Methods for test-validation of flow induced vibrations

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Abstract
Highly sensitive components, e.g., in highly precise systems, are often developed considering dynamical behavior. One of the sources of possible disturbances in actively cooled systems are flow induced vibrations (FIV). Therefore, FIV levels have to be specified for the systems and subsystems. In very early development stages, when the design is not complete, specifications must be defined in some abstract but meaningful way, e.g., with respect to the center of gravity (CoG) of the subcomponent in question. These early definitions can later lead to practical difficulties during the design verification stage, when real measurements must be performed to verify that FIV disturbances are within specification. This paper describes three methods to evaluate FIV disturbance forces using acceleration measurements performed at some point other than the CoG that resolve the above-mentioned difficulties.

1 Introduction

For many high precision machines that require active cooling flow induced vibrations of the coolant are a significant disturbance source. Maximum allowed disturbance forces for the complete system and its submodules have to be specified based on top-down budgeting at project start and later validated by tests.

Early on in a project, when only few detailed design information is available, such force specifications can often only be defined as acting on the center-of-gravity of individual submodules. During subsequent detailed development, these specification points can end up being unreachable for later validation purposes.

Several frequency response function (FRF) based approaches can then be applied to still be able to perform meaningful validation tests. This contribution describes three such methods in a case study. All three are based on the same fundamental test setup: a free-free submodule with coolant supply and accelerometers attached to points that are not the center-of-gravity.

1) “Mass Scaling”: In some cases, if the geometry of the subsystem is stiff with respect to the frequency range of interest, and if the point of interest lies at the center of gravity, acceleration measurements can be multiplied by the subsystem mass. This method is limited through the prerequisite of stiffness - only the frequency range between decoupling frequencies (rigid body modes) and first flexible mode of a substructure of interest can be evaluated. However often the frequency range that includes some of the first flexible modes of a substructure is of interest and is specified. Thus, this simplified method cannot always be applied and has to be used with care.
2) “Measurement Conversion”: More often than not, the subsystem cannot be simplified in such a manner as described above due to the complexity and high number of eigenfrequencies in the bandwidth of interest. For these cases we propose in this paper a hybrid method, which is based on FEM calculation of frequency response function combined with acceleration measurements at 6 degrees of freedom. To estimate force using accelerations and FRFs an "inverse" matrix multiplication is used. The resulting forces can be checked with the specifications for the subsystem.
3) “Specification Conversion”: An alternative approach described in this contribution is specification inversion. In this method direct matrix multiplication is used to convert force specifications at the center of gravity into acceleration specifications at given measurements points. These can then be directly validated with tests. While this method gives the quickest feedback concerning test

pass/fail, namely immediately after test completion, drawing conclusions for design improvements is less obvious in case of test failure.

The described methods allow for FIV validation even under sub-optimal specification conditions inherited from early project decisions.

1 Mathematical background

1.1 Mass scaling

The most common approach to evaluate FIV forces on CoG is mass scaling. For that the sensors during the verification measurement must either be positioned near the CoG or the system can be simplified for the frequency range of interest as absolutely stiff. It is also possible to verify if the system is really stiff in a given frequency band – for this, an eigenfrequency measurement and/or a modal analysis can be performed to verify that there are no resonances in the range.

This stiffness assumption and/or verification allows scaling of the acceleration measurements with a mass of the system and calculate forces this way. For PSDs the acceleration PSD must be scaled with the squared mass.

To calculate PSD of torque using acceleration PSD one would need to take into account the moment of inertia and use the formula (2):

$$\tau = I \cdot \ddot{\varphi} \tag{1}$$

where *I* – moment of inertia, $\ddot{\varphi}$ – angular acceleration.

Time variant angular acceleration is defined as:

$$\ddot{\varphi} = \frac{d}{dt}\left(\frac{v_\perp}{r}\right) = \frac{1}{r}\frac{dv_\perp}{dt} - \frac{v_\perp}{r^2}\frac{dr}{dt} \tag{2}$$

where $v_\perp$ - cross-radial component of the instantaneous velocity, *r* – distance from the origin.

In the special case where the point undergoes circular motion about the origin, *dv/dt* becomes just the tangential acceleration $a_\perp$, and *dr/dt* vanishes (since the distance from the origin stays constant), so the above equation simplifies to

$$\ddot{\varphi} = \frac{a_\perp}{r} \tag{3}$$

with that torque would be:

$$\tau = \frac{I}{r} \cdot a_\perp \tag{4}$$

For PSDs, then all terms must be squared:

$$PSD_{torque} = \frac{I^2}{r^2} \cdot PSD_{acc} \tag{5}$$

A common way to specify FIV disturbances is the cumulated 3σ value than can be calculated as follows:

for forces

$$3\sigma_{force} = 3 \cdot \sqrt{\int_{f_1}^{f_2} PSD_{force} df} \tag{6}$$

for torques

$$3\sigma_{torque} = 3 \cdot \sqrt{\int_{f_1}^{f_2} PSD_{torque} df} \tag{7}$$

where f_1 – the smallest frequency, due to measurement system and/or decoupling of the structure, f_2 – the highest frequency which is limited through the first flexible mode of the structure – at this point the structure cannot be considered stiff.

This method provides fast estimation of the forces and torques. Nevertheless, it is limited through the initial assumption. If the system is somewhat more complex and has eigenfrequencies in the frequency range of interest the method can still be used, but the results can be used only to roughly estimate the possible forces. Any comparison to the specifications would have an elevated level of uncertainty.

1.2 Measurement conversion

Prerequisite for this method are six measured acceleration signals, that are positioned in such a way that all six degrees of freedom at point 1 can be derived (Figure 1). In the example Y2 signal could be used to derive rotational degree of freedom (DoF) Rz at point 1, Z2 for Ry at point 1, Z3 for Rx at point 1.

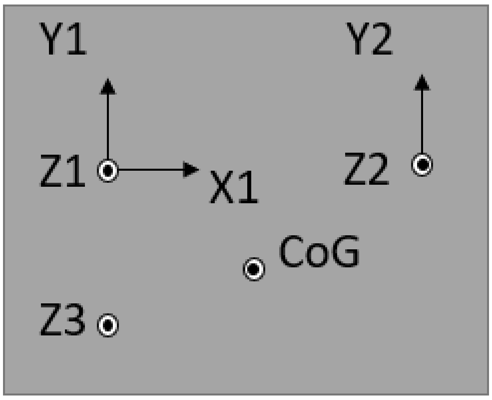

Figure 1: Accelerators positions example

Measured acceleration signals are correlated to forces on defined points, e.g., CoG through the transfer function as follows:

$$a_j = H_{ji} f_i \tag{8}$$

where a_j – acceleration (complex in frequency domain), H_{ji} – transfer function, f_i – force (complex in frequency domain), all variables are complex values.

The force and acceleration PSDs (cross-spectra) at *n* force DoFs (e.g., X, Y, Z, Rx, Ry, Rz at CoG) and m acceleration DoFs (e.g., Y1, Y1, Z1, Y2, Z2, Z3 at sensor points) can be summarized as follows:

$$PSD[force]_{i_1 i_2} = f_{i_1 i_2},\ i_1 = 1,..n,\ i_2 = 1,..n, \tag{9}$$

$$PSD[acceleration]_{i_1 i_2} = a_{j_1 j_2},\ j_1 = 1,..m,\ j_2 = 1,..m, \tag{10}$$

where i_n – force/torque DoFs (sensor signals), j_m – acceleration DoFs (output force e.g., at CoG)

The matrices of acceleration and force PSDs for n = 6 outputs(acceleration) and m = 6 inputs (forces) is:

$$PSD[force\, matrix] = \mathrm{F} = \begin{bmatrix} f_{11} & f_{12} & .. & f_{16} \\ f_{21} & f_{22} & .. & f_{26} \\ .. & .. & .. & .. \\ f_{61} & f_{62} & .. & f_{66} \end{bmatrix}, \tag{11}$$

$$PSD[acceleration\, matrix] = \mathrm{A} = \begin{bmatrix} a_{11} & a_{12} & .. & a_{16} \\ a_{21} & a_{22} & .. & a_{26} \\ .. & .. & .. & .. \\ a_{61} & a_{62} & .. & a_{66} \end{bmatrix}, \tag{12}$$

Frequency responses (or frequency response functions) can be summarized in a similar matrix. These values can be acquired using FEM Simulation. They could also be measured (given, that the CoG is accessible at least for such FRF measurements).

$$FRF[matrix] = \mathrm{H} = \begin{bmatrix} H_{11} & H_{12} & .. & H_{16} \\ H_{21} & H_{22} & .. & H_{26} \\ .. & .. & .. & .. \\ H_{61} & H_{62} & .. & H_{66} \end{bmatrix}, \tag{13}$$

Then acceleration and force are connected through the FRF matrix as follows:

$$A = H \cdot F \cdot \bar{H}^T \tag{14}$$

The diagonal matrix elements contain so called auto-spectra, the non-diagonal matrix elements- contain the cross-spectra. Auto-Spectra are real numbers, cross-spectra are complex numbers where the matrix element mirrored on the diagonal is its own complex conjugate.

$$a_{j_1 j_2} = \bar{a}_{j_2 j_1} \tag{15}$$

$$f_{i_1 i_2} = \bar{f}_{i_2 i_1} \tag{16}$$

Equation 9 can be transformed by multiplying from the left with H^{-1} and from the right with $\bar{H}^{T-1}$

$$F = H^{-1} \cdot A \cdot \bar{H}^{T-1} \tag{17}$$

If both forces and torques on the CoG have to be extracted from one set of measurements with six degrees of freedom as described above, the FRF matrix contains mixed units For equation (17) to yield correct results, the stringent use of a consistent set of units, preferably SI units, is mandatory for all quantities, since the mixed units FRF matrix can otherwise lead to erroneous results that cannot be corrected by subsequent rescaling.

1.3 Specification inversion

An alternative method to evaluate the acceleration measurements is the inversion of the initial force specification by directly applying equation (14). If the forces/torques specification is given (e.g., at CoG) then F is a known matrix; the FRFs in H are directly multiplied with the forces matrix F resulting in an acceleration matrix at the measurement points which can be then used as specification for the specific design verification configuration (Figure 2).

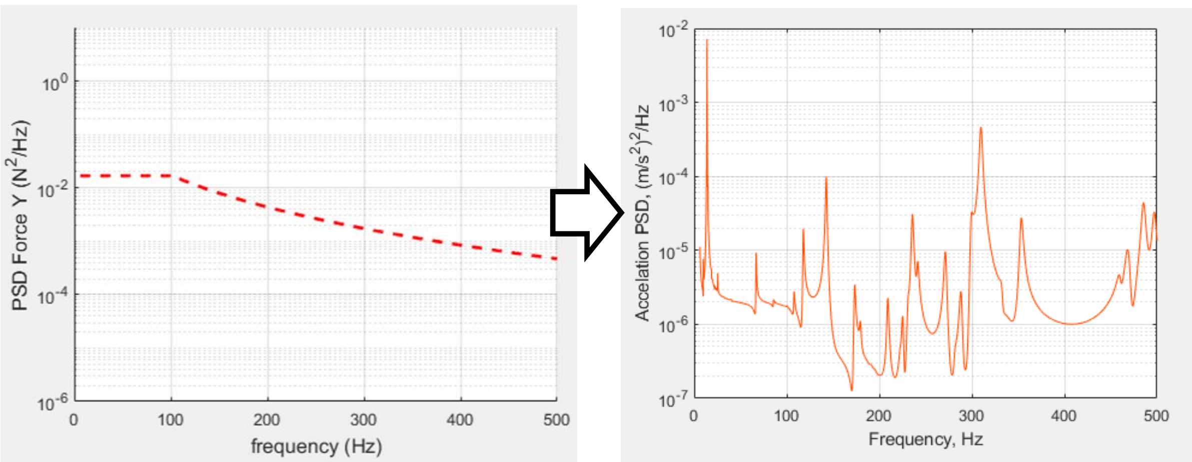

Figure 2: Transformation of force specification to acceleration specification.

2 Measurement Data Acquisition and Processing

2.1 Test setup and procedure

The above described method of "Measurement Conversion" was applied in a case study, in which a water-cooled system was measured with six accelerometers as described in Figure 1. The sensors were placed on the system using petroleum wax and the structure under test was suspended on rubber bands to achieve a decoupling frequency of about 3 Hz. To excite the structure the water flow with defined flow rate was used. First the water flow was turned on for 30 minutes to get it settled and to allow any air to get out.

Thereafter the measurement was performed with the following parameters:

- frequency range 6,25-500 Hz,
- FFT Lines 3200,
- frequency resolution 0.15625Hz,
- 100 averages,
- Hanning window.

Then Power auto-spectra and cross-spectra of the accelerations (PS) were acquired.

2.2 Post-processing

The acquired power spectra of acceleration were converted to power spectral densities for further calculations using following equation (12):

$$PSD = \frac{PS}{k_H \cdot df} \tag{18}$$

where *PS* – power spectra, m/s^2, k_H = 1,5 – Hanning pre-factor [1], *df* = 0, 15625 Hz - frequency resolution.

The resulting auto-spectra PSDs can be seen in Figure 3 – they represent the diagonal matrix elements in the acceleration matrix A (see equation (6), $a_{11}=X_1$, $a_{22}=Y_2$ etc.). Non-diagonal values are the cross-spectra and were calculated in the same manner.

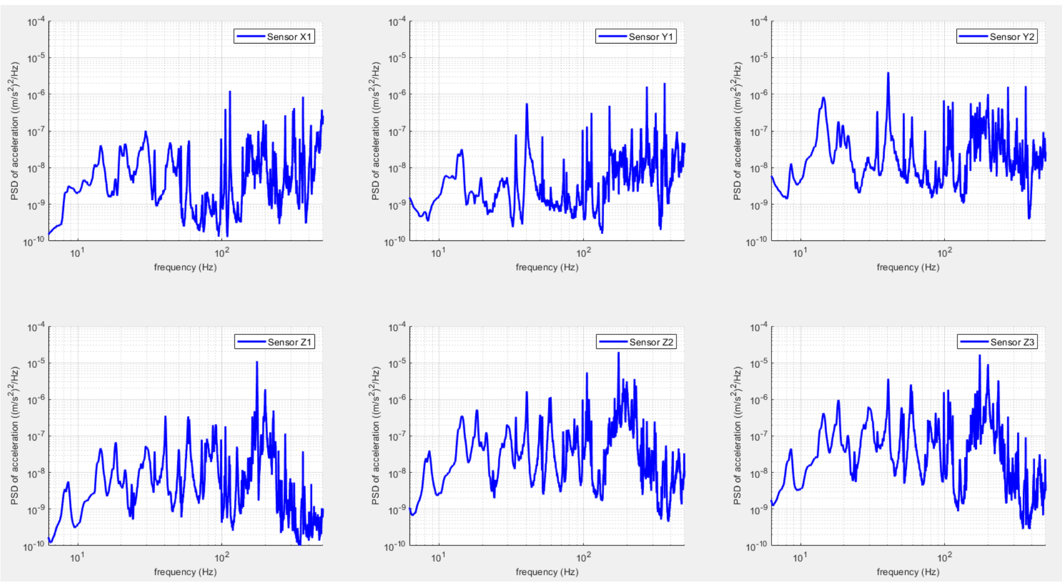

Figure 3: Auto-spectra PSDs for all diagonal matrix elements of acceleration matrix A

To calculate the frequency response functions a FEM model of the device under test was used (Figure 4). The mixed forces-torque matrix was created as described in chapter 2.2.

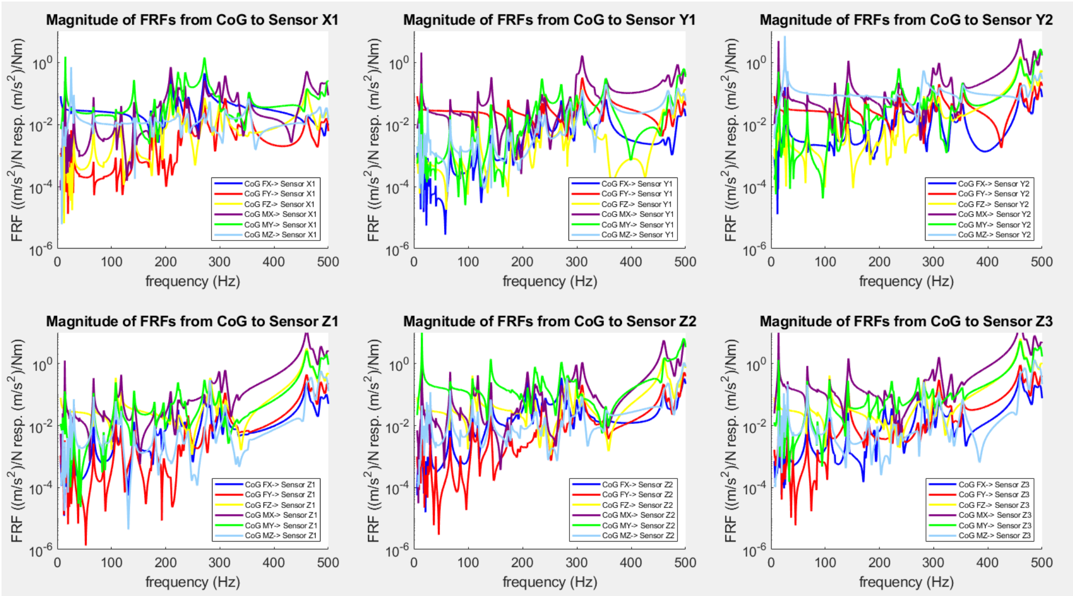

Figure 4: FEM modeled FRFs

Using FRF matrix H and acceleration matrix A, the force/torque matrix F was calculated with equation (11).

To check the plausibility and correctness of the implemented calculation algorithm, a backward calculation using equation (9) was performed; relative differences of less than 0.025% between the initial acceleration measurement data and the reconstructed accelerations after passing twice through the algorithm have been found (Figure 5).

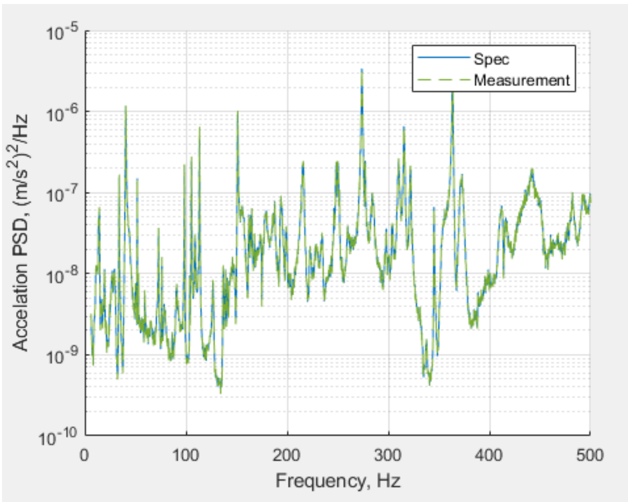

Figure 5: Measured and backwards calculated acceleration auto-spectrum PSDs on sensor Y1

3 Results and discussion

The resulting forces can then be directly compared to the specified spectrum (Figure 6). In this particular case, while the forces are mostly far below the spec line, some narrow peaks violate the specification. This then necessitates discussions concerning possible tolerancing, adjustment of error budgets or even design changes; in short, the measurements can then be used to make decisions with respect to design validity. The alternative method of converting the “force”-specification line into a measurement specific “acceleration”-

specification line using formula (9) yields acceleration curves, that can be directly compared with the measurements results (Figure 7 showing the example for one of the sensor locations). The measured acceleration at sensor location Z2 shows clearly more peaks than the corresponding inverse specification line. This can be caused by two factors. Firstly, the FEM model used to derive the transfer functions did not have all the elements of the actual structure used in the measurement campaign – some elements were simplified in the FE model as mass-points to reduce the calculation time. This means that some of the local eigenmodes of these elements are not in the model, hence cannot be seen in the FRFs and are not present in the converted acceleration specification. Secondly, the flow regime in the test was highly turbulent, leading to vortex shedding inside the test structure channels that could cause further peaks in the measurement data that are not related to structural dynamics. The frequency of vortex shedding is linearly dependent on the local water speed [2] and thus if required could be easily identified by varying the volume flow and comparing peak location with volume flow.

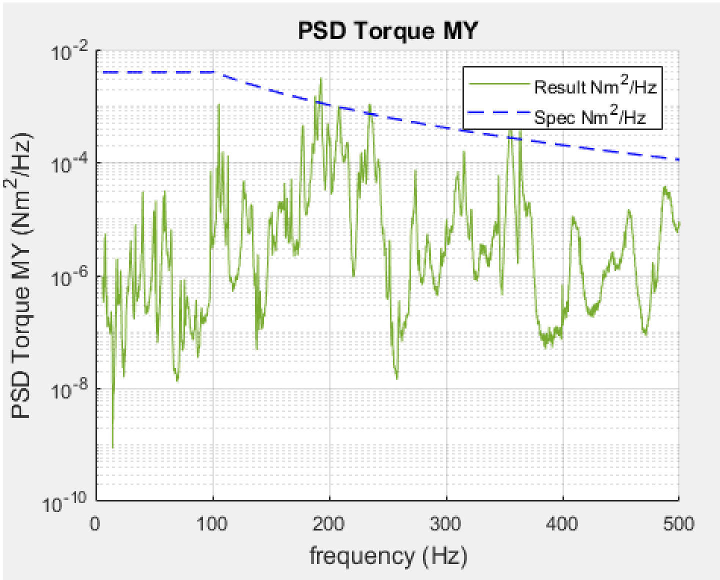

Figure 6: Resulting torque PSD for rotational DOF Ry on CoG vs specification

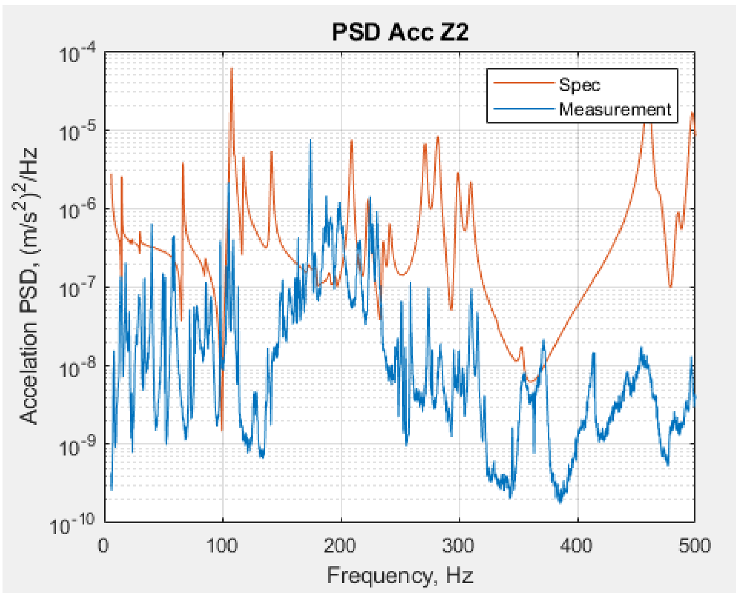

Figure 7: Resulting acceleration specification PSD for sensor point Z2 on CoG vs measurement

One more issue is the sensor position dependent anti-resonances at some frequencies, that are present in the calculated FRFs. After the calculation resulting forces could have resonances at these frequencies. In a real system excitation occurs not only at CoG or other single point but at many different points simultaneously (e.g., due to complex cooling channels geometry). Thereby, many forces with corresponding many transfer-paths result in acceleration at a sensor point – "averaged" at each frequency no anti-resonances can then be seen in acceleration signal. Thus, single forces calculated with this method can be overestimated at these frequencies. Nevertheless, these points can be predicted from the FRFs which are used for calculation – looking at anti-resonances by each single FRF in the H-matrix.

To "globally" improve this issue more points can be either simulated or measured. By simulation more FRFs could be calculated for the "excitation" points near CoG and then averaged to reduce the number of anti-resonances. Alternatively, more sensor points near already planned (e.g., 6 DoF points as in this paper) could be measured and then averaged in the same manner.

The described method can also be used to calculate forces at some other critical points of interest, for example some interfaces to other sensitive sub-systems. These can be used to evaluate the behavior of the other sub-systems that are dependent on the first given sub-system.

It is important to notice that the method is based on the prerequisite that the FEM model is correct. Therefore, a preliminary test to validate the FEM model has to be performed – e.g., modal analysis of the structure, preferably with FEM-EMA correlation.

Another important prerequisite is the linearity of the system of transfer functions – it can be also verified through reciprocity and linearity experimental tests in the context of the aforementioned modal analysis.

References

[1] A. Brandt, "Noise and Vibration Analysis: Signal Analysis and Experimental Procedures", *John Wiley & Sons,* 2011, ISBN 9780470746448.

[2] T. Kármán, "Aerodynamics" *Dover,* 1994, ISBN 978-0-486-43485-8.