Vibration and acoustic radiation of an impacted plate: parametric study based on isogeometric analysis

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Abstract

The impact of a plate by a sphere has been widely studied in the literature from the mechanical and acoustic point of view. Nevertheless, few studies have been dedicated to understand the effect of some parameters on the vibratory and acoustic response of impacted plates. The purpose of this work is to conduct a more complete parametric study on the parameters influencing both the vibratory and acoustic response of an impacted plate. The contact force has been approximated analytically which allows to reduce the calculation time. The plate is circular and modeled by the Reissner-Mindlin theory. It is surrounded by air and embedded in a rigid baffle. Its acoustic radiation is modeled using the Rayleigh integral equation. Numerical simulations were performed using isogeometric analysis based on Bézier extraction. The latter allows considering the exact geometry of the plate while preserving the local character of the basis functions for each Bézier element like the standard finite elements.

1 Introduction

A significant part of our acoustic environment is due to noise radiated by impacted vibrating structures. The geometries of these vibrating structures are often complex which prevents, in most cases, their accurate modeling. An effective way to capture the essence of their vibroacoustic response is to simplify the complex geometries by studying the vibratory behavior and acoustic radiation of plates.

The great interest in plates is clearly observed in the quantity of results and models published on these structures. Most of the studies conducted are either semi-analytical [1-2] or experimental [3-4]. The first analytical studies were based on Hertz's law for the calculation of contact force. However, this approach based on the elastic deformation due to the impact of a sphere on a massive plane does not reflect the situations actually encountered. A better approximation of the impact force of a sphere on flexible planes has been tested and validated experimentally [2]. More comprehensive models including calculations for impact force [5-6] as well as the acoustic response [5] in the case of the inelastic impact between a sphere and a rectangular plate have been developed.

On the other hand, although the finite element method (FEM) has been widely used for the simulation of the vibratory behavior of plates, its use in the vibroacoustic response seems limited [7]. Isogeometric analysis (IGA) [8], an emerging numerical method, can also be used within the finite element framework to alleviate computational time and make the results more accurate [9]. Indeed, this is possible in IGA thanks to the use of the exact geometry of the impacted structures. Thus, we can allow ourselves to roughly mesh the structure without losing precision [9-10].

The main purpose of this parametric study is to examine the relevance of the mechanical properties and geometric characteristics on the quality of the noise induced by an impacted plate. A model based on Bézier interpolation [11] and Bézier extraction [12] as developed in [13] has been used here. Several numerical simulations are performed to study (1) the sensitivity of the initial deformation of the plate as well its bending waves with respect to the characteristics of the two impacting bodies (such as materials, velocity of impact, dimensions) and (2) the effect of these characteristics on the initial transient acoustic wave and ringing noise.

2 Theory

Let us consider a circular plate defined in xy-plane so that the z-axis is an axis of symmetry. It is assumed to be embedded in a rigid baffle and impacted at its center O by a small sphere. The impact, without friction, is perpendicular to the plate. Moreover, the plate is elastic made from an homogeneous and lossless metallic material and its behavior is governed by the Hook's constitutive law ($\sigma = C\varepsilon$). Its vibrations are governed by Reissner-Mindlin theory in which the degrees of freedom w, β_1, and β_2 at cartesian coordinates x, y , and z are independent from $z(\in \left[-\frac{h}{2}, \frac{h}{2}\right])$ as shown in Eqs.(1) [14].

$$\begin{aligned} u_x(x,y,z) &= z\beta_2(x,y) \\ u_y(x,y,z) &= -z\beta_1(x,y) \\ u_z(x,y,z) &= w(x,y) \end{aligned} \tag{1}$$

The equations governing the vibratory behavior of the plate are given by:

$$\begin{aligned} div\, \sigma(u_p) - \rho_p \frac{\partial^2 u_p}{\partial t^2} = 0, \quad in\ V_p \\ \sigma(u_p).n = F(t), \quad on\ S_p^f \\ u_p = 0,\ on\ S_p^u \end{aligned} \tag{2}$$

Here and in what follows the notations $\bullet_i$ ($i = p, s$) denotes a quantity for the plate ($i = p$) or the sphere ($i = s$).

At $t = 0$, the plate is at rest. A sphere of an initial velocity V_0 impacts the plate at its center. The contact is assumed to be punctual with a contact force given by [2]:

$$F(t) \approx F_0 \left\{ \left(\frac{1.1}{1+\Gamma+2\Gamma^2}\right) \sin(0.97T)^{1.5}\, e^{-(0.4T)^4} + \left(\frac{1+2/\Gamma}{1+\Gamma}\right)\left(\frac{T}{T+1/\Gamma}\right)^{1.5}\ e^{-T/\Gamma} \right\} \tag{3}$$

where F_0 is the Hertzian contact force [1,15], $T = \frac{\pi t}{\tau}$ is the normalized time with respect to the duration τ of the Hertzian contact [1,15], and Γ is a parameter related to the impact energy and the flexibility of the plate [2]. These parameters depend, on the impact velocity, as well as, on the mechanical and geometrical properties of the impacting bodies.

Since the plate is embedded in a rigid baffle, we use the Rayleigh integral equation to express the acoustic pressure due the plate vibration:

$$p(X,t) = \int_{S_p} \frac{\rho_f}{2\,\pi d}\, a_n(Y, t - d/c_f) dS \tag{4}$$

where d is the distance between an observation point X in the surrounding fluid (air) and the source point Y located on the plate.

By applying the variational method for the problem described by Eqs.(2), one can solve it numerically using FEM. In this paper, the IGA is applied to simulate the vibratory behavior of the plate and its acoustic response [10,13]. The two types of approximation usually used in IGA are NURBS or Bsplines. According to the order of approximation, their basis functions span multiple elements. The numerical scheme, adopted in this work is based on Bézier interpolation [11] as well as the Bézier extraction concept [12]. These two methods allow considering the exact geometry of the plate while the basis functions are localized in each Bézier element just as the standard FEM does. More details on this Bézier-based IGA scheme can be found in [13].

3 Parametric study and discussion

In this first simulation, a Plexiglas sphere hits an aluminum plate. The mechanical parameters used to perform this simulation are summarized in Table (1). The used geometrical parameters are given in the first line of Table (2) (see case (1a)).

Table 1: Mechanical properties

	$\rho(kg/m^3)$	$E(GPa)$	ν	$c_f(m/s)$
Aluminium (AL)	2714	71.7	0.34	–
Steel (STL)	7800	200	0.3	–
Plexiglass (PLX)	1180	3.17	0.4	–
Air	1.204	–	–	343

Table 2: Parameters used in the parametric study

Effect	# case	Plate	Sphere	V_0(m/s)	R_s(mm)	R_p(mm)	h(mm)	τ(ms)	F_{max}(N)
Impact velocity	1a	AL	PLX	0.20	2	240	4	0.0357	0.7106
	1b	AL	PLX	0.23	2	240	4	0.0347	0.8402
	1c	AL	PLX	0.25	2	240	4	0.0342	0.9285
Sphere radius	2a	AL	PLX	0.20	4	240	4	0.0714	2.7843
	2b	AL	PLX	0.20	3	240	4	0.0536	1.5854
	2c	AL	PLX	0.20	2	240	4	0.0357	0.7106
Plate radius	3a	STL	PLX	0.20	3	240	4	0.0530	1.6193
	3b	STL	PLX	0.20	3	200	4	0.0530	1.6193
	3c	STL	PLX	0.20	3	160	4	0.0530	1.6193
Plate thickness	4a	STL	PLX	0.20	3	240	4	0.0530	1.6193
	4b	STL	PLX	0.20	3	240	3	0.0530	1.6122
	4c	STL	PLX	0.20	3	240	2	0.0530	1.5925
Sphere material	5a	STL	PLX	0.20	3	240	4	0.0542	1.5961
	5b	STL	AL	0.20	3	240	4	0.0244	7.9044
Plate material	6a	STL	PLX	0.20	2	240	4	0.0353	0.7218
	6b	AL	PLX	0.20	2	240	4	0.0357	0.7106

Fig.1a represents the variation, with respect to time, of the transverse displacement of the plate at the impact point O for different impact velocities. As we limit ourselves to case (1a) of Table (2), let us then look only at the variation of the variables for the impact velocity of 0.20m/s i.e. at the continuous curves of Fig.1. The transverse displacement of the point O shows 3 phases. First, in the initial deformation phase, the transverse displacement at the impact point decreases as the impact force is applied until it reaches a constant value u_0 at the end of the contact. Second, the impact point maintains this position for short time after the end of the contact. Finally, it starts to oscillate. This means that the impact point O is reached by the bending waves reflected on the edge of the plate.

Fig.1b represents the variation, with respect to time, of the acoustic pressure at a point situated at 30mm from the point O on-axis. As the transverse displacement, the pressure on-axis shows 3 phases [Fig.1b]: 1) the transient acoustic wave which is due to the initial deformation wave of the plate, 2) a silent phase during which the bending waves propagate toward the edge of the plate and 3) the ringing noise caused by the arrival, at the field point, of the bending waves reflected on the edge of the plate.

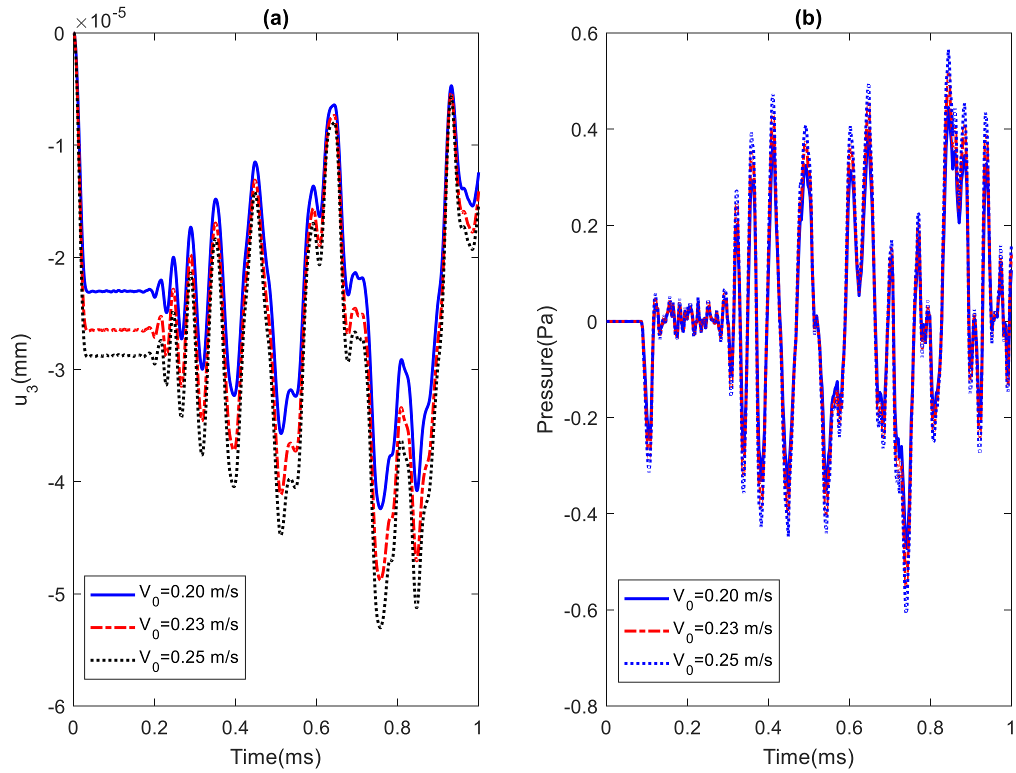

Figure 1: Effect of the impact velocity on (a) the transverse displacement of the impact point and (b) acoustic pressure on-axis at a point located at 30mm from the impact point

In the following, the goal is to study the effect of some parameters on the variation of the transverse displacement at the impact point and acoustic pressure on-axis when a sphere hits a plate. The mechanical properties of the different materials used in simulations are given in Table (1). Table (2) summarizes the parameters used to conduct the parametric study. To carry out this study, the parameters were chosen to get almost Hertzian contacts. This means that the maximum value of the force as well as its duration is very close to the values predicted by the Hertz theory. Otherwise, the non-Hertzian contacts will give rise to forces of lower amplitude and longer duration than those of the Hertz contact, which certainly complicates the comparison later. Indeed several parameters come into consideration and their influence is quite different in terms of amplitude and duration of the force. This explains the choice of low impact speeds and the materials of the sphere and the plate.

3.1 Effect of the sphere parameters

From the mechanical point of view, increasing the impact velocity 1) causes the constant value u_0 to be higher, and 2) increases the amplitude of the free transverse vibration of the plate after the impact [1]. This is due to the increase of the force with the impact velocity. From the acoustic point of view, increasing the impact velocity increases both the pressure amplitude of the transient acoustic wave and the ringing noise level [Fig.1b]. However, the duration decreases slightly by increasing the impact velocity [Table. 2]. Note that due to the small variation in impact velocity taken in the present simulations, the increase in pressure amplitudes and the decrease in the duration of the initial transient wave are also very small.

When it comes to study the influence of the sphere radius, the variation of the transverse displacement depicted on [Fig.2a] shows that the dynamic response is sensitive to this parameter. Indeed, by increasing the radius of the sphere, the duration of the initial deformation and its amplitude increases [Table 2]. It is the same for the amplitude of the bending wave. This is due to the increase in force. However, the duration of the initial wave of the initial deformation increases by increasing the radius.

According to Fig.1 and Fig.2 the shape of the bending wave is independent from the value of the sphere parameters (sphere radius and impact velocity). Indeed, the bending waves actually depend on the parameters of the plate and not the sphere as it will be shown in the next subsection.

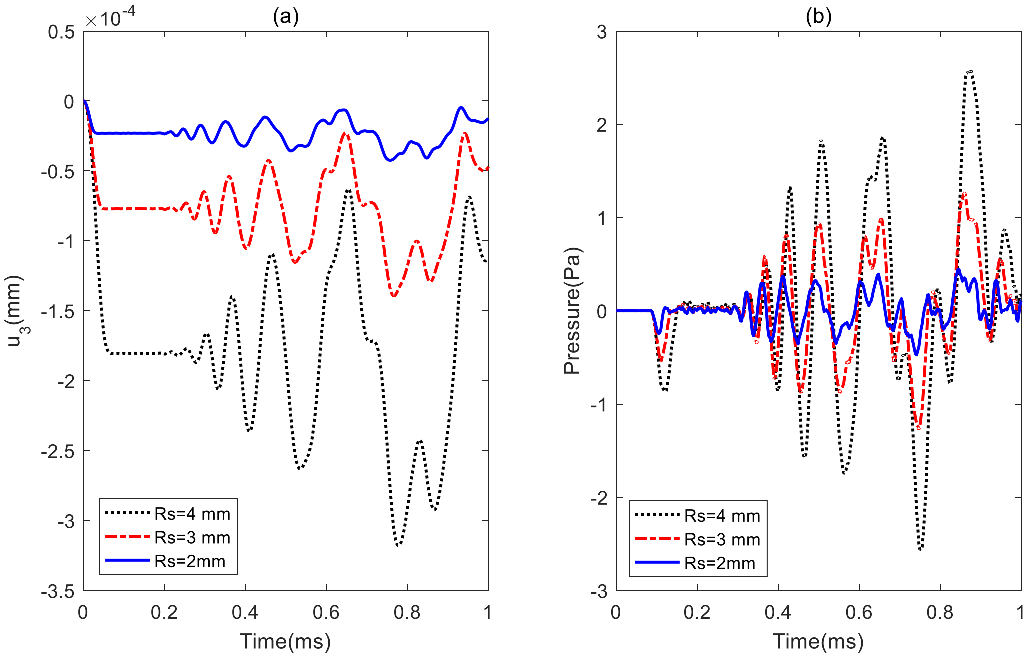

Figure 2: Effect of the sphere radius on (a) the transverse displacement of the impact point and (b) acoustic pressure on-axis at a point located at 30mm from the impact point

3.2 Effect of the plate parameters

As expected, the amplitude of the initial deformation wave u_0 and the initial transient acoustic wave are completely independent of the plate radius. This is due to the fact that the contact force is the same whatever the value of R_p. Indeed, the force expression remains valid for any size of the plate provided that its extent is large enough that the boundary conditions do not affect the contact force. However, the bending waves and consequently the silent zone and ringing noise depend strongly on the size of the plate. Indeed, the larger the plate, the longer it takes for the bending waves to reach the edge of the plate and then return to the impact point. Thus, reducing the radius of the plate reduces the silent zone in pressure and the plateau in dynamics.

However, even if the amplitude of the contact force and its duration are almost constant by changing the thickness of the plate, we notice that the thicker the plate the lower the value of u_0. The same is true for the amplitude of the bending and acoustic waves. This can be explained by the fact that the transverse displacement and the acoustic pressure are inversely proportional to the thickness as shown in the theoretical expressions in [1 Akay]. In other words the thinner the plate the greater its vibration and acoustic radiation.

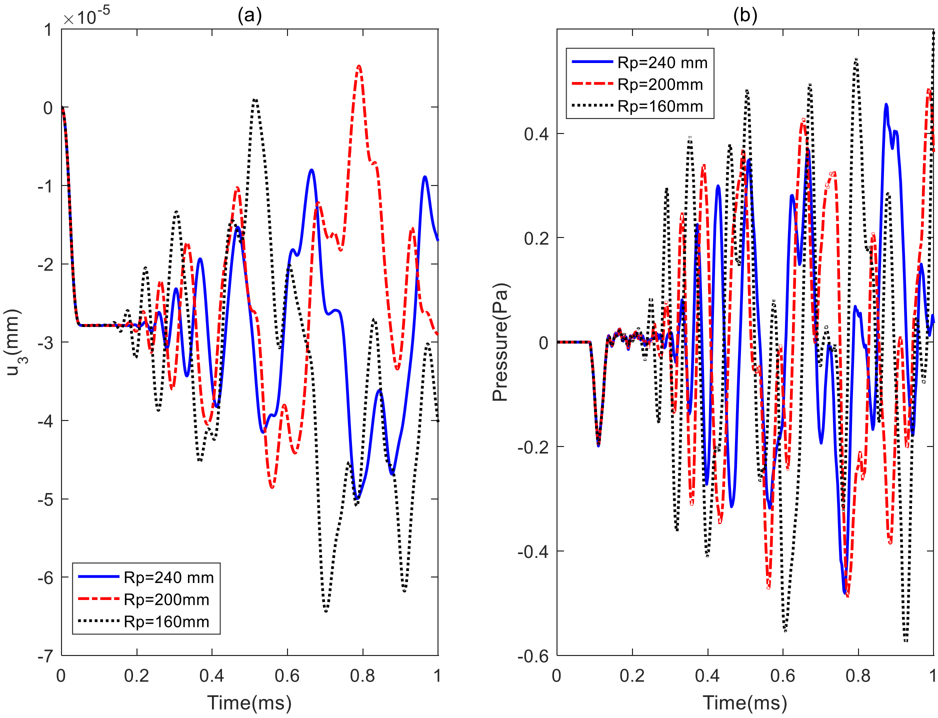

Figure 3: Effect of the plate radius on (a) the transverse displacement of the impact point and (b) acoustic pressure on-axis at a point located at 30mm from the impact point

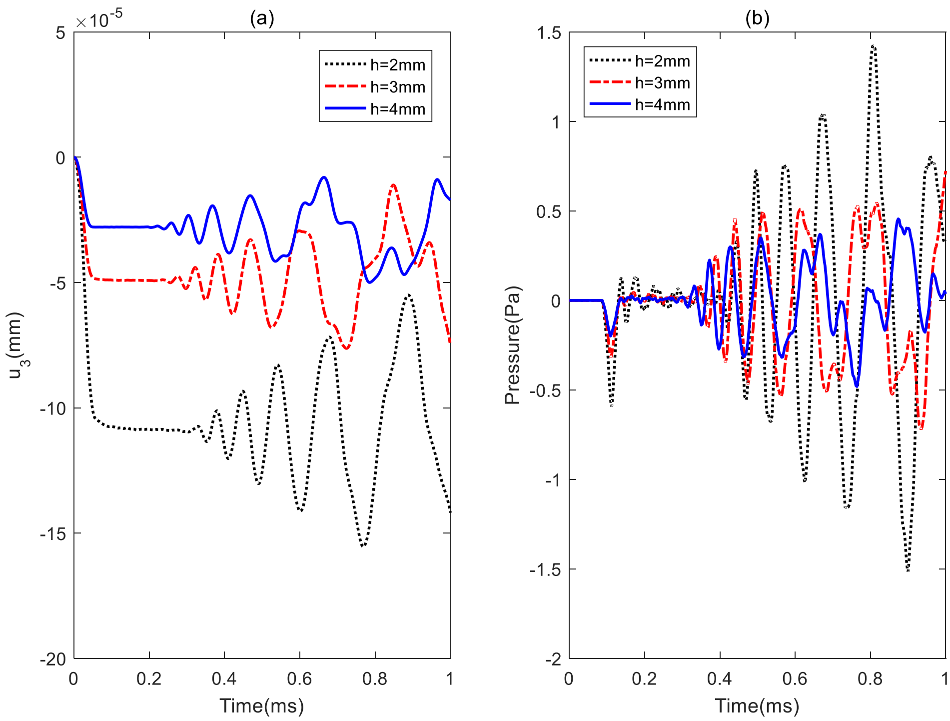

Figure 4: Effect of the plate thickness on (a) the transverse displacement of the impact point and (b) acoustic pressure on-axis at a point located at 30mm from the impact point

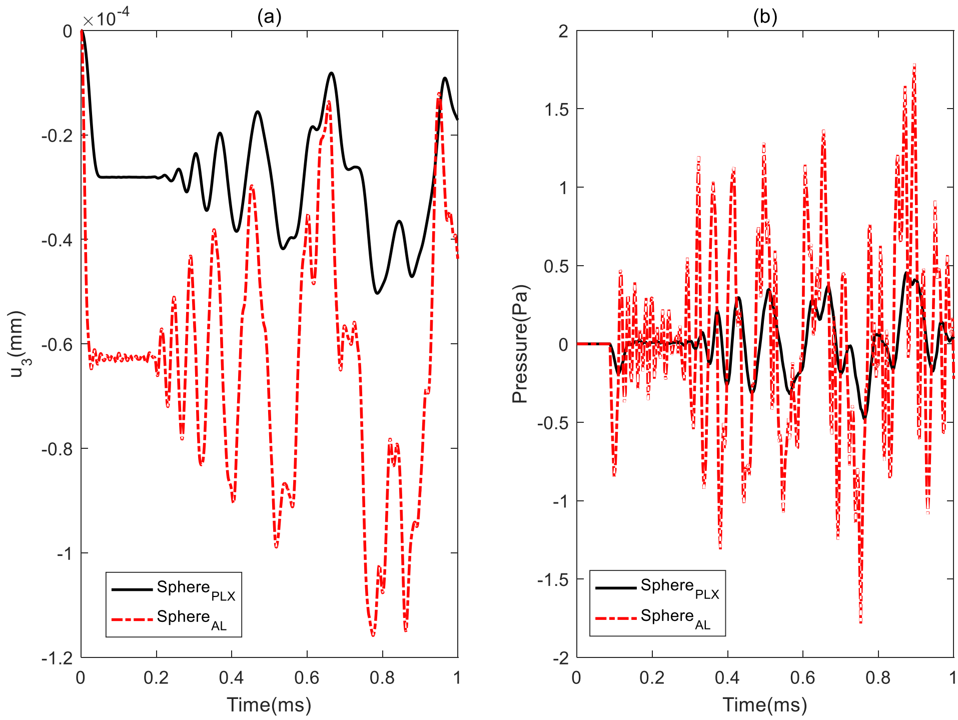

Figure 5: Effect of the sphere material on (a) the transverse displacement of the impact point and (b) acoustic pressure on-axis at a point located at 30mm from the impact point

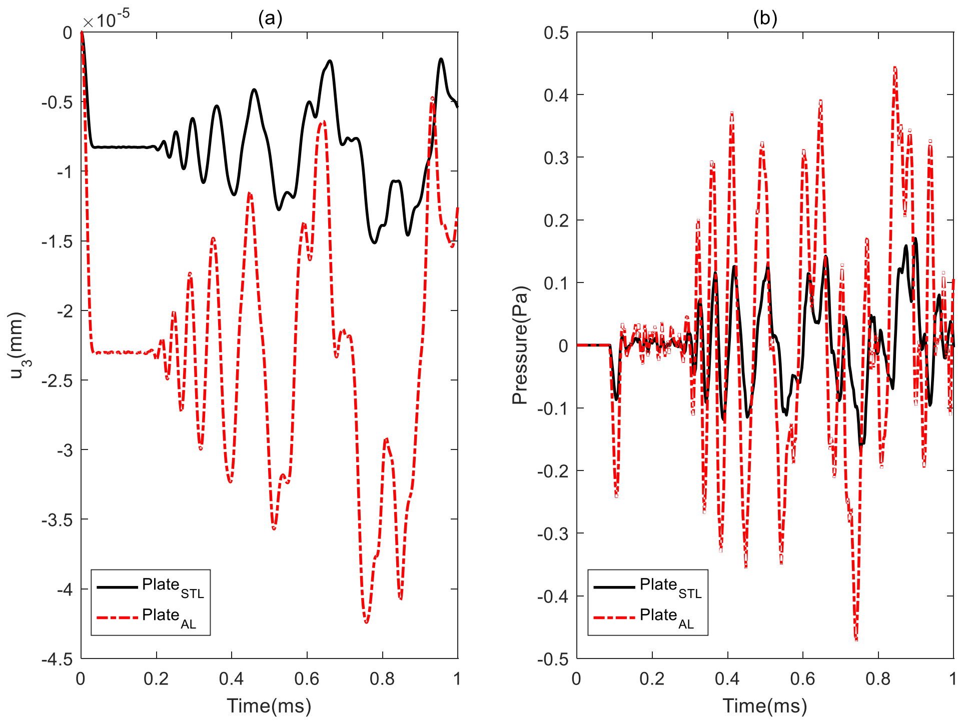

Figure 6: Effect of the plate material on (a) the transverse displacement of the impact point and (b) acoustic pressure on-axis at a point located at 30mm from the impact point

3.3 Effect of the material

Even though it is difficult to show the effect of the variation of the mechanical properties of the sphere and plate, it is interesting to see globally how the transverse displacement and acoustic pressure vary in the presence of a lighter/heavier and/or flexible/stiff plate/sphere.

Compared to Plexiglas, an aluminum sphere is heavier and deforms the plate more, which radiates acoustic waves of greater amplitude [Fig. 5]. In this case, the contact force is greater and of shorter duration. Now, when a Plexiglas sphere impacts an aluminum plate, the latter deforms and radiates more than if it were steel-based, although the force in the case of aluminum is slightly weaker than in the case of steel [Fig.6]. Thus, a low force does not necessarily mean a low deformation or low acoustic radiation (compare the maximum of the force of case (6a) and case (6b) in Table 2). Like thickness, transverse displacement and acoustic pressure are inversely proportional to density. Thus, the heavier the plate, the less easy it is to vibrate it.

4 Conclusion

The vibration and acoustic radiation of a plate embedded in a rigid baffle and impacted in its center by a small sphere have been investigated within a parametric study. This one concerned the variation of the parameters of the sphere (radius and impact speed), the plate (radius and thickness) and the influence of the material constituting both the sphere and plate. It has been found that the variation in the radius of the plate only affects the bending waves and consequently the silent zone and ringing noise. Reducing the thickness of the plate or using a light plate promotes its deformation and acoustic radiation. The increase in impact velocity only increases the radiated sound pressure without changing its shape. Finally, the increase in the mass of the sphere (through its radius or density) increases the sound pressure radiated and affects the duration of the initial transient wave. This study shows the extent of the possibilities offered by the variation of these and probably other parameters to meet the need for noise reduction at the source.

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Appendix

A Nomenclature

a_n	Normal acceleration of point on the plate
C	Elasticity matrix
c_f	Longitudinal waves speed in the surrounding fluid (air)
E	Young's modulus
F_{max}	The maximum of the impact force
h	Thickness of the plate
n	Normal vector the to plate
p	Acoustic pressure
r	The first polar coordinate in xz-plane
R_i	Radius of impacting bodies (with $i = p, s$)
S_p	Surface of the plate
S_p^f	Boundary for prescribed force
S_p^u	Boundary for prescribed displacement
t	Time
u_0	Transverse displacement value reached by the impact point just after the contact
u_p	Displacement of the plate
u_j	Displacement of the plate in x, y, and z direction (with $j = x, y, z$)
V_0	Impact velocity
V_p	Volume of the plate
w	Displacement of the plate in z direction
β_1, β_2	Rotations of the normal to the reference plane of the plate
ε	Strain tensor
ν	Poisson's ratio
ρ	Density
σ	Stress tensor
ψ	The second polar coordinate in xz-plane