# BEMAO: A novel adaptive and high-order BEM solver for steady-state acoustics – part 2: applications

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## Abstract

This paper presents a novel adaptive and high order implementation of the Boundary Element Method (BEM) for steady-state acoustics, which we refer to as BEMAO. The method uses a priori error estimators to automatically adapt the p-order of the elements based on the target accuracy, edge lengths and input frequency. Using high order interpolation shape functions (up to order 6) compresses the model size substantially. In addition, BEMAO is also able to use a single non-homogeneous mesh to cover a wide frequency range while it provides optimal model size per frequency without the need for remeshing. The assembly cost is reduced via the use of multipole approximations. A direct solver is used to solve the compressed models. All these properties make BEMAO a novel and competitive BEM solver. The Part 2 of the BEMAO papers focuses on industrial applications: noise radiation from aircraft propellers, automotive pass-by noise, submarine TES (Target Echo Strength), loudspeaker sound.

## 1 Introduction

Since decades harmonic acoustic simulation is a part of standard product engineering and becomes more and more integrated early in the design process through digital twin modeling. The classical technologies for low- and mid-frequency are well known, accurate, and mature, as variations of the Finite Element Method (FEM) and the Boundary Element Method (BEM). Choosing between FEM and BEM may depend on several factors. For instance, FEM is the method of choice when the fluid in which the sound propagates is non-uniform. Furthermore, for some physical systems BEM would lead to smaller but denser equation systems compared to the larger but sparser system matrices in FEM. However, BEM does present some advantages over FEM. Especially, model preparation is easier in BEM. As opposed to FEM, BEM only requires a surface mesh on the geometric boundaries. In addition, BEM does not require ad-hoc treatment for exterior acoustics radiation of unbounded domains, as the Sommerfeld radiation condition is directly handled through the formulation. Also, for applications with large dimensions between the acoustic scatterer and reflective surfaces, BEM allows to keep the model size reduced whereas FEM would require filling the space with numerous 3D elements, making the model excessively large. So, altogether, BEM is generally an elegant option for free-field acoustic radiation analysis.

A difficulty that many acoustic simulation methods face, and BEM as well, is the exponential growth of the computational requirements with the model size and frequencies. For several applications, BEM had to be complemented with other technologies to cover the full audible range. Growing model size became problematic for applications with larger geometries, such as automotive pass-by noise or aircraft landing noise analyses. Solutions have been developed, for instance through significant progress made in terms of hardware with much more powerful computers available on the market and at less prohibitive prices. At the same time, the classical BEM and FEM approaches have been enhanced to technically address these limitations related to larger model size. In practice, the first consideration in terms of efficient modeling regards the acoustic mesh: it must be fine enough to correctly describe the geometry and accurately represent the field at the solving frequency (the classical rule-of-thumb requires 6 linear elements per acoustic wavelength). Using a single fine mesh for the full frequency range results in unnecessarily expensive computations at lower frequencies, whereas using multiple meshes for different frequency bands impacts the preparation time negatively, even if this process is automated, since a repeated re-meshing process would cost additional time. None of these options are ideal.

The authors propose an alternative technique, the Boundary Element Method with Adaptive Order (BEMAO), following the model of the Finite Element Method with Adaptive Order (FEMAO) [1]. This approach provides engineers with an elegant method that automatically adapts the necessary model size to the calculation frequency. As a result, the solution is much faster for the lower frequencies and more efficient for the higher frequencies, in comparison to standard methods. BEMAO is based on using high-order polynomial shape functions in the discretization of the indirect BEM formulation. Consequently, a single lean coarse mesh can be used to solve for a broad frequency range by relying on the polynomial order adaptivity: at each frequency, the solver automatically selects the appropriate order within each edge and element, based on the local size and speed of sound, and an a-priori target error estimator. This ensures an optimal model size to guarantee both sufficient accuracy and best performance. In practice, engineers only need to input a single coarse mesh and a desired accuracy, further helping to democratize the usage of acoustic simulations. An ideal model size is hence obtained for each frequency. As a result, broadband acoustics analysis can be performed with optimal performance, for the full frequency range, and with minimum user intervention. The technical details explaining this novel adaptive and high-order BEM solver for steady-state acoustics, can be found in the first part of this dual paper [2]. In this paper here, the authors focus on showing some industrial applications to illustrate the potential of the BEMAO approach.

In Section 2, a short summary of the BEMAO theory is described, which can be completed with reviewing Part 1 [2]. Then, the following sections will present several test cases covering different industries: a business jet in approach phase at the height of 100 m and the computation of the sound radiated to the ground; a submarine hit by acoustic plane waves to compute the Target Echo Strength (TES); a car pass-by noise application to calculate the transfer functions between the acoustic sources by the vehicle to the microphone side lines; and finally, a loudspeaker application with the sound radiation from a vibrating diaphragm.

## 2 BEMAO – Summary

A short summary of the key aspects of the BEMAO theory is shared in this section. More details can be found in the first part of this duo of papers [2]. The BEMAO implementation uses an indirect variational formulation of BEM. The main objective of BEMAO being to optimize the model size for a given mesh and a given frequency, two main concepts are used: a non-isoparametric approach and an a-priori error estimator. With a non-isoparametric technique, the polynomial orders of the geometry and field interpolation shape functions are disconnected. This allows to keep the geometry definition unchanged in a frequency sweep, while the orders of the interpolation shape functions are adjusted to the frequency per element. In BEMAO, this automatic $p$-adaptivity (with $p$ the polynomial order of the interpolation shape functions) uses an a-priori error estimator with adaptivity rules based on the element size, the acoustic wavelength, and the target accuracy. Based on these inputs, tabulated values are scanned, and the optimal order is returned by the solver.

A difficulty inherent to the BEM technique is due to numerical singularities in its kernel. Special quadrature rules are applied, in particular with advanced coordinates transformations with Jacobians cancelling out these kernel singularities [3]. A main advantage of these Sauter rules is that they allow integration on linear and quadratic geometric elements, for both triangular and quadrangular shapes. Therefore, BEMAO supports all combinations of those elements, and hybrid meshes (TRIA3, TRIA6, QUAD4, QUAD8). To balance accuracy and computational costs, a semi-empirical specific quadrature order rule is applied to

BEMAO, ensuring that the quadrature order does not exceed the interpolation error imposed by the order of the shape functions.

Finally, one more challenge associated with BEM is that all the boundary elements interact with each other, resulting in an $O(N^2)$ complexity for the standard assembly procedure. BEMAO fully exploits the power of high order shape functions and compresses the size of the model, therefore substantially reducing the matrix solving time of the dense system. But using high order shape functions still requires a costly system assembly step. Exploiting the property that the contribution from far-away elements decreases with increasing distance, a multi-level fast-multipole based acceleration is used for BEMAO system assembly. This algorithm significantly reduces the assembly cost while robustly controlling the error of the multipole approximation.

# 3 Uncoupled Acoustic Applications

In this section, a couple of applications are described, which involve uncoupled acoustics only. Section 4 will show an application with fluid-structure coupling. In the following, 3 examples are shown, starting with the business jet case, followed by the submarine scattering example, and finally the car pass-by noise application. Note that all the computations are run with the Simcenter 3D software, on a Windows workstation with Intel(R) Xeon(R) Silver 4210R CPU @ 2.40 GHz processors and 256 GB RAM.

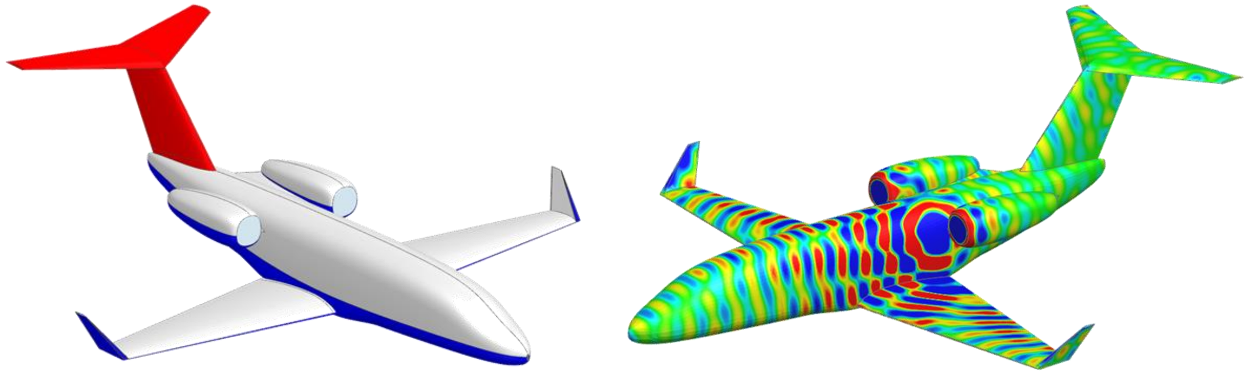

Figure 1: Business jet ground noise – Geometry and example of acoustic pressure contours.

## 3.1 Business Jet Ground Noise

This first example focuses on a business jet in approach phase at the height of 100 m to the ground. The airplane is 15 m long, spans over 15.5 m, with a vertical stabilizer reaching the height of 4 m. Figure 1 shows the geometry of the business jet and an example of the acoustic pressure field around the aircraft at 500 Hz. The sound sources are 2 acoustic monopoles of unit amplitude (1 N/m), each of which is placed in front of one of the propellers. In addition, a symmetry plane boundary condition is added to the model to account for the sound reflections on the ground. A broadband frequency analysis is run between 20 Hz and 500 Hz, with a frequency step of 1 Hz, meaning 481 solving frequencies. To compute the acoustic pressure field 2 sets of microphones are located near the ground, at a height of 1.5 m: 2 lines of microphones at 450 m on either sides of the airplane, and 1 square plane of microphones of 30 m side. A total of 42805 microphones is present in the model.

Next, discretizing the geometry with an appropriate mesh is very important. Two different meshes are produced for this example, one that is suited for classical methods such as the standard BEM and H-Matrix BEM techniques, and one mesh that is fit for BEMAO (see Figure 2). The first mesh is composed of 41834 linear triangular elements and 20919 nodes, with a characteristic mesh size of 100 mm with smaller elements near the more complex geometry areas, which is fine enough to solve up to 500 Hz following the classical rule of thumb of 6 linear elements per acoustic wavelength. Note that the air properties are taken at 15 °C, with a speed of sound of 340 m/s and a mass density of 1.225 kg/m$^3$. The second mesh is much coarser and contains 3614 parabolic triangular elements and 7230 nodes: the element size in this mesh varies between 50 mm and 1000 mm. In order to damp internal resonances which may occur with closed meshes, a characteristic impedance boundary condition is also applied on the inner side of 20 % of the acoustic elements.

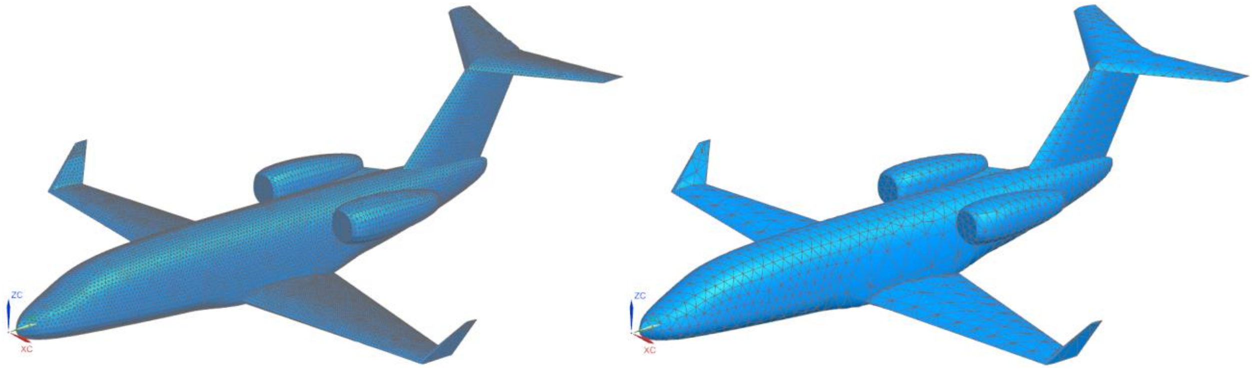

Figure 2: Business jet ground noise – Fine (standard BEM, H-Matrix BEM) and coarse (BEMAO) acoustic meshes.

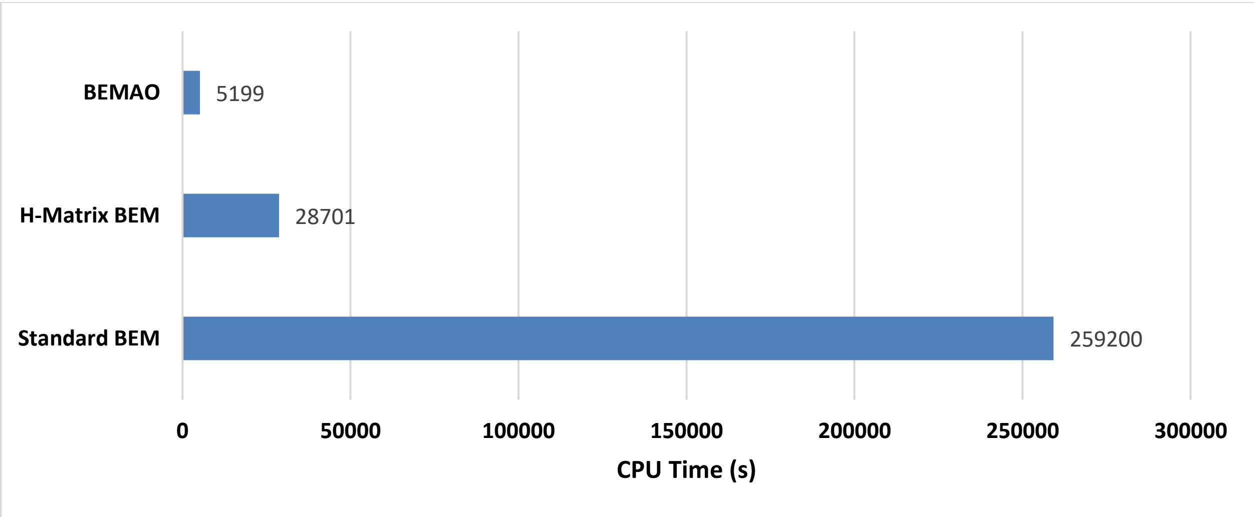


Figure 3: Business jet ground noise – Computational times.

The computational times for this scenario are reported in Figure 3. These are obtained with parallel processing, with 5 threads and 4 processors. With the standard BEM method, the solution is computed in about 72 hours (estimation based on the time spent to solve 66 frequencies). This performance is improved with the H-Matrix BEM technique which solves the job in 7 h 58 min 21 s. A drastic improvement is then obtained with BEMAO, which solves this business jet scenario in 1 h 8 min 12 s, i.e., 5.5 times faster than the H-Matrix BEM run, and 50 times faster than the standard BEM run. The performance gain is remarkable and allows to drastically reduce the time spent in computing the solution, and for engineers to fully focus on results analysis. A comparison of the acoustic pressure field along the microphone line on the right side of the airplane at 500 Hz is shown in Figure 4. The acoustic pressure at the microphone on the front side of the aircraft near the ground is also displayed on the right-hand side graph, for the full frequency range. Some very fine oscillations can be observed, which can be explained by interference phenomena. A good agreement between the solutions can be found. Note that an attempt was made to solve for this scenario with FEMAO; the difficulty here is that the space between the aircraft and the ground, which is 100 m in this case, needs to be filled with 3D elements. This leads to an acoustic mesh of 561048 nodes, and polynomial orders up to 8 at the maximum frequency 500 Hz. But the solve could not proceed because of lack of memory. This shows the benefits that BEMAO could bring on some applications where FEMAO exhibits certain limitations.

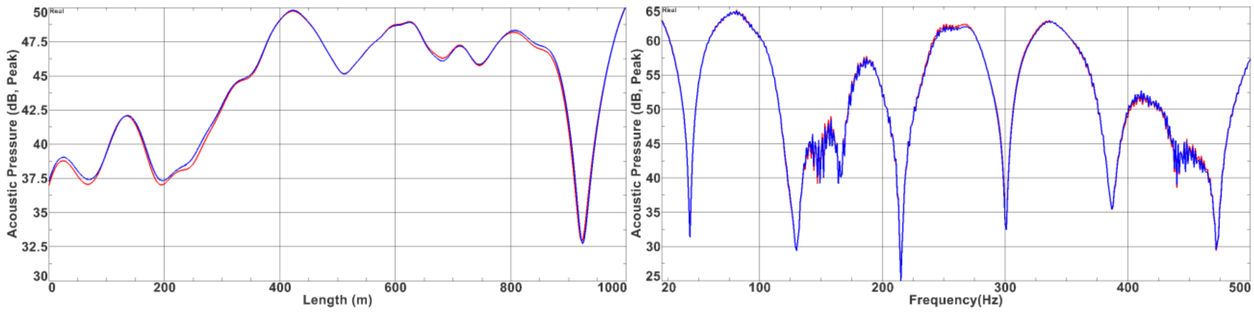


Figure 4: Business jet ground noise – Left: Acoustic pressure along the microphone line on the right side of the aircraft at 500 Hz. Right: Acoustic pressure at the microphone on the front side of the aircraft. Red: BEMAO. Blue: H-Matrix BEM.

## 3.2 Submarine Target Echo Strength

The second test case is a submerged submarine about 62 m long, and of radius 3.5 m on its main body. The bow and the tail cone follow ellipse and circle segment profiles, respectively. The fin of the submarine is a structure of 13 m long and 3.5 m high. The rudders complete the submarine geometry, as shown in Figure 1. An example of the acoustic pressure field obtained from an acoustic plane wave impinging on the submarine is also displayed at 1 kHz. In this scenario, 360 acoustic plane wave excitations with an amplitude of 1 Pa each are distributed at 0 ° elevation on a circle of radius 1000 m centered on the submarine at every angle all around the submarine. The solution is computed at 10 frequencies, from 100 Hz to 1000 Hz with a step of 100 Hz, and the acoustic pressure is calculated on a circular microphone mesh centered on the submarine and at a distance of 100 m. 360 microphones are present in the model, 1 at each angle from 1 ° to 360 °.

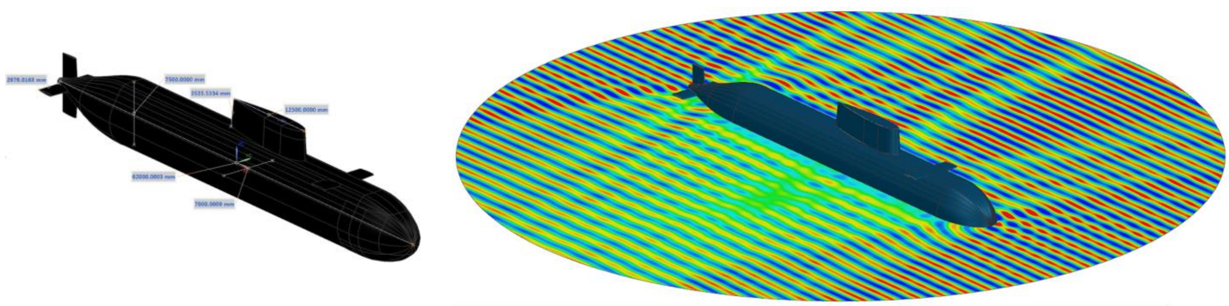

Figure 5: Submarine target echo strength – Geometry and example of acoustic pressure contours.

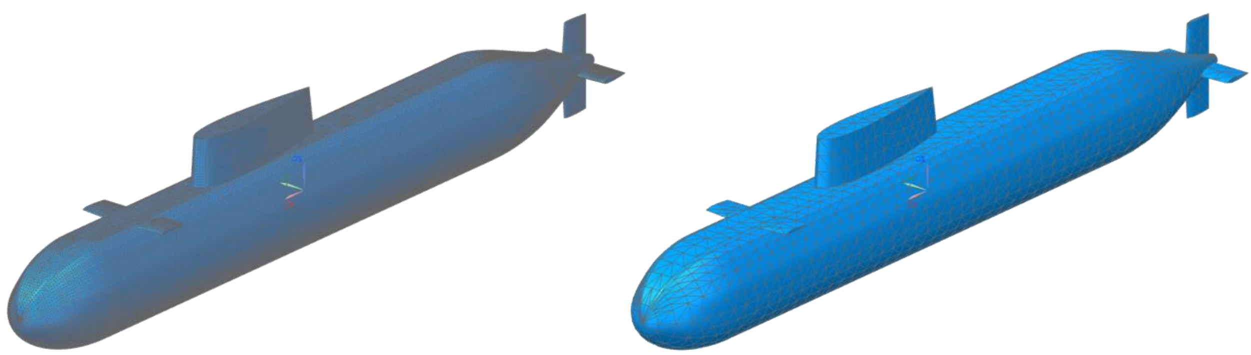

Figure 6: Submarine target echo strength – Fine (standard BEM, H-Matrix BEM) and coarse (BEMAO) acoustic meshes.

Like in Section 3.1 for the business jet, two meshes are produced, one fine mesh suitable for the low-order H-Matrix BEM technique and one coarse mesh appropriate for BEMAO (see Figure 6). The fine mesh is composed of 61034 linear triangular elements and 30519 nodes, with a characteristic mesh size of 250 mm, which is fine enough to solve up to 1000 Hz according to the classical rule of thumb of 6 linear elements per acoustic wavelength. Note that the water properties are taken with a speed of sound of 1500 m/s and a mass density of 1000 kg/m$^3$. The coarse mesh contains 2110 parabolic triangular elements and 4222 nodes, with a characteristic mesh size of 1500 mm, which is the acoustic wavelength in water at 1000 Hz. A characteristic impedance boundary condition is also applied on the inner side of 20 % of the acoustic elements to damp potential internal resonances.

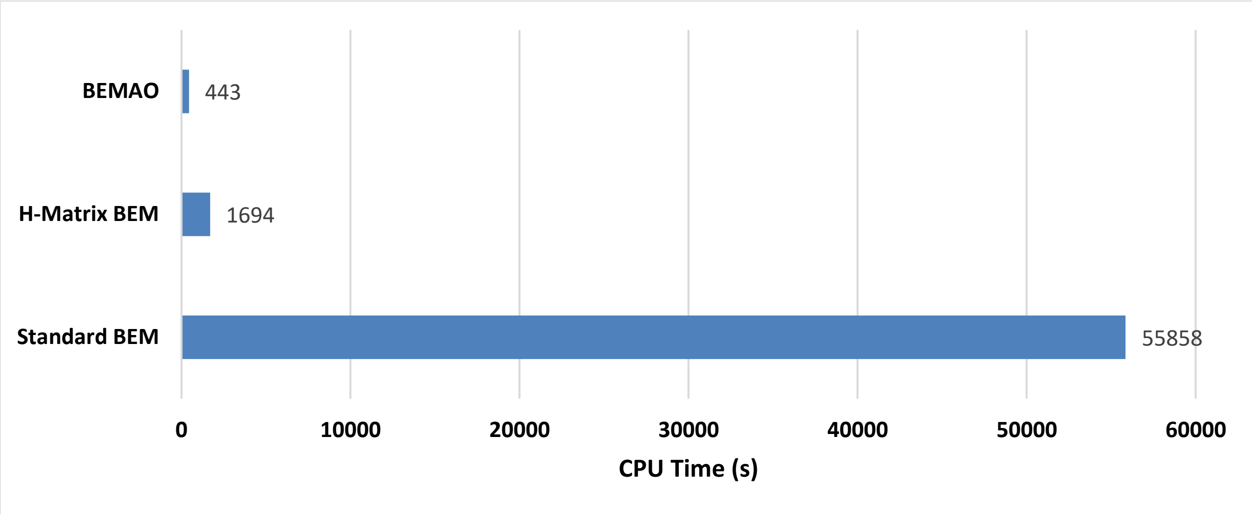


Figure 7: Submarine target echo strength – Computational times.

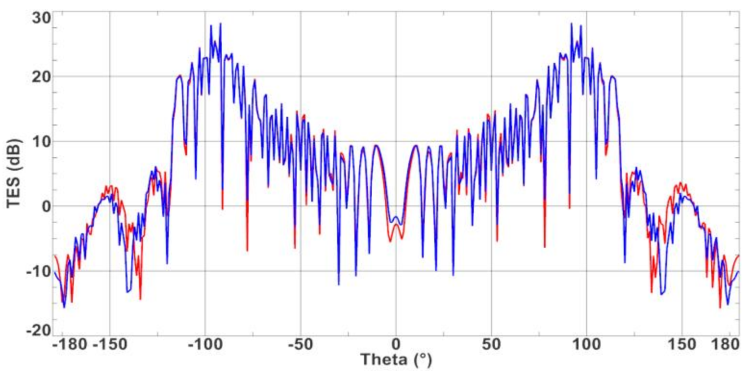


Figure 8: Submarine target echo strength – TES. Red: BEMAO. Blue: H-Matrix BEM.

The calculations are done using multi-threading with 20 threads and Figure 8 reports the computational times for the different solvers. BEMAO only takes 7 min 23 s to solve for the 10 frequencies, with 22 s to solve at 100 Hz (only order 1 elements) and 110 s to solve at 1000 Hz (with elements mainly of orders 3 and 4). With the H-Matrix BEM solver, the solving time is 28 min 14 s, whereas it is 15 h 30 min 58 s with the standard BEM method. For this scenario, BEMAO is then 4 times faster than the H-Matrix BEM and 126 times faster than the standard BEM run. In terms of results, the Target Echo Strength (TES) is calculated based on the acoustic pressure field obtained at the microphone points on the circle mesh around the submarine. The TES is a measure of the submarine stealth and is defined based on the incident and scattered pressure fields, $p_{\mathrm{i}}$ and $p_s$ respectively [4]:

$$\mathrm{TES} = 20\log_{10}\left(\frac{r}{r_0}\frac{|p_{\mathrm{s}}|}{|p_{\mathrm{i}}|}\right), \tag{1}$$

With $r$ the radial distance between the target and the receiver and $r_0 = 1$ m the reference radius. Figure 9 shows the TES found with BEMAO in red and with H-Matrix BEM in blue. The results match well over the whole range of azimuth angles.

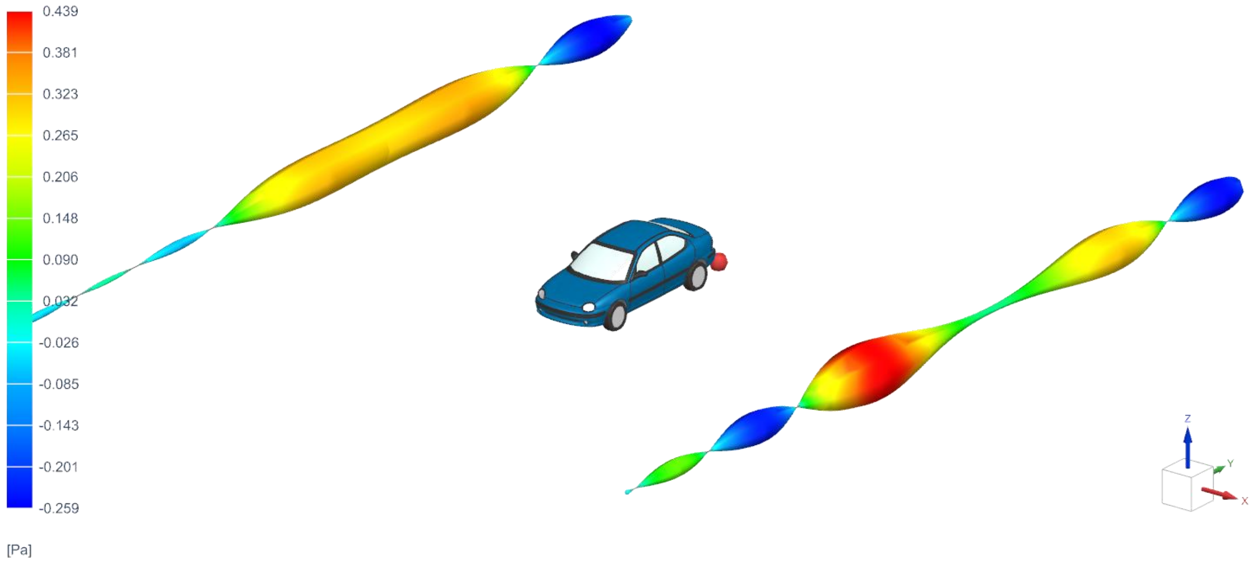


Figure 9: Car pass-by noise – Geometry and example of acoustic pressure along the microphone lines.

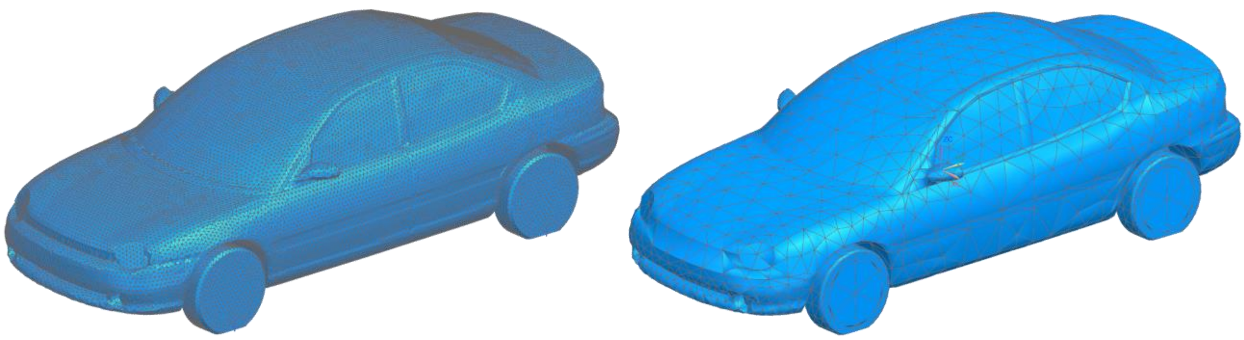

Figure 10: Car pass-by noise – Fine (H-Matrix BEM) and coarse (BEMAO) acoustic meshes.

## 3.3 Car Pass-By Noise

A third uncoupled acoustic example is presented in this section. In the automotive industry, car manufacturers are interested in estimating the pass-by noise of the vehicles under certain conditions. This can be done through measurements in open air or in wind tunnels, or through simulation. It consists in obtaining the acoustic transfer functions between sound sources and a series of microphones. The sound sources are typically represented by unit-strength acoustic monopoles, and placed where the relevant noise generation is observed, for instance near the tires or by the exhaust. The microphones consist of 2 lines on either side of the vehicle, at a height of 1.2 m and a distance of 7.5 m from the center line of the car, according to the ISO standards [5]. A car of about 4.5 m long and 1.3 m high is considered in this pass-by noise scenario, as shown in Figure 9 where the real part of the acoustic pressure along the side microphone lines is displayed. The simulations are run for 5 load cases, each of which with a single unit-strength acoustic monopole source placed at different locations: near the front left tire, the front right tire, the back left tire, the back right tire, and the exhaust. The acoustic transfer functions are obtained for 251 frequencies, logarithmically distributed between 20 Hz and 2000 Hz. A symmetry plane boundary condition is applied to mimic the ground reflections, and the potential internal resonances are damped with a characteristic impedance boundary condition applied on the inner side of 20 % of the acoustic elements.

As shown in Figure 10, two meshes are created: a fine mesh for the low-order H-Matrix BEM technique contains 65998 linear triangular elements and 33067 nodes, whereas the BEMAO coarse mesh is composed of 3226 quadratic triangular elements and 6506 nodes. The characteristic mesh sizes are derived from the acoustic wavelength at the maximum solving frequency: hence at 2000 Hz, with an acoustic wavelength of 170 mm, the mesh size with H-Matrix BEM is 25 mm, whereas with BEMAO it is 250 mm. The microphone 1D lines are composed of 4002 nodes in total, hence 1 microphone every 10 mm, over a distance of 20 m.

Figure 11 reports the computational times obtained with the 2 techniques, using parallel processing, with 5 threads and 4 processors. With the H-Matrix BEM approach, the solution is computed in 6 h 16 min 51 s. Again, a drastic improvement is then obtained with BEMAO, which solves this car pass-by noise scenario in 36 min 9 s, i.e., 10 times faster than the H-Matrix BEM run. Results are displayed in Figure 12, with the acoustic pressure at a microphone on the left side of the car for the exhaust noise source and as function of frequency. The acoustic pressure along the microphone line on the right side of the car is also shown. Once more, the BEMAO and H-Matrix BEM solutions provide very similar results.

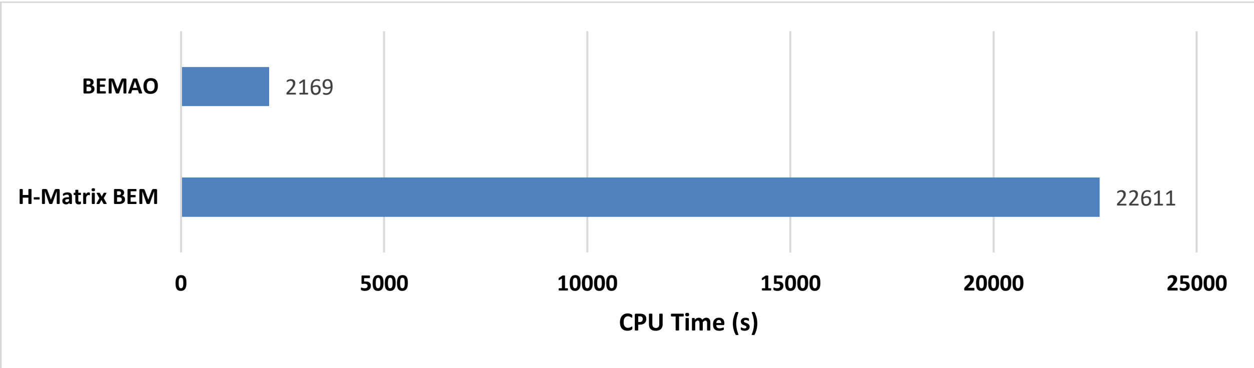


Figure 11: Car pass-by noise – Computational times.

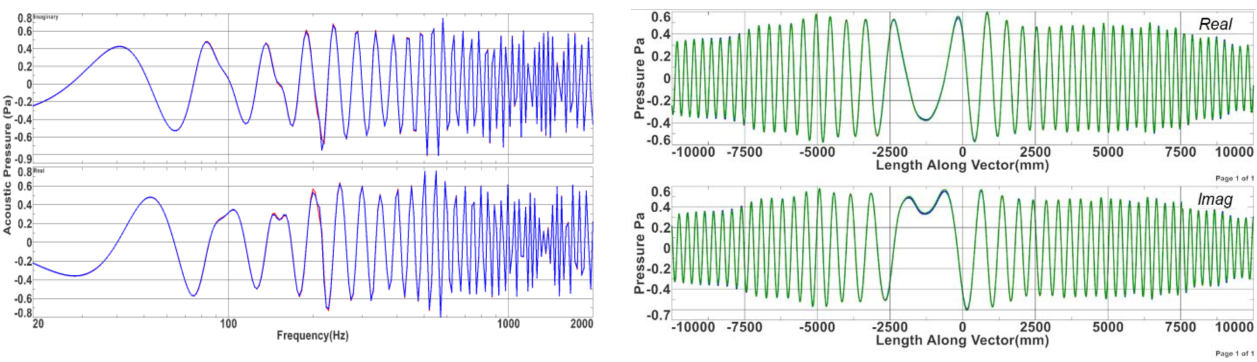


Figure 12: Car pass-by noise – Left: Acoustic pressure at a microphone on the left side of the car. Right: Acoustic pressure along the microphone line on the right side of the car at 1000 Hz. Red/Green: BEMAO. Blue: H-Matrix BEM.

# 4 Vibro-Acoustic Application

In the following two subsections, a vibro-acoustic case is studied, which involves fluid structure interaction. A loudspeaker noise is simulated, as standalone and installed component.

## 4.1 Standalone Loudspeaker Noise

In the electronics industry, the design of loudspeaker is crucial to satisfy users with appropriate noise distribution and quality. In this example, a sound box is considered with a single loudspeaker, including its classical components, i.e., diaphragm, dust cap, voice coil and spider. The vibro-acoustic response is calculated for a force excitation at the end of the coil, with the speaker being constrained at the junction between the diaphragm and the box, as well as around the spider. To compute the structural vibrations coupled with the fluid mesh, a Simcenter Nastran SOL 108 Direct Frequency Response solution is used, and a damping factor of 5 % is applied in the structural model. The vibrations on the diaphragm and the dust cap are then coupled with the fluid mesh in the BEM solutions. The solving frequencies range from 20 Hz to 10000 Hz, with a logarithmic sweep and 1001 frequencies. A symmetry plane boundary condition is applied to mimic the reflections by the support on which the loudspeaker is placed. The microphones at which the acoustic pressure is calculated consist in circles and semi-circles surrounding the loudspeaker, with microphones at every angle, hence a total of 722 microphones.

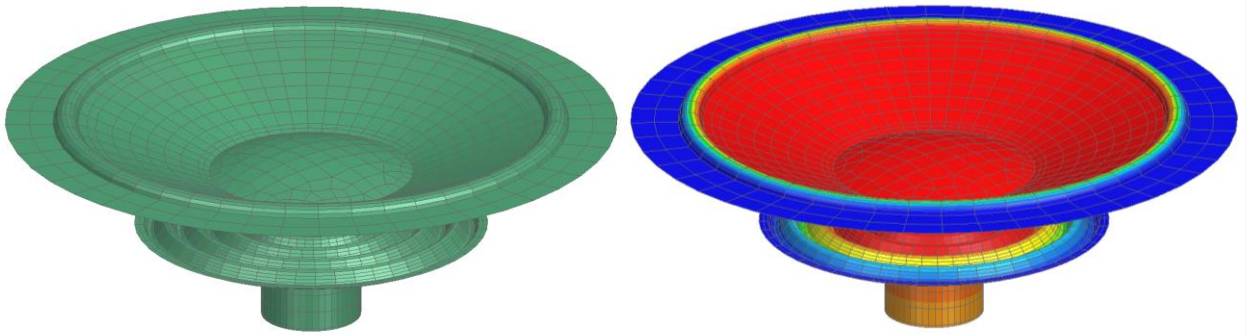

Figure 13: Standalone loudspeaker noise – Structural mesh of the diaphragm and example of the structural deformations.

Two acoustic meshes are created, to couple with the structural vibrations, as shown in Figure 14. The fine acoustic mesh for the low-order H-Matrix BEM technique contains 38758 linear triangular elements and 19396 nodes, and the BEMAO coarse mesh is composed of 1384 quadratic triangular elements and 2793 nodes. At 10000 Hz, the acoustic wavelength is 34 mm: the characteristic mesh size used in the BEMAO mesh is 35 mm, and it is 5 mm in the H-Matrix BEM mesh.

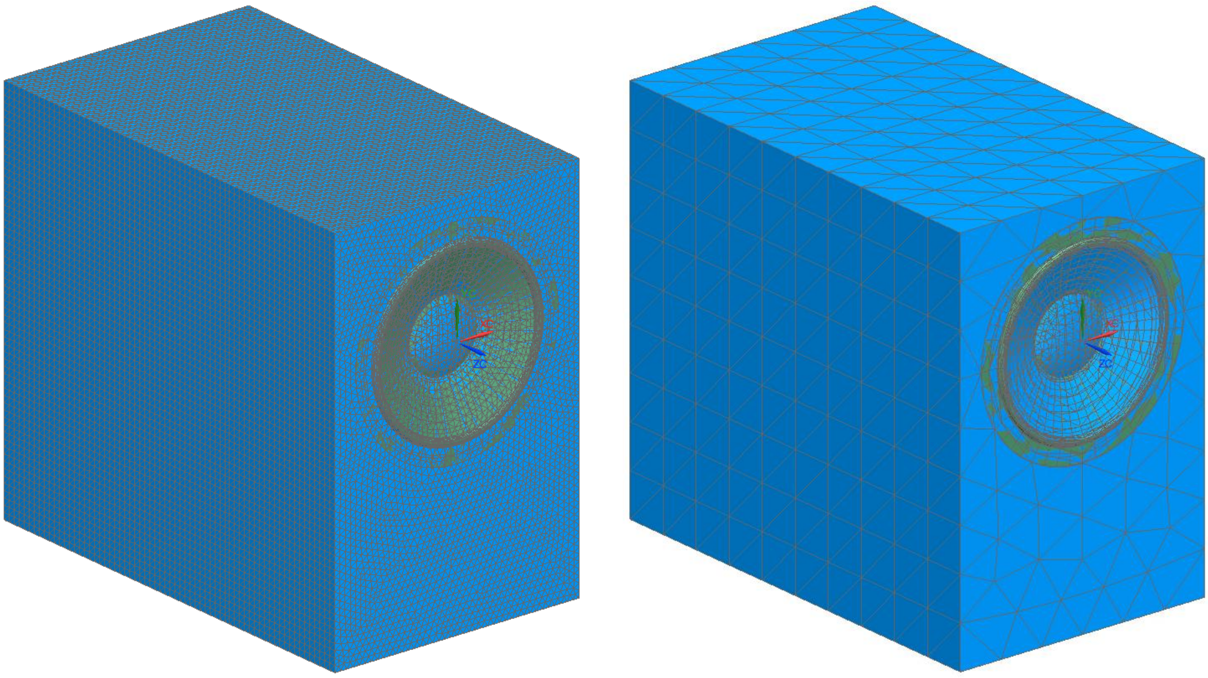

Figure 14: Standalone loudspeaker noise – Fine (H-Matrix BEM) and coarse (BEMAO) acoustic meshes.

The computational times reported in Figure 15 are obtained using parallel processing, with 10 threads and 2 processors. With the standard BEM solver, the timing is derived from 2 frequencies which solved in 23 min 15 s: extrapolating to 1001 frequencies, the estimation is that this scenario would solve in almost 200 hours. With the H-Matrix BEM approach, the solution is computed in 33 h 54 min 46 s, hence about 6 times faster than the standard BEM technique. In comparison, BEMAO solves way faster, in 30 min 57 s, i.e., 66 times faster than the H-Matrix BEM run.

The results displayed in Figure 16 show the acoustic pressure at a microphone on the front side of the loudspeaker and as function of frequency. Very similar results are obtained between BEMAO and H-Matrix BEM, with some discrepancies at higher frequencies. Note that the red and blue curves are the results obtained with weak coupling, for a fair comparison between the 2 solvers, since strong coupling is not available with the current implementation of the H-Matrix BEM solver in Simcenter 3D. To give a view on the impact that the fluid has on the membrane of the loudspeaker, the green curve shows the acoustic pressure obtained with strong fluid-structure coupling. To do so, the structural part of the loudspeaker is represented with mode shapes obtained from a Simcenter Nastran SOL 103 Real Eigenvalues solution, and then loaded within the BEMAO vibro-acoustic model. The effects at low frequencies are remarkable, with the 2 peaks around 60 Hz and 70 Hz being completely damped out.

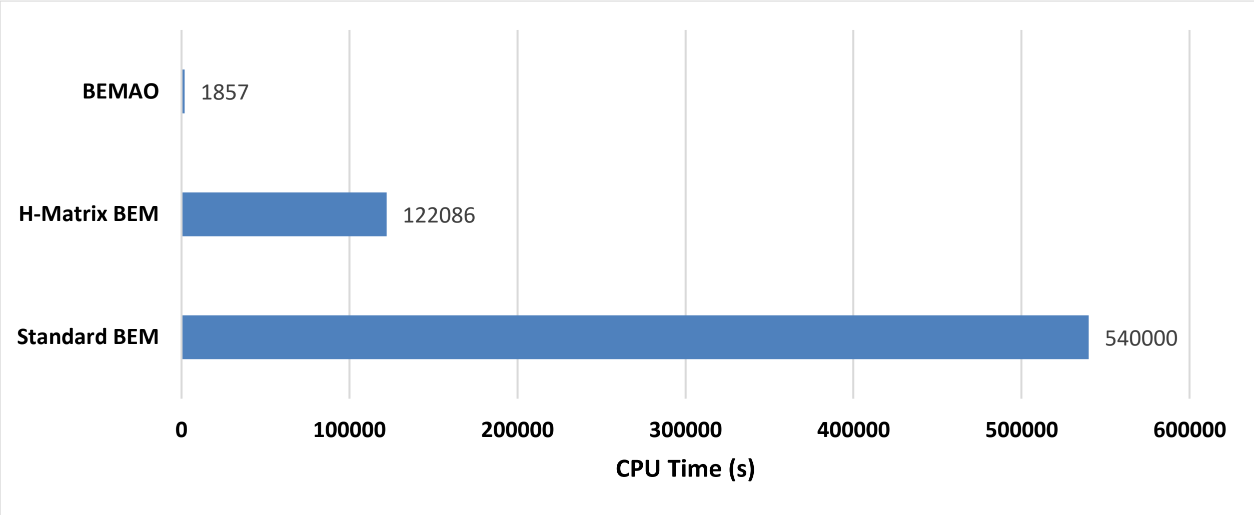


Figure 15: Standalone loudspeaker noise – Computational times.

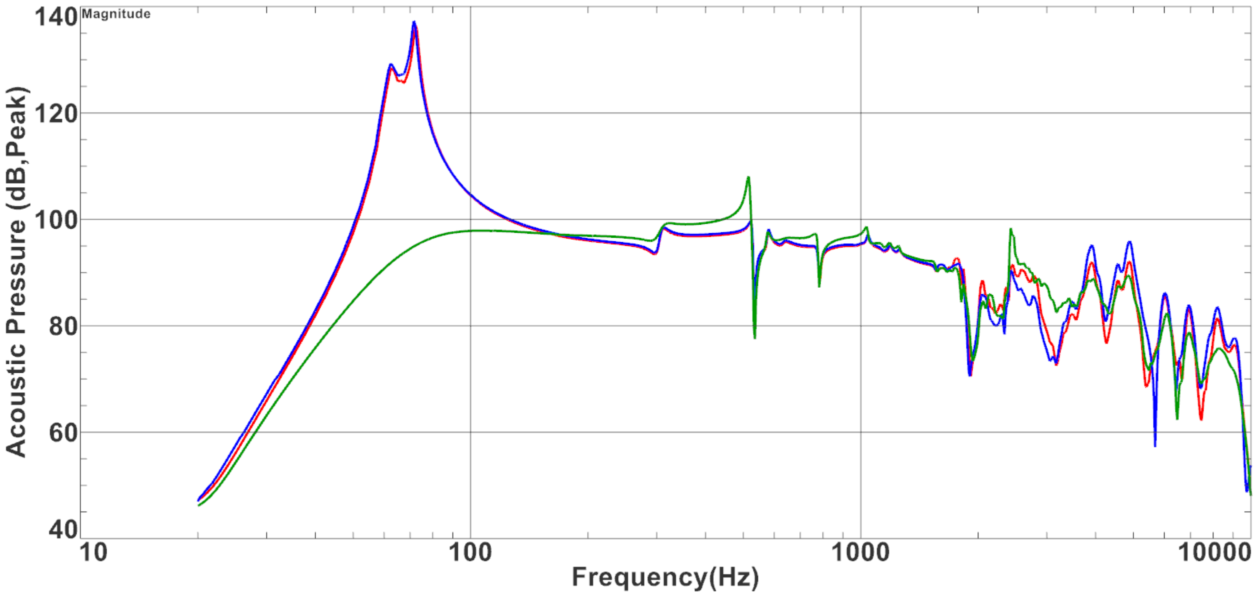


Figure 16: Standalone loudspeaker noise – Acoustic pressure at a microphone on the front side of the loudspeaker. Red: BEMAO (weak coupling). Blue: H-Matrix BEM (weak coupling). Green: BEMAO (strong coupling)

### 4.2 Installed Loudspeaker Noise

Finally, one last scenario is presented here, which consists in computing the sound radiated from 2 loudspeakers installed on a table. The loudspeakers and their structural excitations are the same as in Section 4.1. The table is discretized with an acoustic mesh, and orthogonal symmetry planes are inserted to represent the reflections from the ground and from the wall against which the table is placed. Three microphone planes are created for the visualization of the results: one plane parallel to the ground and near the ground, one plane parallel to the wall and near the wall, and one vertical plane cutting the model in half. A total of 74270 microphones is present in the model. An example of the real part of the acoustic pressure field is shown in Figure 17. The reflections against the wall and the ground are visible, as well as the radiation pattern in front of the table. The results are computed between 20 Hz and 5000 Hz, with 100 logarithmic intervals. The acoustic meshes are also different in the 2 methods studied with this example: the H-Matrix BEM mesh is composed of a total of 147490 linear triangular elements and 74077 nodes, whereas the BEMAO mesh contains 7455 parabolic elements (mix of triangles and quadrangles) and 20266 nodes. Figure 18 shows the coarse mesh used for the BEMAO solution.

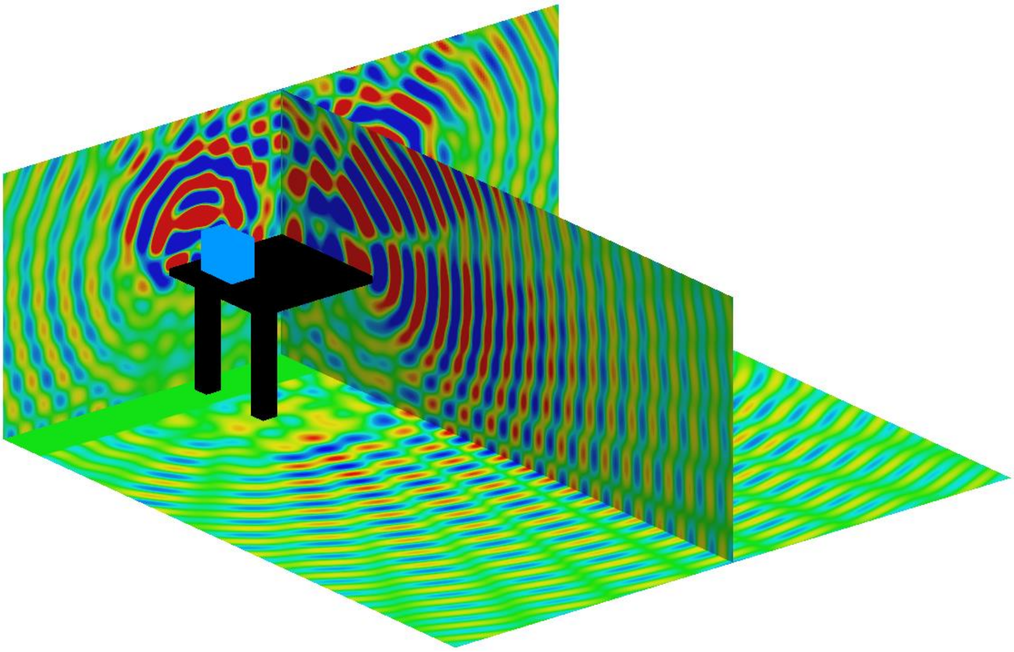

Figure 17: Installed loudspeaker noise – Example of the real part of the acoustic pressure field.

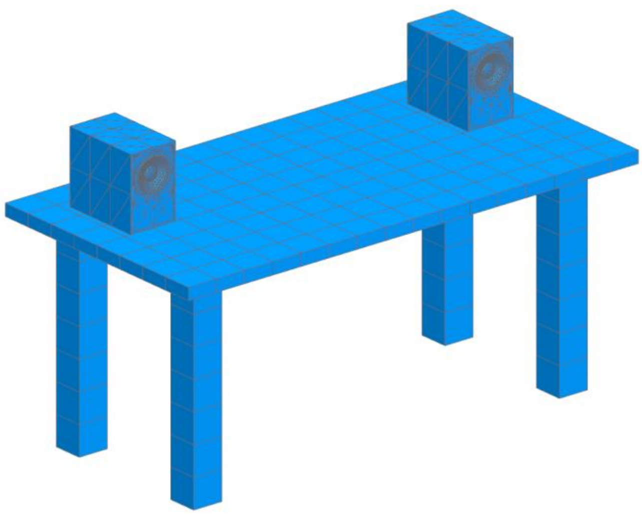

Figure 18: Installed loudspeaker noise – Coarse acoustic mesh used in BEMAO.

For this example, the solutions are obtained using parallel processing, with 10 threads and 2 processors. The H-Matrix BEM solver provided the results after 29 h 14 min and 46 s, and the BEMAO solver took 2 h 25 min 55 s to compute the acoustic pressure, hence a speed-up of the order of 12 (see Figure 19). Note that for the lowest frequencies the distribution of microphones in an octree with the H-Matrix BEM solver was too high for the solver to provide a solution, hence the solution was solved from 100 Hz onwards. Figure 20 shows the acoustic pressure against frequency at a microphone in front of the left-hand-side loudspeaker. A good agreement is observed between the two solutions, between 100 Hz and 3000 Hz. Some artefacts are observed between 3000 Hz and 5000 Hz with H-Matrix BEM.

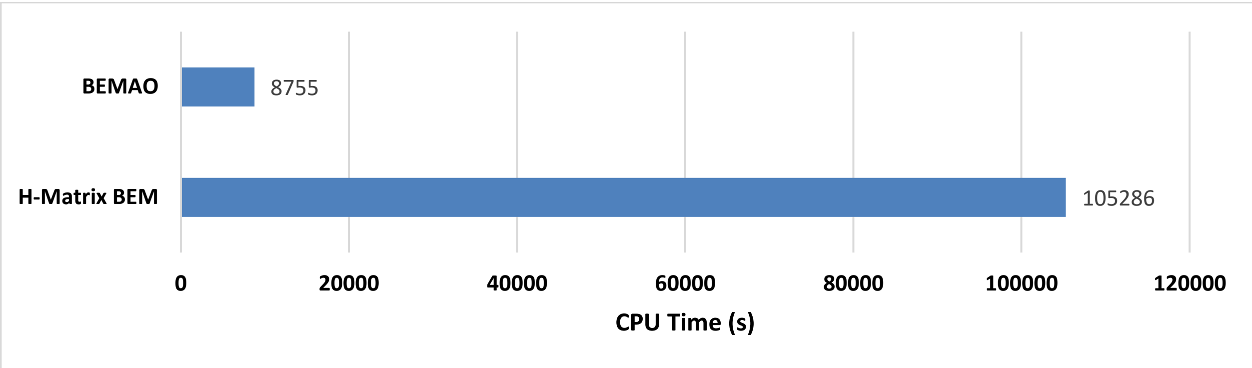


Figure 19: Installed loudspeaker noise – Computational times.

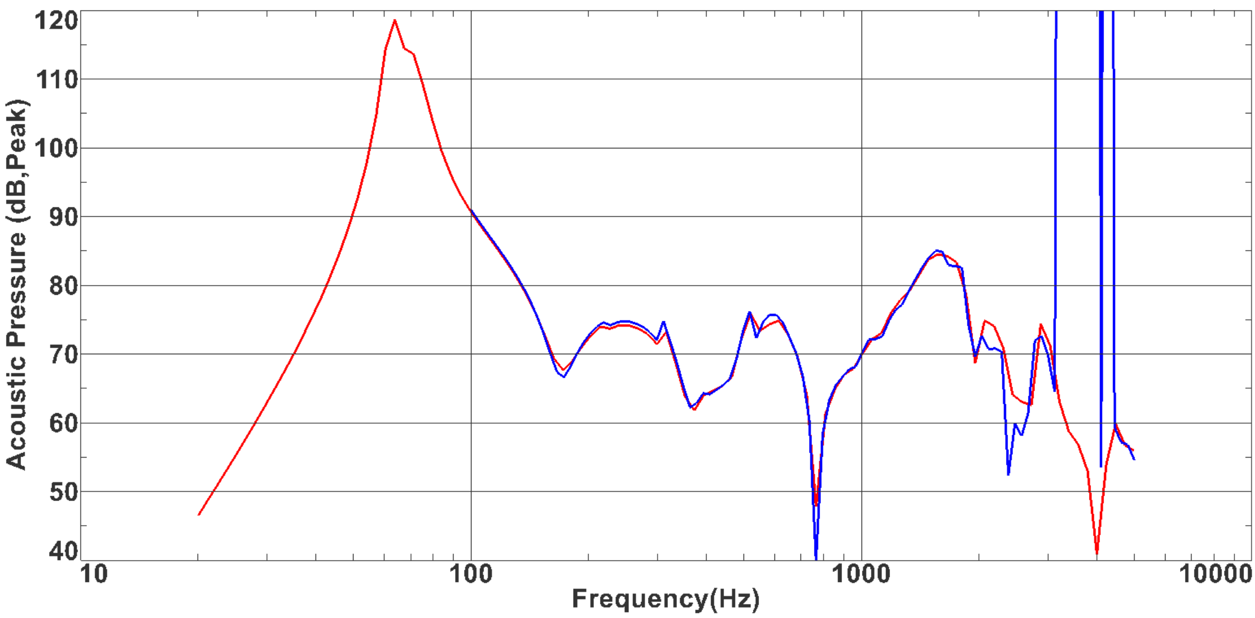


Figure 20: Installed loudspeaker noise – Acoustic pressure at a microphone in front of the left-hand-side loudspeaker. Red: BEMAO. Blue: H-Matrix BEM.

## 5 Conclusion

Realistic examples of acoustic simulations for industrial applications have been used in this paper to illustrate the Boundary Element Method with Adaptive Order, BEMAO. The advantages of a BEM approach are taken to the next level with the addition of adaptive high-order shape functions and of an a-priori error estimator. This way, BEMAO helps obtaining accurate acoustic results for business jet ground noise, submarine TES, car pass-by noise or loudspeaker noise in a very efficient way in comparison to the other classical BEM techniques. The benefit in terms of computational time for frequency sweep analyses is remarkable with speed-up factors of the order of tens. Other applications could also be targeted, such as wind turbine noise, encapsulation, etc.

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