Efficient design of viscoelastic damping treatments

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Abstract
In the context of noise and vibration mitigation, passive damping treatments generally lead to low-cost and robust structural vibration control. In many engineering problems, constrained viscoelastic patches are added to the vibrating structure to increase the amount of damping. The efficiency of such damping treatments depends on the fraction of strain energy in the core layer and the inherent damping properties of the viscoelastic material. To design optimal damping treatments, the placement and the dimensions of the viscoelastic patches, as well as the material properties of the viscoelastic material must be optimised. The goal of this work if to present simple cost-effective techniques for the design of optimal constrained viscoelastic treatments.

1 Introduction

The use of constrained viscoelastic materials has been regarded as a low-cost and robust strategy to reduce noise and vibrations in many types of industrial applications [1]. In this configuration, the core viscoelastic layer undergoes high shear deformations, which combined with the inherent damping properties of the viscoelastic material, results in heat dissipation. The efficiency of this passive damping treatment depends on material and geometric parameters, and is usually estimated through the structural modal properties: eigenfrequencies and modal loss factors. However, the frequency dependency of the viscoelastic material properties leads to a non-linear eigenvalue problem that cannot be solved by conventional numerical methods. Some methods allows to compute the exact solution of the nonlinear eigenvalue problem, such as the asymptotic numerical method, but generally at great computational cost [2]. Therefore, in practice, the modal parameters are estimated through approximate methods such as the Modal Strain Energy (MSE) method or the Iterative Complex Eigensolution (ICE) method [3], or indirectly computed from frequency response functions [4].
To design optimal damping treatments, the placement and the dimensions of the viscoelastic patches, as well as the material properties of the viscoelastic layer must be optimised, usually by maximising the modal loss factors. In a first approach, strain maps or strain energy maps can be successfully used to identify the potential optimal placement of constrained viscoelastic layers [5]. A more precise placement of the viscoelastic patches and its dimensions can be determined using topology optimisation via SIMP, PSO, ESO or level-set methods [6]. While a large number of approaches are proposed in the literature to optimise the distribution of viscoelastic treatment for given materials, the determination of optimal material properties for a given configuration is rarely investigated. More efficient and adaptive damping treatments may be obtained by designing and manufacturing new elastomeric compounds and tailoring composite constraining layer. Some parametric studies have demonstrated that a change in the moduli of the viscoelastic core and the elastic faces can reshape the efficiency curve, while the loss factor of the viscoelastic material is directly proportional to the damping treatment efficiency when the material damping of the elastic faces can be neglected in comparison to the loss factor of the core layer [7].
The goal of this work is to investigate simple cost-effective techniques for the design of optimal constrained viscoelastic treatments from a material perspective. The first step is to highlight the existence of an optimal viscoelastic material. Then, a material optimisation is performed to maximise the efficiency of a viscoelastic damping treatment on the whole frequency range.

The paper is organised as follows. The implementation of linear viscoelasticity into the finite element method is described in section 2. Section 3 presents some methods to evaluate the efficiency of damping treatments through the calculation of modal loss factors. A numerical study is carried out in Section 4 to optimise the properties of the viscoelastic core layer. Finally, conclusions and perspectives are discussed in Section 5.

2 Finite element modelling of viscoelastically damped structures

The behavior of viscoelastic materials is usually characterised by frequency- and temperature-dependent properties. Under the assumption of harmonic excitation and constant temperature, the one-dimensional relationship between stress $\sigma^*(\omega)$ and strain $\varepsilon^*(\omega)$ is expressed as:

$$\sigma^*(\omega) = G^*(\omega)\varepsilon^*(\omega) \tag{1}$$

where ω is the angular frequency, $q^*(\omega)$ represents the Fourier transform of a variable $q(t)$, and $G^*(\omega)$ is the complex shear modulus, which can be written as:

$$G^*(\omega) = G'(\omega) + \mathrm{j}G''(\omega) = G'(\omega)(1 + \mathrm{j}\eta_v(\omega)) \tag{2}$$

where $G'(\omega)$ is the storage shear modulus, $G''(\omega)$ is the loss shear modulus and $\eta_v(\omega) = \dfrac{G''(\omega)}{G'(\omega)}$ is the material loss factor.
The extension of the one-dimensional constitutive equation to a fully three-dimensional behaviour law is easily performed by assuming isotropy and constant Poisson ratio:

$$\boldsymbol{\sigma}^*(\omega) = \mathbb{C}^*(\omega)\boldsymbol{\varepsilon}^*(\omega) \tag{3}$$

where $\boldsymbol{\sigma}^*(\omega)$ and $\tilde{\boldsymbol{\varepsilon}}^*(\omega)$ are the Fourier transform of the stress and strain tensor, in Voigt notation, and $\mathbb{C}^*(\omega)$ is a complex, frequency-dependent elasticity matrix:

$$\mathbb{C}^*(\omega) = \frac{G^*(\omega)}{0.5-\nu}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5-\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-\nu \end{bmatrix} \tag{4}$$

Considering this representation, the stiffness matrix of the viscoelastic layer is complex and frequency-dependent:

$$\mathbb{K}_v^*(\omega) = G^*(\omega)\mathbb{K}_v^0 \tag{5}$$

where $\mathbb{K}_v^0$ is a constant stiffness matrix evaluated with unit shear modulus. Therefore, the discretized problem associated to the dynamics of a structure with constrained layer damping in the frequency domain is:

$$\left(\mathbb{K}_e + G^*(\omega)\mathbb{K}_v^0 - \omega^2\mathbb{M}\right)\mathbf{U}^*(\omega) = \mathbf{F} \tag{6}$$

The advantage of decomposing the stiffness matrix into an elastic and a viscoelastic part is that the complex frequency-dependent shear modulus is in factor of a constant matrix.

3 Prediction of modal loss factors

The efficiency of a damping treatment is commonly evaluated through modal loss factors. For structures with constant damping, modal loss factors can be identified by solving the associated complex eigenvalue problem. However, in case of viscoelastic damping, the frequency dependency of the stiffness matrix leads to a non-linear eigenvalue problem:

$$\left(\mathbb{K}_e + G^*(\omega_k)\mathbb{K}_v^0 - \lambda_k^*(\omega_k)^2\mathbb{M}\right) \mathbf{\Phi}_k^* = \mathbf{0} \tag{7}$$

where the eigenfrequencies $\lambda_k^{*2} = \omega_k^2(1 + \mathrm{j}\eta_k)$ and the normal modes $\mathbf{\Phi}_k^*$ are complex. The resolution of such a non-linear eigenvalue problem requires the use of non conventional methods, such as the asymptotic numerical method [2], which are generally associated with a great computational cost. In practice, modal loss factors are estimated through approximate methods such as the Modal Strain Energy (MSE) method or the Iterative Complex Eigensolution (ICE) method [3], or indirectly computed from frequency response functions [4]. The key features of those methods are presented in this section and compared in the next section. In the following, the damping of the elastic faces is neglected in comparison with the loss factor of the core layer.

3.1 Modal Strain Energy (MSE) method

The modal strain energy (MSE) method has been introduced by Johnson et *al.* [9] to estimate the damping ratio associated to the r-th mode of the structure, η_r^{MSE}, from the real eigenvector of the undamped structure:

$$\eta_r^{MSE} = \frac{\eta_v(\omega_r^0)}{2} \frac{\mathbf{\Phi}_r^{0T} G'(\omega_r^0)\mathbb{K}_v^0\mathbf{\Phi}_r^0}{\mathbf{\Phi}_r^{0T}(\mathbb{K}_e + G'(\omega_r^0)\mathbb{K}_v^0)\mathbf{\Phi}_r^0} \tag{8}$$

where $(\omega_k^0, \mathbf{\Phi}_k^0)$ are solutions of:

$$\left(\mathbb{K}_e + G_0\mathbb{K}_v^0 - \left(\omega_r^0\right)^2\mathbb{M}\right) \mathbf{\Phi}_r^0 = \mathbf{0} \tag{9}$$

with $G_0 = G^*(\omega = 0)$.
Equation (8) supposes that the eigenfrequencies and the eigenvectors are not modified by the frequency dependency of the viscoelastic properties or the material loss factor.

3.2 Iterative Complex Eigensolution (ICE) method

The iterative complex eigensolution method was developped to extend the modal strain energy method, by iteratively seeking a better approximation of the non-linear eigenvalue problem. The complex eigenfrequencies λ_k^* and eigenvectors $\mathbf{\Phi}_k^*$, solutions of Equation (7) are estimated from the converged values eigenfrequencies of the following eigenproblem:

$$\left(\mathbb{K}_e + G^*(\omega_p)\mathbb{K}_v^0 - \lambda_k^{*2}(\omega_p)\mathbb{M}\right) \mathbf{\Phi}_k^*(\omega_p) = \mathbf{0} \tag{10}$$

For each mode k, convergence is achieved when:

$$\frac{\omega_p - \Re(\lambda_k^*(\omega_p))}{\Re(\lambda_k^*(\omega_p))} < \varepsilon_{tol} \tag{11}$$

The r-th damping ratio of the damped structure, η_r^{ICE} can be estimated from the converged solutions of the iterative algorithm:

$$\eta_r^{ICE} = \frac{\Im(\lambda_r^*)}{\Re(\lambda_r^*)} \tag{12}$$

While this method provides a good approximation of the modal loss factors, it also requires a large computational effort as a complex eigenvalue problem need to be solved at each iterative before convergence.

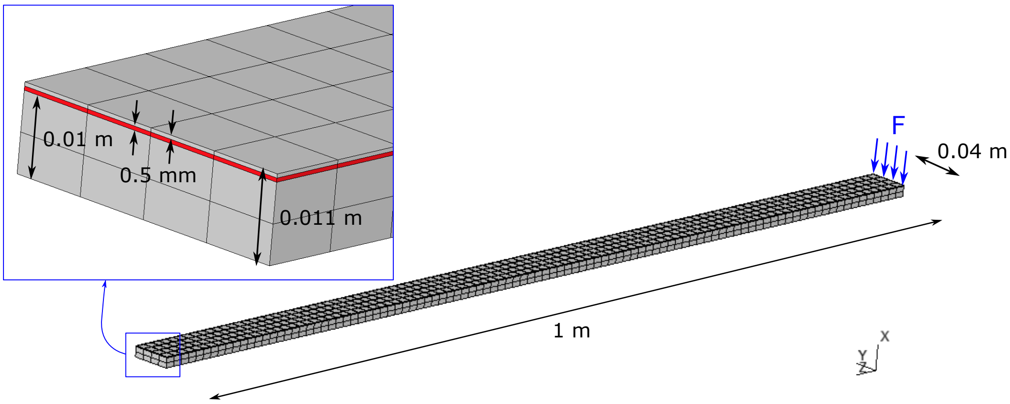

Figure 1: Geometry and mesh of the structure with CLD treatment.

Table 1: Material parameters

Material	Shear modulus	Poisson ratio	Density
Aluminum	$G = 26.7$ GPa	$\nu = 0.33$	$\rho = 2700$ kg/m^3
Viscoelastic	$G^*(\omega) = \dfrac{G_0 + G_\infty (\mathrm{j}\omega\tau)^\alpha}{1 + (\mathrm{j}\omega\tau)^\alpha}$	$\nu = 0.4985$	$\rho = 1460$ kg/m^3
	$G_0 = 1.4$ MPa		
(Deltane 350)	$G_\infty = 0.54$ GPa		
	$\tau = 0.52\ \mu$s		
	$\alpha = 059$		

3.3 Rational Fraction Polymial (RFP) method

Input-output dynamic identification approaches consist in identifying modal parameters from single or multiple input-output frequency response functions. In this work, we focus on single input-output receptance functions, in which the dynamic displacement from Equation (6) is computed using a multi-modal reduction method [10]. This approach allows a quick evaluation of the receptance function and is adapted to structures with viscoelastic damping.
To estimate the modal parameters of the structure from the computed receptance functions, a rational fraction polynomial (RFP) method is used [4]. The modal loss factors estimated from this approach are denoted η_r^{RFP} and will be considered as reference in the following section.

4 Material optimisation

4.1 Description of the simulation setup

The structure under study us a free rectangular aluminum beam equipped with a constrained layer damping (CLD) treatment, subjected to a harmonic load at one end [11]. The CLD treatment consists of a viscoelastic core layer made of Deltane350 and an aluminum constraining layer. The geometry and the mesh of the structure are represented in Figure 1. The frequency dependency of the viscoelastic properties of Deltane350 is described by a fractional derivative model, whose parameters are given in Table 1. The storage modulus and the loss factor of the damping material are plotted as a function of frequency in Figure 2.

4.2 Comparison of modal loss factors estimations

The receptance functions of the undamped (by considering $G^*(\omega) = G_0$) and the damped structure (with Deltane350) are computed and compared in Figure 3. The CLD treatment with Deltane350 leads to a signif-

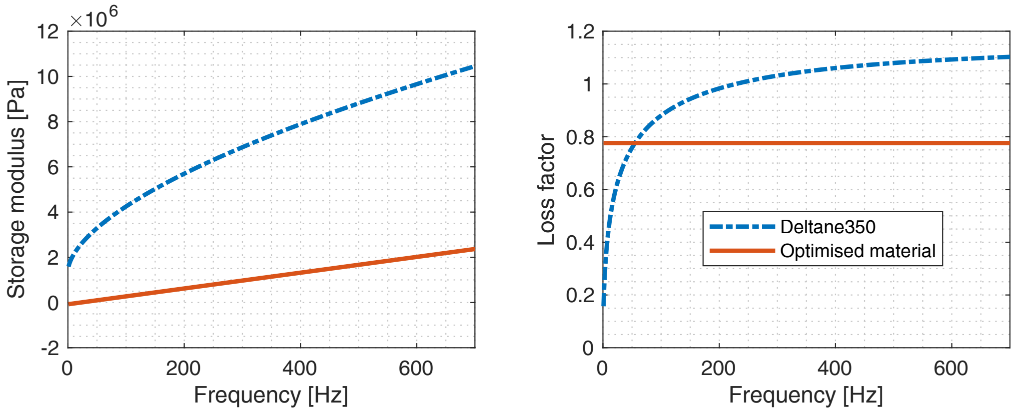

Figure 2: Storage modulus and loss factor of the viscoelastic material (Deltane350) and the optimised viscoelastic material.

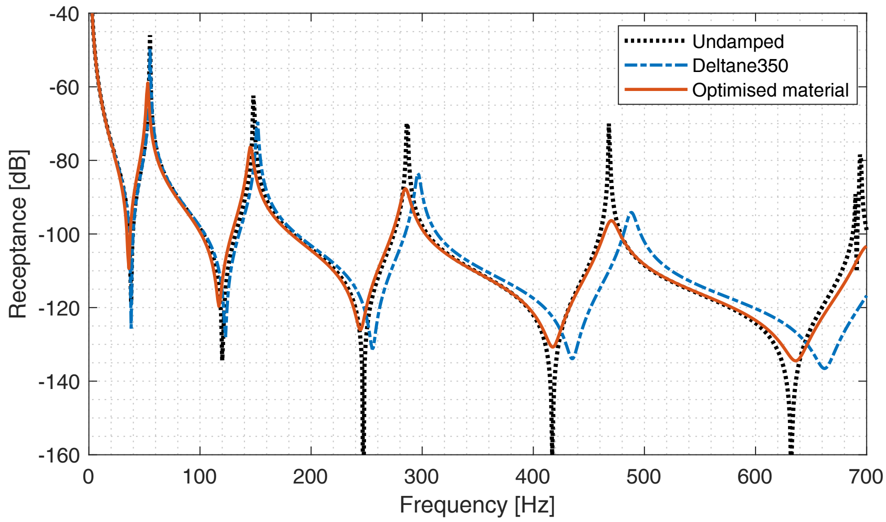

Figure 3: Comparison of receptance functions for various viscoelastic behaviour: undamped ($G^*(\omega) = G_0$, $\eta_v = 0$), damped with Deltance 350, damped with the optimal material.

icant reduction in the vibration levels. The eigenfrequencies and the modal loss factors of the damped structure are computed using the three methods described in the previous section, and reported in Table 2. One can notice that the modal strain energy method greatly overestimates the modal loss factors, indicating that the frequency dependence of the viscoelastic properties and the material loss factors modify significantly the mode shapes of the structure. The modal loss factors computed with iterative complex eigensolution method are very close to those identified with the rational fraction polynomial method from the receptance function plotted in Figure 3. However, the latter method is computationnally more efficient to estimate the modal parameters. Therefore, in the following, only the RFP method will be considered.

4.3 Optimization of the material properties

In this section, the properties of the viscoelastic layer are optimised to maximise the modal loss factors of the four modes excited in the given frequency range. The proposed method is to find an optimal storage shear modulus for each vibration mode. The material loss factor of the damping layer is taken as constant as it is directly proportionnal to the modal loss factors [7]. Two values are considered for each mode r: $\eta_v = \eta_v(\omega_r)$, and $\eta_v = 0.2$ (arbitrary value). The receptance function is computed for a viscoelastic storage shear modulus varying from 10^5 Hz to 10^9 Hz, and for each frequency response function, the modal loss

Table 2: Modal parameters estimated by the Modal Strain Energy (MSE) method, the Iterative Complex Eigensolution (ICE) method and the Rational Fraction Polynomial (RFP) method

Eigenfrequencies			
Method	*MSE*	*ICE*	*RFP*
Mode 1	54.8 Hz	55.2 Hz	55.2 Hz
Mode 2	149.2 Hz	151.6 Hz	151.6 Hz
Mode 3	284.7 Hz	296.5 Hz	296.2 Hz
Mode 4	463.9 Hz	487.6 Hz	488.0 Hz

Modal loss factors			
Method	*MSE*	*ICE*	*RFP*
Mode 1	1.42%	0.37%	0.41%
Mode 2	4.13%	0.63%	1.01%
Mode 3	5.57%	1.27%	1.48%
Mode 4	5.89%	1.95%	1.86%

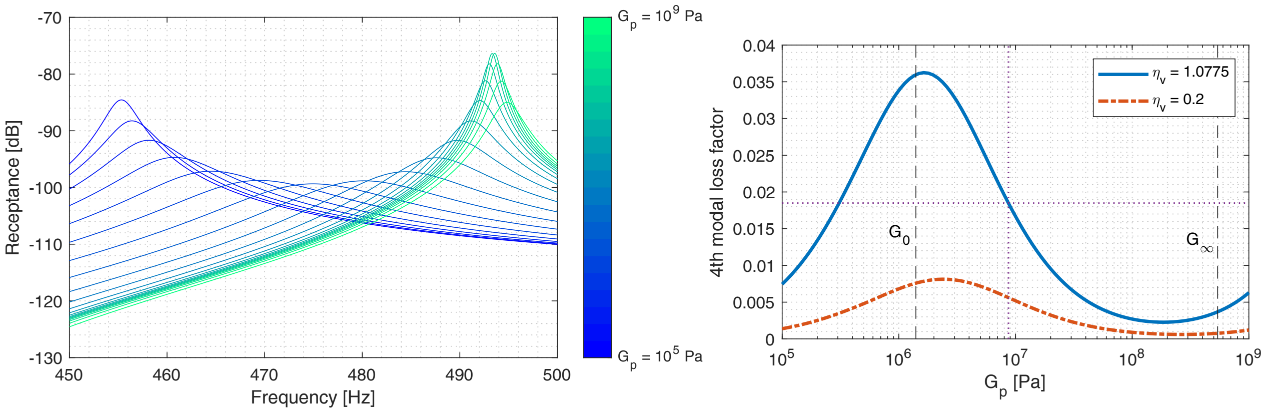

Figure 4: Optimisation of the storage modulus of the viscoelastic layer for the 4th mode of vibration.

factors are computed using the RFP method previously described. Results are presented only for the fourth mode in Figure 4. Note that we retrieve the value of $\eta_4^{FRP} = 1.86\%$ for $G' = G'(\omega_4) = 8.7$ MPa and $\eta_v = \eta_v(\omega_4) = 1.0775$. Figure 4 indicates that the storage modulus has a strong impact on the vibrations amplitudes and that there is an optimal value of storage modulus which maximises the 4-th modal loss factor of the structure ($G'_{opt} = 1.65$ MPa for $\eta_v = \eta_v(\omega_4) = 1.0775$, and $G'_{opt} = 2.4$ MPa for $\eta_v = 2$).

In a similar way, an optimal value of storage modulus is identified for the first three vibration modes. It should be noted that the optimal value of storage modulus is increasing with the eigenfrequency of the mode, which is coherent with the physical variations of the storage modulus expected for viscoelastic materials. Another important observation is that for a given value of material loss factor, the maximal value of modal loss factor is the same for all the vibration modes considered. An ideal viscoelastic material is then defined from optimal values of storage shear modulus and an arbitrary material loss factor $\eta_v = 0.776$. The properties of this optimal material are compared to Deltane350 in Figure 2. The receptance function of the structure with this optimal material as core layer of the CLD treatment is plotted in Figure 3. The material optimisation performed leads to a further reduction of the amplitude of vibrations on the whole frequency range, and the modal loss factors of the four modes are all equal to $\eta = 2.7\%$.

5 Conclusion

The goal of this work was to design an optimal constrained viscoelastic treatment from a material perspective. To do this, efficient computations of frequency response functions through a multi-modal approach is combined with a modal parameter identification method (here a rational fraction polynomial method) to identify an optimal value of storage modulus for each vibration mode. A numerical analysis carried out on a aluminum beam with CLD treatment evidenced the existence of a maximum achievable modal loss factor, common to all vibration modes, for a given material loss factor. The identification of an optimal viscoelastic material would be of great help in the ealy-stage design of new elastomeric compound for vibration reduction.

Moreover, the concept of maximum achievable modal loss factor is very similar to the notion of coupling factor, which drives the efficiency of piezoelectric shunts [12]. For piezoelectric shunts, the coupling coefficient is computed explicitly from two real-valued eigenvalue problems associated with short circuit and open circuit conditions. The next step would be to find a similar way to efficiently identify the maximum achievable modal loss factor for viscoelastic damping treatments. Some preliminary studies have been carried out in [8] to propose an explicit optimality condition for structures with partial viscoelastic treatments, and need to be further investigated. Linking the maximum achievable modal loss factor for viscoelastic damping treatment to the coupling factor defined for piezoelectric shunts would also enable an objective comparison between those two passive damping treatments [13].

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