damping ratio. All of these phenomena are exhibited by the two-mass model, which justifies modeling the powder as a lumped mass sliding within the structure with dry friction interface. These similarities show that by calibrating the two-mass model to a specific structure with embedded particle dampers, it can predict the structure's dynamic response to prescribed excitation near a single mode of vibration.

6 Summary and conclusions

In this paper, we presented a simple two-mass model for the dynamic behavior of structures containing embedded particle dampers. The proposed model exhibited both linear and nonlinear dynamics regimes, depending on its response amplitude. Due to the nonlinear behavior, we adopted Hilbert-transform-based analysis tools for the characterization of the model's behavior. We followed the numerical investigation of the model by an experimental study of structures fabricated using additive manufacturing. The experimental results were analyzed using the same analysis tools to estimate the instantaneous model parameters of the structures. The investigated structures exhibited the same dynamic phenomena as the model: The structure exhibited linear behavior up to a certain vibration acceleration threshold. Above the threshold, the particles inside the structure started moving vigorously, causing the structure's natural frequency to increase with increasing vibration amplitude and the damping ratio to increase and then decrease. The similarities drawn between the model and the experimental system show that the model can be used for modeling the response of a structure containing an embedded particle damper by calibrating the model for the specific damper. While in this work we calibrated the model based on experiments, relating the model's parameters to the damper's properties (such as particle size, cavity geometry, filling ratio, etc.) is still a challenge, which is left for future work.

Acknowledgments

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Appendix

A Derivation of the model's equations of motion

The equations of motion of the two-mass system described in section 2 and depicted in Figure 1 are formulated using the constrained Lagrange method. The vector of degrees-of-freedom:

$$q = \begin{pmatrix} x_1 & x_2 \end{pmatrix}^T . \tag{A1}$$

The sums of kinetic energies T, potential energies V, and the dissipation function D are:

$$T = \frac{1}{2} m_1 x_1^2 + \frac{1}{2} m_2 x_2{}^2, \qquad V = \frac{1}{2} k x_1^2, \qquad D = \frac{1}{2} c \dot{x}_1^2 . \tag{A2}$$

To calculate the friction force, a kinematic constraint enforcing rigid-body motion is formulated, setting the masses' velocities to be equal. The constraint is expressed using the holonomic matrix $W = \begin{pmatrix} 1 & -1 \end{pmatrix}$ as:

$$W\dot{q} = 0 . \tag{A3}$$

Substituting the above equations into the constrained Lagrange equation, we obtain:

$$\frac{d}{dt}\left(\frac{dT}{d\dot{q}_i}\right) - \frac{dT}{dq_i} + \frac{dD}{d\dot{q}_i} + \frac{dV}{dq_i} = Q_i + \sum_{j=1}^{n_{constraints}} w_{ij}\left(q\right)\lambda_j \tag{A4}$$

from which the system's constrained equations of motion are obtained:

$$\underbrace{\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}}_{M} \ddot{q} + \underbrace{\begin{pmatrix} c & 0 \\ 0 & 0 \end{pmatrix}}_{C} \dot{q} + \underbrace{\begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}}_{K} q = \underbrace{\begin{pmatrix} F(t) \\ 0 \end{pmatrix}}_{Q} + \underbrace{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}_{W^T} \lambda \tag{A5}$$

the Lagrange multiplier λ represents the friction force. Since the time-derivative of the constraint equation (A3) is zero, it can be used to calculate the friction force as a function of the system state:

$$\frac{d}{dt}\left(W\dot{q}\right) = 0 \rightarrow \dot{W}\dot{q} + W\ddot{q} = 0 \tag{A6}$$

Substituting (A5):

$$WM^{-1}\left(W^T\lambda + Q - C\dot{q} - Kq\right) = 0 \tag{A7}$$

Solving for the constraint force λ:

$$\lambda = \left(WM^{-1}W^T\right)^{-1}\left(WM^{-1}\left(C\dot{q} + Kq - Q\right)\right). \tag{A8}$$

To ensure the friction force between the masses is below its limiting value of $f \le \mu N$, and to allow the transition from rigid-body to sliding motion, an "IF" condition is used in the numerical integration:

$$\begin{matrix} if & \lambda > \mu N \; or \; \left|\dot{x}_2 - \dot{x}_1\right| > tol & f = \mu N \operatorname{sgn}\left(\dot{x}_2 - \dot{x}_1\right) \\ else & & f = \lambda \end{matrix} . \tag{A9}$$

The expression $\left|\dot{x}_2 - \dot{x}_1\right| > tol$ is used to avoid a transition from sliding to rigid-body motion when the velocities of both masses are different, but equation (A8) yields a value lower than μN. tol is some numerical tolerance.

The two-mass system is simulated by numerical integration of:

$$M\ddot{q} + C\ddot{q} + Kq = Q + W^T f \tag{A10}$$

And using equations (A8) and (A9) to calculate the friction force.