Probabilistic learning on manifolds for design optimisation of aero-acoustic liner impedance

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Abstract

We address the problem of noise reduction in modern aircraft engines, targeting the low frequency tonal noises by means of tailored acoustic liners in the nacelle. Due to the prohibitive cost of high fidelity computational models for design optimisation, we use Probabilistic learning on Manifolds (PLoM) for constructing statistical meta-models (surrogate models) of the liner acoustic impedances . This allows a learned set to be generated from a given training set whose points are realisations of a non-Gaussian random vector whose support of its probability distribution is concentrated in a subset (a manifold). This approach preserves the concentration of the probability measure for the learned set and has been developed for the case of small training sets as opposed to big data. We then construct a probabilistic meta-model of the liner impedance for which the training set is constructed with CAA performed by Airbus. Conditional statistics of the random impedance are estimated. This allows a robust meta-model of the liner impedance to be constructed.

1 Introduction

Air traffic has grown significantly over the past decade. With ever increasing environmental concerns, green aviation has come into prominence. In particular, aircraft noise is of significant interest to the aviation community, whether it be the question of external or internal noise relative to the aircraft. This has led to increasingly stringent requirements by the certification authorities.

In modern turbofan engines (UHBR - Ultra high bypass ratio), fan noise is one of the main contributors to the overall aircraft noise. Fan noise can be characterised by broadband and tonal noise components. Acoustic liners (acoustic treatments) can be designed to tackle both the components. Tonal noise is attenuated by resonance effect and broadband noise is attenuated by viscous dissipation. Noise attenuation by acoustic treatments is tuned for the blade passing frequency (BPF), whereas dissipating as much as possible the broadband component, by modifying the liner geometry or intrinsic properties. In order to be effective in absorbing fan noise liners have to be studied in their operating environment i.e. different flight conditions. In this work, the study only concerns the low frequency tonal noise and how impedance liner models can be improved to correctly model such phenomena.

Many experiments have been carried out in the past for identifying uncertainties related to liner impedance (see for instance [1, 2, 3, 4]). High fidelity models have also been developed for predicting the liner performances (see for instance [5, 6, 7, 8, 9, 10]). The design and robust design methodologies of liners are of prime interest and many works have been published on this subject (see for instance [11, 12, 13, 14, 15, 16, 17]). Machine learning tools for statistical inference such as Bayesian approaches have recently been used for statistical inverse problem related to liners impedance (see for instance [18, 19]).

The design and performance of the liner depend on the quantities which are highly dependent on the operating conditions (ex. velocity, mean pressure, density of the fluid). Any external variation directly impacts the environment of the liners and thus the acoustic response of the liners system. This requires to take into account uncertainties in the high fidelity computational model of the liners system in order to perform robust design and optimization. A robust design of the liners system is presented by Dangla [16], where the study of uncertainty quantification is undertaken in liner performance aeroacoustic models. It lays several of the foundations for the present work and provides some insight into non-parametric probabilistic approach of modelling errors.

A liner is characterised by its acoustic absorption. The absorption value can be quantified by its local impedance. In the framework of this paper, the liner acoustic impedance is estimated using a high fidelity CAA (computational aero-acoustics) model, via direct numerical simulation of the impedance resulting from the interaction acoustic-liner. One evaluation with such a CAA model is computationally expensive and consequently, cannot be used for performing the liner design optimisation, as a large number of computations with the CAA model has to be done. We thus propose to develop a statistical meta-model of the liner impedance based on the use of a probabilistic learning tool and a training set made up of a small number of points must be constructed using the high fidelity CAA model. The learning tool will be the probabilistic learning on manifolds (PLoM). The data for the training set will be generated based on Lavieille's work [5] for numerical prediction of acoustic impedance of typical nacelle single degree of freedom (SDOF) perforated plate liners, accounting for high sound pressure levels. We use the standard SDOF liners to validate the workflow, before applying the workflow to liners which target low frequencies.

1.1 Summary of the PLoM methodology

Probabilistic learning on manifolds (PLoM) is a data sampling technique preserving the concentration proposed by Soize & Ghanem [20, 21, 22]. This methodology is used to generate new realisations of a non-Gaussian random vector $\mathbf{X}$, which are statistically consistent with the points of the training set. The steps involved in PLoM are as follows : (i) Multidimensional kernel density estimation (MKDE) method to estimate the pdf of $\mathbf{X}$ using the training set (small data). (ii) Construction of an MCMC (Markov Chain Monte Carlo) generator of realisations based on a matrix valued non-linear ISDE (Itô stochastic differential equation). (iii) Construction of a diffusion-maps basis using the training set, which allows for the geometry of the manifold to be characterised and to preserve the concentration of the learned probability measure. (iv) Construction of a reduced-order MCMC by projecting the matrix valued ISDE on the diffusion maps basis. (v) Generation of big data using the reduced-order MCMC generator.
In this work, the PLoM approach will be used in the following manner. The PLoM starts with training set $\mathcal{D}_d = \{\mathbf{x}_d^j, j = 1,\ldots,N_d\}$ with a relatively small number N_d of data points. For the supervised learning case, the $\mathbb{R}^n$ valued random vector $\mathbf{X}$ is written as $\mathbf{X} = (\mathbf{Q},\mathbf{W})$ in which $\mathbf{Q}$ is the $\mathbb{R}^{n_q}$-valued random vector of the quantities of interest and $\mathbf{W}$ is the $\mathbb{R}^{n_w}$-valued random control parameter defined on a probability space $(\boldsymbol{\Theta},\boldsymbol{\mathcal{T}},\boldsymbol{\mathcal{P}})$, where $n = n_q + n_w$. The random variable $\mathbf{Q}$ is written as $\mathbf{Q} = \mathbf{f}(\mathbf{W})$ in which the measurable mapping $\mathbf{f}$ is unknown. The probability measure of $\mathbf{X}$ is represented by a smoothed probability measure admitting a pdf that is estimated with MKDE based on the points of $\mathcal{D}_d$. In PLoM, it is not assumed that the manifold defined by the graph $\{(\mathbf{f}(\mathbf{w}),\mathbf{w}), \mathbf{w}\in\mathcal{C}_w\}$ can be directly described, in which $\mathcal{C}_w$ is the support of the probability measure of $\mathbf{W}$. The only available information is the cloud of points of the training set. The non-Gaussian probability measure of $\mathbf{X}$ is concentrated on the region defined by this cloud of points. From this training set $\mathcal{D}_d$, the learned set $\mathcal{D}_{\text{ar}} = \{\mathbf{x}_{\text{ar}}^\ell, \ell = 1,\ldots,N_{\text{ar}}\}$ is generated with $N_{\text{ar}} \gg N_d$ additional realisations also called learned realisations. The generation of the learned set is performed while preserving the concentration of the probability measure [20, 21]. The concentration is quantified by using a distance (defined by the mean-square norm) between the learned set and the training set [22]. Statistical conditioning using non-parametric statistics and the learned set generated with PLoM is then used to construct the statistical meta-model.

2 Methodology

In the present work a quarter-wavelength resonator is modelled from a perforated plate liner. Using this single resonator a CAD (computer aided design) model and a mesh have been generated as shown in fig. 1. High fidelity computational fluid dynamics is used to generate the points $\{\mathbf{x}_d^j\}_j$ of the training set . Navier-Stokes (NS) equations with Direct Numerical resolution of turbulence (DNS) have been solved in the vicinity of the mesh refinement zone while Euler equations have been used elsewhere [5].

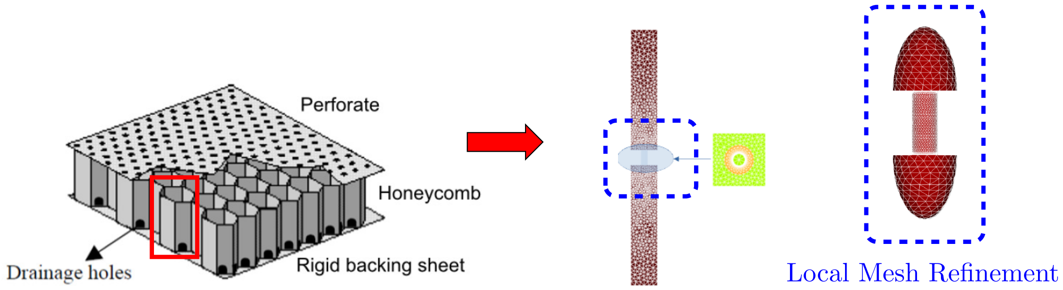

Figure 1: Perforated plate liner [5] (left). A single quarter-wavelength resonator is modelled and meshed(centre) along with the local mesh refinement (right).

In this paper, the results presented will be limited to Mach number equal to 0 (the methodology presented is general and not limited to this case). The parameters used to control the liner system are percentage of open area (POA) and sound pressure level (SPL). These two parameters (POA, SPL) are used as components of vector-valued random variable $\mathbf{W}$. The frequency axis ω (rad/s) is sampled as the finite set of discrete frequencies $\{\omega_1, \ldots, \omega_{n_\omega}\}$ in which n_ω is the number of frequency sampling points. Let $\mathbf{Z} = (Z(\omega_1), \ldots, Z(\omega_{n_\omega}))$ be the $\mathbb{C}^{n_\omega}$-valued random variable corresponding to the frequency sampling of the random impedance. Random vector $\mathbf{Z}$ is written as:

$$\mathbf{Z} = \mathbf{R} + i\mathbf{V} \tag{1}$$

where $\mathbf{R}$ is the vector-valued random variable representing acoustic resistance and $\mathbf{V}$ is the vector-valued random variable representing acoustic reactance. The random quantity of interest $\mathbf{Q} = (\mathbf{R}, \mathbf{V})$ depends on the control parameter $\mathbf{W} = (W_1, W_2) = (\mathrm{POA}, \mathrm{SPL})$ whose prior probability distribution is defined as follows. The support of this probability distribution is the admissible set $\mathcal{C}_w \subset \mathbb{R}^{n_w}$ defined by,

$$\mathcal{C}_w = \{\mathbf{w} \in \mathbb{R}^2 \,|\, w_1^{\text{min}} \leq w_1 \leq w_1^{\text{max}} \,,\, w_2^{\text{min}} \leq w_2 \leq w_2^{\text{max}}\} \tag{2}$$

Let $\mathbf{w}_d^1, \ldots, \mathbf{w}_d^{n_d}$ be N_d independent realisations of random variable $\mathbf{W}$ generated with its prior probability distribution. For $j = 1, \ldots, N_d$, let $\mathbf{q}_d^j = \mathbf{f}(\mathbf{w}_d^j)$ the corresponding realisation of the quantity of interest computed with the high fidelity CAA model. The training set is then written as

$$\mathbf{x}_d^j = (\mathbf{q}_d^j, \mathbf{w}_d^j) \quad , \quad j \in \{1, \ldots, N_d\}\,, \tag{3}$$

Using PLoM, a learned set $\{\mathbf{x}_{\text{ar}}^\ell, \ell = 1, .., N_{\text{ar}}\}$ is generated from the training set with $N_{\text{ar}} \gg N_d$. It can then be deduced the learned realisations of $\mathbf{Q}$ and $\mathbf{W}$ such that

$$(\mathbf{q}_{\text{ar}}^\ell, \mathbf{w}_{\text{ar}}^\ell) = \mathbf{x}_{\text{ar}}^\ell \quad , \quad \ell = 1, .., N_{\text{ar}}\,. \tag{4}$$

The statistical meta-model is constructed by estimating the conditional mathematical expectation and the conditional confidence interval (CI) of $\mathbf{R}$ and $\mathbf{V}$, given $\mathbf{W} = \mathbf{w}$ in which $\mathbf{w}$ browses through $\mathcal{C}_w$. These statistics require the estimation of the joint probability density function of (i) $\mathbf{R}$ and $\mathbf{W}$ and (ii) $\mathbf{V}$ and $\mathbf{W}$. This estimation is performed using the Gaussian Kernel density estimation (GKDE) and the learned set.

As an illustration, we give the algebraic expression of the conditional mathematical expectation of $\mathbf{Q} = (\mathbf{R}, \mathbf{V})$ given $\mathbf{W} = \mathbf{w}_\mathrm{O}$ in which $\mathbf{w}_\mathrm{O} \in \mathcal{C}_w$,

$$\mathrm{E}\{\mathbf{Q}|\mathbf{W} = \mathbf{w}_\mathrm{O}\} = \frac{\sum_{\ell=1}^{\mathrm{N_{ar}}} \mathbf{q}^\ell \times \exp(-\frac{1}{2\mathrm{s}^2}\|\mathbf{w}_\mathrm{O} - \mathbf{w}^\ell\|^2)}{\sum_{\ell=1}^{\mathrm{N_{ar}}} \exp(-\frac{1}{2\mathrm{s}^2}\|\mathbf{w}_\mathrm{O} - \mathbf{w}^\ell\|^2)} \tag{5}$$

where 's' is Silverman's bandwidth. In order not to make the presentation cumbersome, we do not give the algebraic expression of the conditional cumulative distribution function estimated using the GKDE and the learned set, but its derivation is straightforward.

The upper $(\mathbf{q}^+)$ and lower $(\mathbf{q}^-)$ bounds corresponding to the 98% confidence interval(CI) is estimated using this conditional cumulative distribution function for each $\mathbf{w}_\mathrm{O}$ and are such that

$$\begin{aligned} \mathbf{q}^+ &: Prob\{\mathbf{Q} \leq \mathbf{q}^+ | \mathbf{W} = \mathbf{w}_\mathrm{O}\} = 0.98 \\ \mathbf{q}^- &: Prob\{\mathbf{Q} \leq \mathbf{q}^- | \mathbf{W} = \mathbf{w}_\mathrm{O}\} = 1 - 0.98 \end{aligned} \tag{6}$$

3 Numerical values of the problem parameters and results

(i) The mesh is made up of $41\,781$ vertices, $206\,560$ elements, and $278\,514$ degrees of freedom. The mesh outside the refinement zone (see 1) changes according to POA. The statistics presented here are an average, presented for information only. The training set is constituted of $N_d = 48$ points. The number of frequency points is $n_\omega = 7$. For each of the N_d point and for each frequency point, the CPU time is 448 hours (using a 64 cores computer). The computation has been done by Airbus using the SANUMO software [5].

(ii) The learned set is generated with the PLoM software [23] , $N_{\mathrm{ar}} = 48\,000$ learned realisations have been computed. The distance measuring the concentration of the learned set with respect to the training set is 0.407 showing that the concentration is preserved (reminder: without PLoM, with a standard MCMC algorithm, the learned realisations are scattered and the distance is then equal to 2).

(iii) Figure 2 displays the dimensionless resistance and reactance of the impedance as a function of the dimensionless frequency. For $\mathbf{W} = \mathbf{w}_\mathrm{O}$ with $\mathbf{w}_\mathrm{O} = (\mathrm{POA_O}, \mathrm{SPL_O})$ in which $\mathrm{POA_O} = 0.045$ and 0.075, and $\mathrm{SPL_O} = 133$ and 141, the values obtained with the high fidelity CAA model are labelled as R_O and V_O. The values of the conditional expectation computed with PLoM are denoted by $\underline{R}_{\mathrm{PLoM}}$ and $\underline{V}_{\mathrm{PLoM}}$, while the 98% CI computed with PLoM is the red zone.

(iv) It can be seen that the dispersion of the resistance is larger than the reactance. This dispersion is directly correlated to the contents of the training set that exhibits a similarly larger dispersion for resistance with respect to the reactance. For the considered values of the control parameters the dispersion of the reactance always stays small, while, the dispersion of the resistance strongly depends on the control parameters values. For given $\mathbf{w}_\mathrm{O}$, the estimated conditional expectation of the resistance and reactance yields a good prediction in comparison to the values obtained from high fidelity CAA model.

4 Conclusions and future works

The results obtained are good taking into account the very small number of points in the training set, constructed with the high fidelity CAA model. The method and statistical conditioning yield a robust metamodel and, for each point of frequency, the confidence interval (CI) is able to encompass the resistance and reactance values from CAA. The conditional mathematical expectation is also estimated and belongs to the

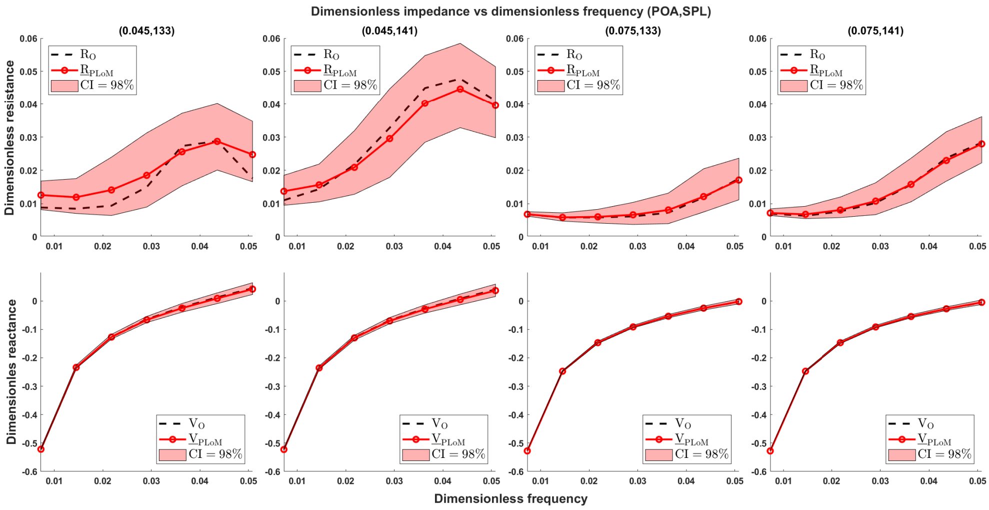

Figure 2: Dimensionless impedance vs dimensionless frequency (resistance top and reactance bottom). For $\mathbf{W} = \mathbf{w}_{\mathrm{O}}$ with $\mathbf{w}_{\mathrm{O}} = (\mathrm{POA}_{\mathrm{O}}, \mathrm{SPL}_{\mathrm{O}})$, CAA ($R_{\mathrm{O}}$ and V_{O}) (black broken lines), conditional expectation $(\underline{R}_{\mathrm{PLoM}}, \underline{V}_{\mathrm{PLoM}})$ from PLoM (red line), and 98% CI from PLoM (red zone).

CI. The random resistance show a much greater dispersion than random reactance. Using the conditional expectation and CI a robust meta-model of the liner can be made which can be used for optimisation of design parameters under uncertainty. The presented methodology that has been validated will be used for the design optimisation of the liners for which the training set will be constructed with high fidelity CAA model. The work in progress consists in, firstly, performing the analysis for non-zero Mach numbers and secondly, implementing the probabilistic learning methodology with constraints in order to perform the fusion of experimental data with CAA data.

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