Damping characterisation of an elevator rope

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Abstract

The ropes of an elevator transmit the vibrations from the motor to the elevator car, and consequently have a great influence on comfort. Therefore, it is of major interest to analyse the dynamic behaviour of these elements. In the present work, the characterisation of the damping of an elevator rope was carried out. Three type of damping estimation methods were analysed. First, the Half Power Bandwidth (HPB) method was used. In addition, due to the low signal quality obtained in some results, the logarithmic decrement method was employed. Finally, Experimental Modal Analysis (EMA) algorithms were carried out. According to the results, similar damping values were obtained with the three methods. Thus, it is confirmed that the HPB method is the most interesting procedure. On the one hand, it is not necessary a previous signal conditioning that requires the definition of the filtering conditions by the analyst. On the other hand, instead of measuring the acceleration response of several points, only one acceleration response signal is enough.

1 Introduction

Ropes are the structural elements of diverse engineering applications, such as elevators, suspension bridges and cranes since they resist large axial loads. In the case of elevator installations, they play a significant role in order to achieve and maintain adequate ride quality standards. Excessive vibrations can compromise the ride quality being the ropes particularly affected due to their flexibility and loading conditions [1]. High flexibility and low intrinsic damping are the most important reason of cable vibrations [2].

Damping can be described by several terms, such as the loss factor, η; specific damping capacity, ψ; logarithmic decrement, δ; or damping ratio, ζ. One of the main problems of damping estimation is that it is significantly more sensitive to the environment noise and experimental uncertainty, compared with the mass and stiffness [3].

There are many different methods to identify the damping capacity of a system. The Half-Power Bandwidth method (HPB) has been extensively used for single degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) structures [4]. The use of this method for damping estimation is based on the assumption that the structure can be modelled by a SDOF system or by a series of decoupled SDOF systems. However, this method has been extended to MDOF structures assuming that each peak in the frequency response is affected only by the mode under study [4]. For damping ratios no greater than 0,1 the classical HPB method can provide acceptable results. However, it could bring noticeable errors for slightly higher damping ratios especially when the acceleration is considered to determine the Frequency Response Function (FRF) to damping estimation [5]. In order to overcome this problem, third-order corrections were proposed by Wang [6]. Wu [7] proposed new approximate damping ratio formulas to solve the errors found.

In the case of the ropes analysed, the wires are made of brass coated steel, which provides them a higher wear resistance and a polymeric coating made of TPU. Recently, ropes with polymeric cover are used to avoid wear problems associated to the contact between the ropes and pulleys, and to improve their damping capacity, among others. Due to the viscoelastic behaviour of the polymeric materials, they exhibit a frequency and temperature dependence, and offer superior properties in terms of energy dissipation. Although a damping improvement could be obtained due to the use of a polymeric coating, no high damping ratio values are expected since the predominant material is still the steel and works principally under tensile load.

Another well-known method to determine damping is the logarithmic decrement, widely employed for viscous damping. For MDOF systems, it is typically applied to estimate the damping of the first vibration mode from a free response signal dominated by the first mode, which can be modelled as a SDOF free response. This method exhibits correct results when SDOF systems in association with previous noise filtering is considered. However, the estimation for higher modes is usually much more difficult since it is not straightforward to acquire signals significantly dominated by only one of the higher modes [8]. Therefore, it is inaccurate for MDOF systems [9]. Liao and Wells [8] suggested the use of band-pass filters to isolate the modal components into individual damped sinusoidal waves. Little and Mann proposed a guidance to determine the correct amount of period in order to minimise the uncertainty in damping estimation [10].

Experimental Modal Analysis (EMA) is another method to identify the damping capacity of the system. The majority of modern experimental modal analysis is based on the application of a modal parameter estimation (curve fitting) technique to a set of measured FRFs [11]. The curve fitting methods can be grouped into four groups: local SDOF methods, local MDOF methods, global methods and multi-reference methods. SDOF methods estimate modal parameters one mode at a time, MDOF, Global and Multi-Reference methods can simultaneously estimate modal parameters for two or more modes at a time. In addition, local methods are applied to one at a time while global and multi-reference methods are applied to an entire set of FRFs at once [11]. The SDOF methods are simpler and easier to apply but in many real systems the proximity between natural frequencies makes them unsuitable, and therefore, MDOF methods are required.

The aim of this work is to analyse the suitability of damping estimation methods, such as the HPB method, the logarithmic method and the EMA method; in order to determine the damping of an elevator rope. Therefore, first of all, the three methods to identify damping are described. Then, based on the experimental results obtained in a previous work [12], the loss factor values obtained by the three methods are compared. Finally, the influence of the preload in the damping is analysed.

2 Damping estimation methods

2.1 Half Power Bandwidth (HPB)

HPB method determines damping from the bandwidth between any two points A and B at which the amplitude of the resonance decreases a ratio of $1/n$, where n is greater than 1. The usual convention is to consider points A and B to be located at frequencies where the amplitude of response is $1/\sqrt{2}$ times the maximum response [13]. Substituting the frequency of maximum amplitude, ω_{res} ,into the particular solution of forced response, the amplitude at resonance is described by Eq. (1) when viscous damping is considered.

$$\left(X_p\right)_{res} = \frac{F}{k}\left[\frac{1}{2(c/2\sqrt{km})\sqrt{1-c^2/4km}}\right] \tag{1}$$

where F is the force, k the stiffness, m the mass and c the viscous damping coefficient. This response is equated to $(1/n)$ times the general equation of the particular solution of forced response, Eq. (2).

$$\frac{1}{\sqrt{\left[1-(m\omega^2/k)\right]^2+C^2\omega^2/k^2}}=\frac{1}{(Cn/\sqrt{km})\sqrt{1-C^2/4km}} \tag{2}$$

Solving the quadratic equation, the frequencies at points A and B are given by Eq. (3).

$$\omega_{1,2}^2=\frac{k}{m}\left[1-2\left(\frac{C^2}{4km}\right)\pm 2\sqrt{n^2-1}\left(\frac{C}{2\sqrt{km}}\right)\sqrt{1-\frac{C^2}{4km}}\right] \tag{3}$$

For $C^2/4km \ll 1$ Eq. (3) is rewritten as Eq. (4)

$$\sqrt{m/k}\left(\omega_{1,2}\right)=1\pm\sqrt{n^2-1}\left(\frac{C}{2\sqrt{km}}\right) \tag{4}$$

Therefore, the relation between the bandwidth and the resonance frequency is described by Eq. (5).

$$\frac{\Delta\omega}{\omega_{\text{res}}}=\frac{\omega_2-\omega_1}{\omega_{\text{res}}}=2\sqrt{n^2-1}\left(\frac{C}{2\sqrt{km}}\right) \tag{5}$$

When n is equal to $\sqrt{2}$, the value of the damping is given by Eq. (6).

$$\frac{\Delta\omega}{\omega_{\text{res}}}=2\left(\frac{C}{2\sqrt{km}}\right)=2\zeta \tag{6}$$

where $\zeta=C/2\sqrt{km}$ is the damping ratio. Similar process could be carried out if a hysteretic damping is considered. In this case, the amplitude at resonance is defined by Eq. (7).

$$\left(X_p\right)_{\text{res}}=\frac{F}{k\eta} \tag{7}$$

The frequencies at points A and B are given by Eq. (8).

$$\omega_{1,2}=\sqrt{\left(\frac{k}{m}\right)\left[1\pm\eta\sqrt{n^2-1}\right]} \tag{8}$$

When n is equal to $\sqrt{2}$, damping is determined by Eq. (9).

$$\frac{\Delta\omega}{\omega_{\text{res}}}=\sqrt{1+\eta}-\sqrt{1-\eta} \tag{9}$$

And for $\eta \ll 1$, Eq. (10).

$$\frac{\Delta\omega}{\omega_{\text{res}}}\cong\left(1+\frac{\eta}{2}\right)-\left(1-\frac{\eta}{2}\right)=\eta \tag{10}$$

According to Montalvao et al. [14], although the viscous behaviour of a material is generally described as a proportional relationship between friction stresses and strain rate, experience shows that for many structural materials, the friction stress shows a linear relation with the strain itself, at least for a wide range of frequencies. Therefore, a hysteretic damping was considered.

2.2 Logarithmic decrement method

The logarithmic decrement method is a time domain technique that represents the rate at which the amplitude of a free damped vibration decreases. The free response of a damped system and the damped frequency are described by Eq. (11) and Eq. (12)

$$x_c(t) = X_c e^{-\zeta\omega_n t}\cos(\omega_d t - \phi) \tag{11}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{12}$$

where X_c is the amplitude, ω_n the natural frequency, ω_d the damped natural frequency and ϕ the phase. The logarithmic decrement, δ, is calculated by comparing displacements multiple periods apart, Eq. (13).

$$\delta = \frac{1}{n}\ln\frac{x(t)}{x(t+nT_d)} \tag{13}$$

where n is the number of the periods considered and T_d is the period of oscillation, which corresponds to the time interval between adjacent response peaks. Assuming that the log decrement is $\delta = \zeta\omega_n T_d$ the damping ratio and the latter can be related by Eq. (14).

$$\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \tag{14}$$

Due to the uncertainty found in literature in terms of selecting the correct number of periods [10], in this case, a fitting of the free response curve was realised in order to determine the damping ratio. In Figure 1 the damped free response and the envelope of the curve are shown.

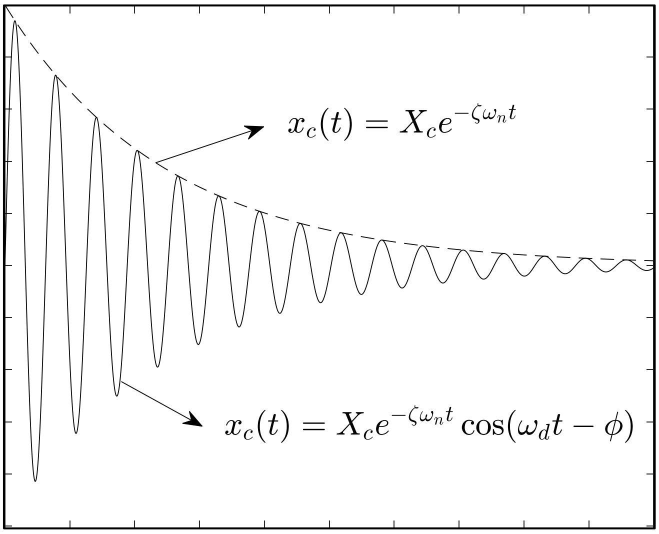


Figure 1: The damped free response and the envelope of the curve

The main issue of the logarithmic decrement method is that it works with the time signal of the frequency component corresponding to the natural frequency considered. Thus, the raw experimental time signal needs to be filtered and the definition of the filter may influence the results obtained.

First, it is necessary to identify the damped frequency. For that purpose, the EMA method was employed to determine the damped frequency associated to the tension-compression mode. In this case, a lowpass filter was applied to the time signals since the natural frequency of the mode of vibration characterised is the lowest. The cutoff frequency is defined according to the natural frequency. For this analysis, the cutoff frequency was set to the natural frequency plus 2 Hz. In Figure 2, the acceleration signal for a 140 kg preload and a 2.5 m length is shown.

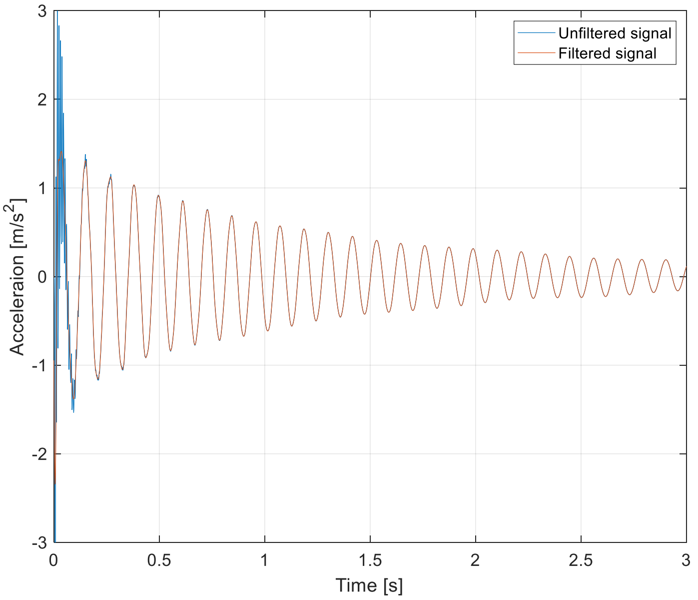


Figure 2: The filtered damped free response and the unfiltered damped free response

Once the signal was filtered, the envelope was obtained, Figure 3, and a decreasing exponential function was fitted to the envelope. Finally, the envelope amplitude $\left(X_c\right)$ and the damping factor (ζ) were estimated from the fitted function considering the damped frequency identified previously.

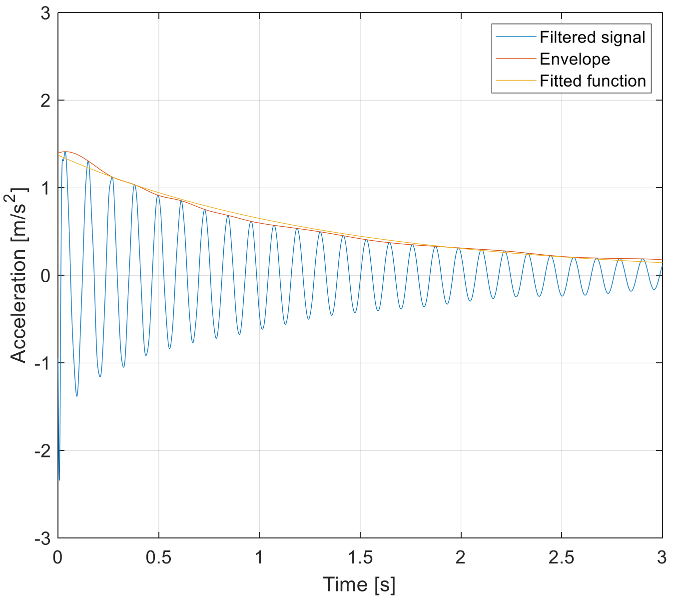


Figure 3: The filtered damped free response, the envelope, and the fitted function

2.3 Experimental Modal Analysis (EMA)

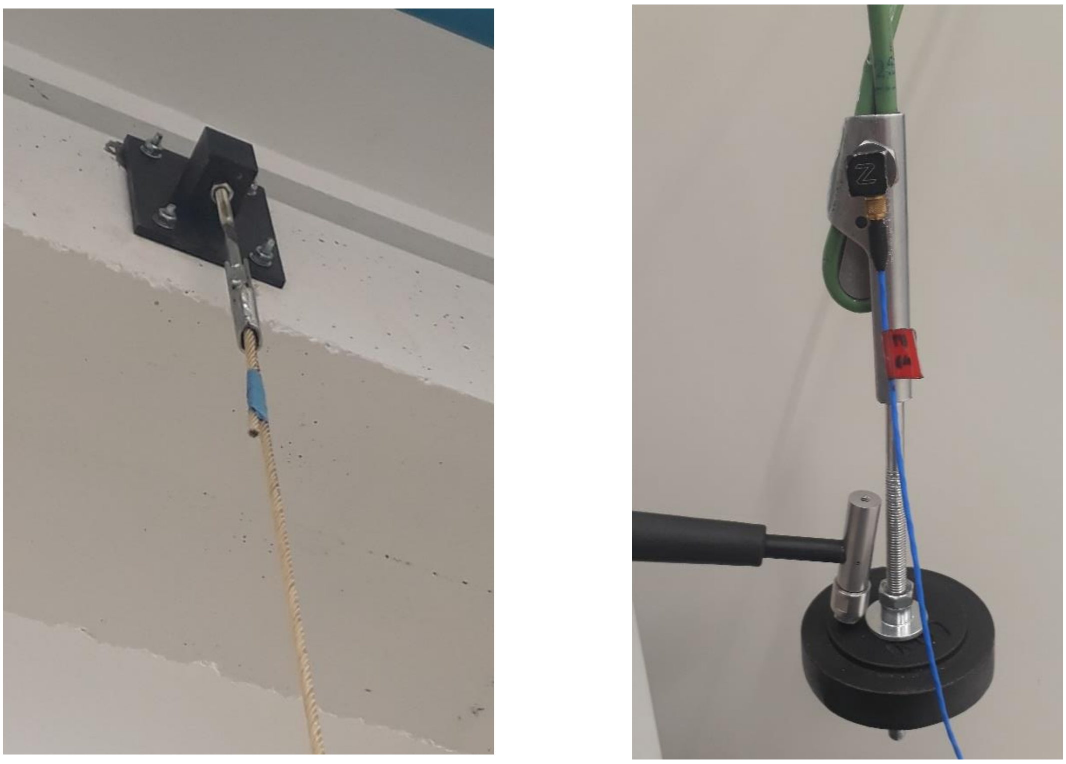
Concerning EMA, the methods to identify modal parameters are divided into two main groups: SDOF and MDOF methods. In the case of the former, they are based on the assumption that at the vicinity of a resonance, the FRF is dominated by the contribution of that vibration mode and the contributions of other vibration modes are negligible [15]. There are many methods such as, the peak-picking method also called the HPB method, the circle fit method, the inverse FRF method and the least-squares method.

In this case, due to the proximity of vibration modes, it was decided to use a MDOF method. The MDOF methods consist of generating the frequency response of the system under study by superposition of normal modes. The difference between the measured and the computed frequency responses is determined and the modal parameters of the superposed modes are modified in order to obtain improved estimates [16]. One of the most widely used MDOF method, is the rational fraction polynomial method. It is based on expressing an FRF in terms of rational fraction polynomials, and through numerical manipulations, the coefficients of these polynomials can be identified. The links between these coefficients and the modal parameters of the FRF can be established [15]. When the structure is lightly damped, it becomes difficult to obtain accurate FRF data near resonances. In these cases, FRF data away from resonances is used. In the present work, considering that the wires are covered by a coating and that the damping could be significant, the method of rational fraction polynomials was employed.

3 Results

3.1 Experimental set-up

The experimental procedure employed to characterise the damping of the rope was described in a previous work [12]. The characterisation procedure consists of suspending the rope from an upper beam while the mass is tied to the joint in the lower end of the rope, Figure 4.



a) b)

Figure 4: Assembly of the rope: a) upper fixed point and b) lower point suspended mass

In order to introduce an excitation to the system, the mass is hit by an impact hammer. The force of the hammer is the excitation force applied in vertical direction downwards, and the acceleration measured by the accelerometers is the response of the system to that excitation. By the response and excitation signals the Frequency Response Function (FRF) of the system is obtained. Before carrying out the characterisation process, it is necessary to identify the peak associated to the tension-compression modes. The peak corresponding to the tension-compression mode could be hard to identify in some cases. Bending modes are assumed to have a negligible longitudinal response, since their response is predominantly in the plane perpendicular to the longitudinal direction, but longitudinal and transversal vibrations are coupled, and it might not always be obvious which peak corresponds to the tension-compression mode. Thus, an EMA was performed. For this purpose, more FRFs need to be measured. If the properties are estimated from the FRFs, only one accelerometer will be enough, whereas to perform an EMA more accelerometers should be used. In this case, 5 of them are used: 2 are placed in the lower joint and the remaining 3 attached to the rope every 0.5 m starting from the upper joint, Figure 5.

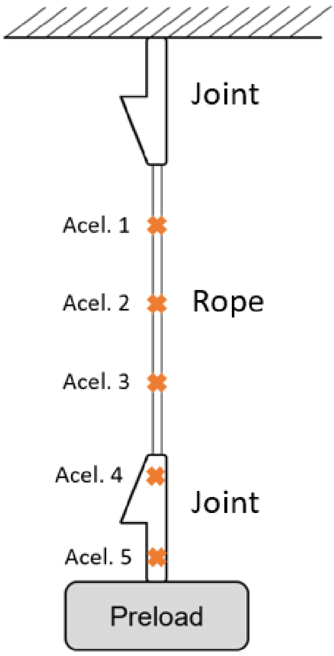


Figure 5: Experimental set-up to perform EMA

This step is of prime importance since it is essential to determine the natural frequency of the tension-compression mode for both the HPB method and logarithmic decrement method.

The characterisation was carried out in terms of preload and length. In the case of the former, preloads from 60 kg to 240 kg were used with a rope length of 2.5 m. In the case of the latter, rope lengths between 0.5 m and 2.5 m were analysed.

3.2 Damping estimation

In Table 1 the results of the loss factor in terms of the preload are shown. Seven different results are presented. The first two associated to the logarithmic decrement method, the first one without any filtering (LDO) and the second one filtering the original signal (LDF). Concerning the HPB method, three different estimations were carried out; the first one considering the original peak and applying the ratio of $1/\sqrt{2}$ (HPB), and the second one and the third one considering either the right (HPBR) side of the peak or the left side (HPBL). Finally, in the case of EMA, the damping was estimated using five accelerometers (EMA5) as shown in Figure 5.

Table 1: Loss factor results in terms of the preload

Preload [kg]	Nat. freq. [Hz]	LDO	LDF	HPB	HPBL	HPBR	EMA5
60	11.726	0.050	0.051	0.044	0.061	0.027	0.051
100	9.789	0.036	0.034	0.027	0.038	0.015	0.026
140	8.743	0.037	0.027	0.023	0.024	0.021	0.016
180	7.868	0.046	0.025	0.022	0.024	0.021	0.015
200	7.499	0.060	0.022	0.021	0.022	0.019	0.014
240	7.031	0.083	0.019	0.017	0.019	0.015	0.015

It was observed that damping exhibited a decrease of 50% approximately, as the preload increased. When the wires of the rope are tensioned, the clearances between them are reduced and their alignment is improved, and consequently smaller damping values are identified. For the purpose of comparison, similar damping values were obtained for the six different methods employed except for HPBR and LDO. As mentioned previously, the HPB method was employed in three different ways. The reason for that was the asymmetry found in the FRFs. The proximity of other modes involved a less symmetrical shape of the peak. In Figure 6, the FRFs measured considering different preloads are shown. These FRFs were obtained with the accelerometer placed on the top of the lower joint. The peaks located between 6 Hz and 14 Hz correspond to the tension-compression mode.

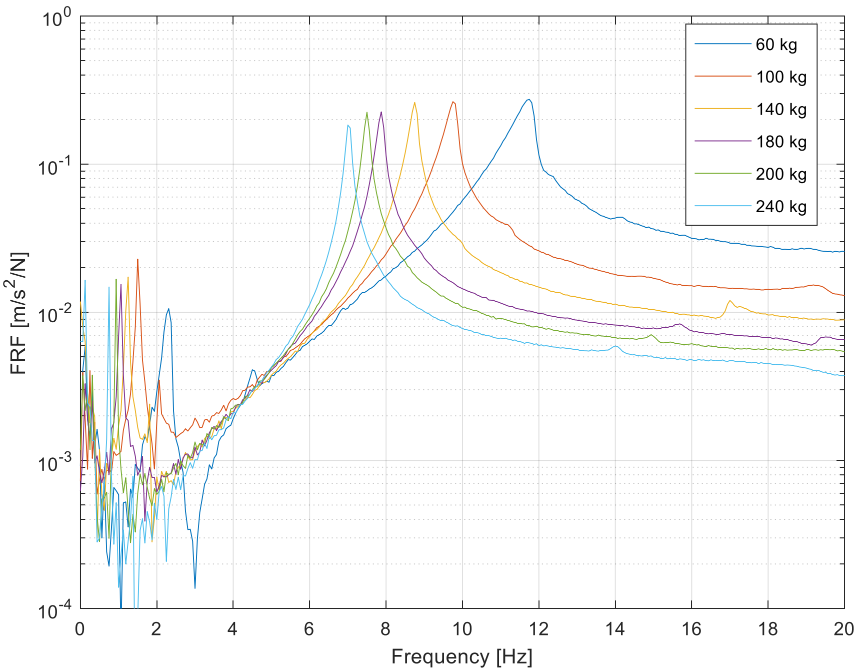


Figure 6: FRFs for different preloads

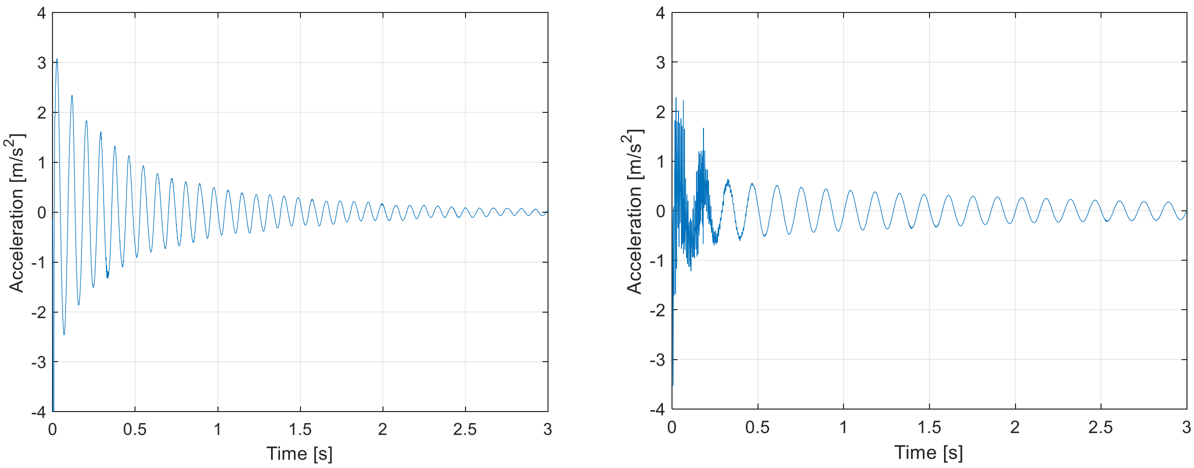
The results show that at higher preloads the FRFs exhibited a more symmetrical peak while in the case of 60 kg preload the asymmetry is significant. This asymmetry may cause either an overestimation or underestimation of damping. Due to the fact that in general it is not evident how this asymmetry is going to occur, it was decided to analyse both sides, right and left. In Eq. 15 and Eq. 16, respectively, the right side HPB method (HPBR) and the left side HPB method (HPBL) are given.

$$\frac{\Delta\omega}{\omega_{\text{res}}} = \frac{2\omega_B}{\omega_{\text{res}}} = \eta \quad (15)$$

$$\frac{\Delta\omega}{\omega_{\text{res}}} = \frac{2\omega_A}{\omega_{\text{res}}} = \eta \tag{16}$$

where ω_A and ω_B are the left and right frequencies in which the amplitudes are $1/\sqrt{2}$ less of the amplitude of the resonance peak. In this case, the asymmetry was located on the right side of the peak resulting in a significantly smaller damping for the case of HPBR. Consequently, the HPBL exhibited greater damping than that of HPB and obviously HPBR.

Concerning the logarithmic decrement, although similar results were obtained without filtering and filtering the signal at low preloads, the difference became substantial as the preload increased. In Figure 7, the free response of 60 kg preload and 240 kg preload are shown.



a) b)

Figure 7: Damped free response, a) for a preload of 60 kg and b) for a preload of 240 kg

Despite the fact that in the case of the 60 kg preload the noise was irrelevant, it became considerable at the preload of 240 kg. Due to the noise, the exponential decay fitting overestimated the damping. Therefore, when a filtering process was carried out the damping decreased as the preload increased, as expected. Thus, in this case, a filtering process was necessary in order to identify the damping values correctly.

The damping estimation in terms of the EMA was carried out considering the five FRFs measured on the longitudinal direction in order to avoid the contribution of the transversal modes. These results agreed well with the values obtained by both the HPB method and the logarithmic decrement method. A maximum deviation of 20% approximately was observed between the logarithmic decrement method and EMA. The former exhibited the greatest damping values and the latter the lowest ones. In Figure 8 the comparison of the three methods is shown. In the case of the HPB method, the results calculated by the original method were considered.

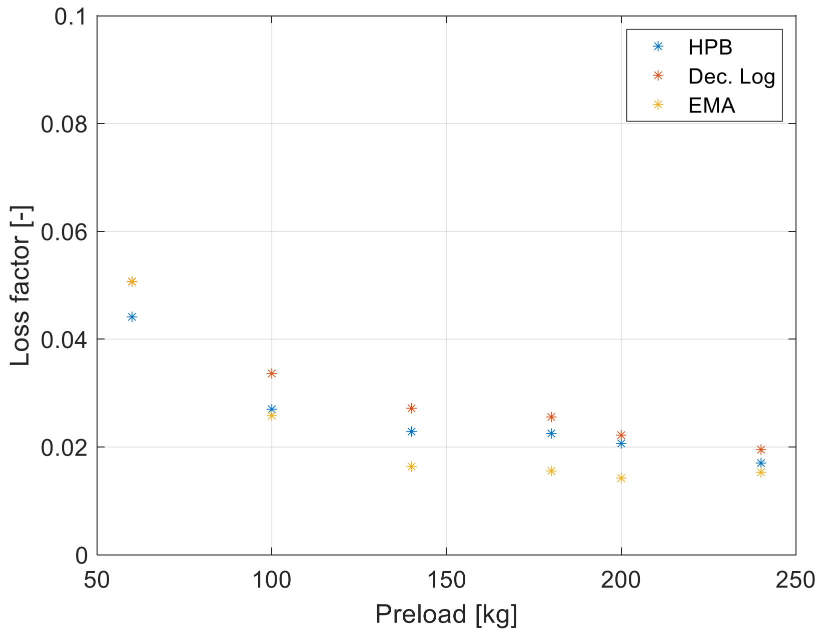


Figure 8: Comparison of the damping estimated by the methods proposed in terms of preload

In addition to the preload analysis, an estimation of damping values in terms of length was carried out. In this case, two methods were employed, the logarithmic decrement method and the HPB method. The preload analysis was carried out with a length of 2.5 m. Therefore, the five accelerometers used to realise the EMA method were placed every 0.5 m. However, this positioning was not feasible for lengths smaller than 2.5 m since a change of the accelerometers' placement would be required. In Table 2 the results of the loss factor in terms of the length are shown.

Table 2: Damping results in terms of length

Length	Natural freq. [Hz]	LDO	LDF	HPB	HPBL	HPBR
0.25	24.213	0.085	0.081	0.076	0.076	0.075
0.5	21.914	0.087	0.065	0.065	0.068	0.062
0.75	19.046	0.068	0.055	0.049	0.054	0.044
1	16.937	0.063	0.049	0.045	0.043	0.046
1.25	15.556	0.060	0.041	0.038	0.044	0.033
1.5	14.570	0.084	0.048	0.050	0.063	0.037
1.75	13.431	0.055	0.037	0.033	0.038	0.027
2	12.723	0.045	0.034	0.031	0.033	0.029
2.25	12.065	0.045	0.032	0.029	0.031	0.027
2.5	11.709	0.051	0.025	0.022	0.027	0.018

The loss factor exhibited a decrease as the length increased. When the length was increased the natural frequency was decreased. Thus, the rope exhibited an increase of the loss factor in terms of frequency. This behaviour could be associated to the viscoelastic nature of the coating. In addition, at smaller lengths the influence of the joints might not be negligible and thus, the increment of the loss factor could be related to both the viscoelastic behaviour and the proximity of joints. In the case of the HPB methods, both the HPBL and HPBR, resulted in an overestimation and underestimation, respectively, at high lengths especially. According to the logarithmic method, a previous filtering process was necessary as in the preload analysis. In Figure 9, the comparison of the two methods is shown where a maximum deviation of 6% is obtained between both methods. Considering that in the case of the EMA method, several points had to be measured and in the case of the logarithmic decrement method a previous filtering process was required, the HPB method seemed to be the most interesting procedure.

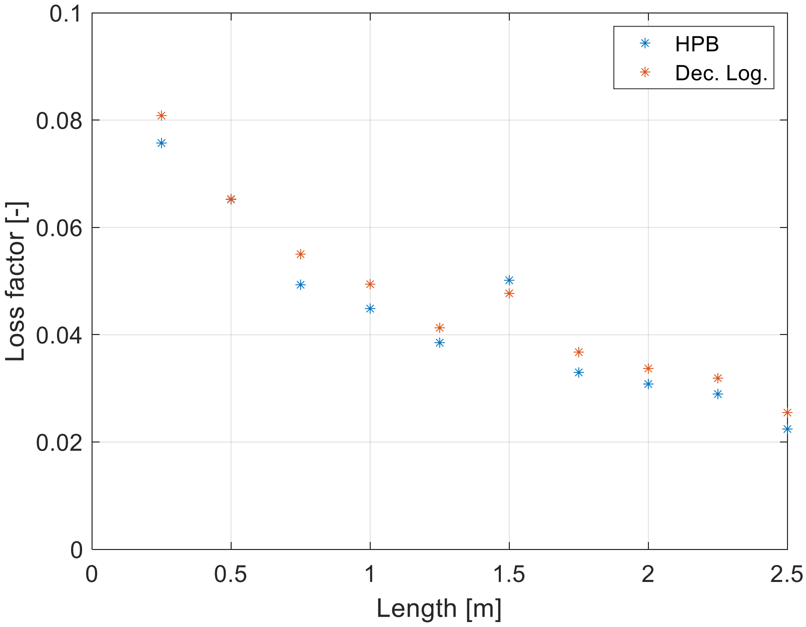


Figure 9: Comparison of the damping estimated by the methods proposed in terms of length

4 Conclusions

In this paper a characterisation of the damping of an elevator rope was carried out. Three methods of damping estimation were compared; the HPB method, the logarithmic decrement method and the EMA method in terms of both preload and length. In the case of the HPB method, it was observed that the asymmetry could result in an overestimation and underestimation of the loss factor. Concerning the logarithmic decrement method, at high preload values, a greater influence of the higher frequency content was found. Therefore, it was necessary to filter the signal previously. Both in the preload analysis and length analysis, the damping exhibited a decrease as the preload or length increased. A maximum deviation of 20% between the three methods was obtained in terms of the preload where the logarithmic decrement method exhibited the greatest loss factor values and the EMA method the lowest ones. In the case of the analysis of the length, the deviation between the HPB method and logarithmic decrement method was not greater than 6%. Therefore, it was concluded that the HPB method is the most interesting method since it does not require any filtering and the response of one accelerometer is enough to estimate the loss factor.

Acknowledgements

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