Additively manufactured composite metastructure for broadband vibration and noise control applications

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Abstract

Artificial synthetic structures as metamaterials have seen tremendous amount of research interest for vibration and noise control and wave manipulation applications. The present study proposes a new type of composite metastructure that is capable of inducing a low-frequency ultrawide bandgap. The metastructure consists of a polymeric housing, constructed by 3-D printing and, rigid steel masses which are embedded inside the housing. The bandgap is generated by the principle of mode separation where the frequency difference between the global resonant mode and local resonant mode are manipulated to widen the bandgap. These locally resonant modes are studied by numerical modal analysis and wave attenuation over broadband frequency range are demonstrated by a frequency response study. The metastructure design morphology and study findings will have a wide range of applications in structural vibration and noise control and elastic wave manipulation.

1 Introduction

Engineered periodic composite structures with architectural design and amazing dynamics properties that can alter acoustic and elastic waves at deep subwavelength scales over a wide frequency range have sparked a lot of research interest in the last decade. For such engineered designs the term phononic crystals [1], acoustic metamaterials [2], architected materials [3, 4], phononic metamaterials [5], mechanical metamaterials [6] and metastructures [7-10] are refereed in literature. A crucial aspect of these designed systems, aside from other remarkable wave manipulation characteristics, is frequency bandgap (BG). The frequency region where wave propagation is prohibited has been seen in these synthetic designs. Further, the frequency BG property resulted into acoustic and elastic waves manipulation in various forms such as waveguiding and localization [6-8], wave focusing and lensing [9], acoustic and elastic cloaks [10, 11], rainbow trapping effects [12], wave multiplexers [13], broadband wave attenuation [14, 15] etc. In the BG frequency range, the peculiar mechanical wave phenomena and governing properties such as negative mass density [16], negative moduli [17], negative refraction [18], negative Poisson ratio/auxetic metamaterials [19], double negative wave medium [20] and kirigami effects [21] are also explored. The mechanical dynamic characteristics of phononic crystals and acoustic metamaterials have been studied extensively in the literature. Further details can be found in Hussein et al. [22] and Wang et al. [5]. For detailed literature survey linking phononic crystals and acoustic metamaterials with photonic crystals and electromagnetic counterparts, one can refer to Muhammad and Lim [23]. To date, the published research and literature support the idea that metamaterial is no longer constrained by pure physics and mechanics theories. Acoustic metamaterials at different length scales are being explored for various applications in acoustic and elastic wave manipulation such as noise cancellation membrane [24], vibration mitigation designs [14, 15], energy harvesters [25], smart devices [26], automobile industry [27], underwater acoustics [28, 29], ambient ground vibration attenuation [30] and seismic metamaterials [31-34]. Despite all of the intriguing discoveries and rapid progress in this field, low frequency vibration and noise control remain a major challenge.

The present research presents a novel metastructure design to address this difficulty and report ultrawide low frequency BG for acoustic and elastic wave control. In the following section, we will use the term metastructure interchangeably for acoustic/elastic metamaterials. The proposed meta-atom/unit cell structures are actually composite designs that resemble the idea of locally resonant sonic crystals initially proposed by Liu et al. [2]. The metastructure is designed in such a manner to make it periodic in all three-directions for opening three-dimensional complete BG. Further details on the design configuration are given in the next section. It should be highlighted that the proposed structure differs from those reported in the literature since it is capable of generating low frequency ultrawide three-dimensional BG that can be used to manipulate/attenuate vibration and noise from all directions. The current stage of the study is limited to FEA based numerical simulations. The experimental validation of reported result is part of future study.

The band structure is obtained by using FEA code COMSOL Multiphysics and resonant modes responsible for opening and closing bounding edges of BG are captured. The numerical findings are divided into two parts i.e (i) band structure (ii) frequency response study. Thanks to COMSOL built-in Floquet-Bloch periodicity condition, and the unit cell structure is assumed infinitely periodic in all three directions to achieve the band structure. The governing mechanism for BG generation is also explained. The BG effectiveness and efficiency are validated by a frequency response study on the finite supercell lattice structure. Both modal analysis and frequency response spectrum validate the low-frequency elastic wave attenuation by the proposed composite metastructure.

The paper is organized as follow. Section 1 is introduction. The unit cell structure and numerical model is discussed in section 2. Section 3 discusses the obtained results. Finally, a conclusion is given in section 4.

2 Modelling Strategy and Metastructure Design

The metastructure design morphology is shown in Fig. 1. The proposed structure consists of two components (i) housing that is made of some 3D printed photosensitive polymer (ii) hard inclusion like steel cylinder that will be embedded inside the printed housing. The material properties are given in Table. 1. The lattice size of the metastructure is $a = 50\text{mm}$ and all geometric parameters are presented with respect to it. The selected geometric parameters are listed at the inset of Figure 1. The proposed unit cell structure is built in commercial FEA code COMSOL Multiphysics v5.4. We applied Floquet-Bloch periodicity condition on all vertical edges and top and bottom boundaries to make the unit cell structure periodic in all three directions. A detailed theoretical formulation is outside the scope of the paper. Interested readers may refer to [22]. Based on these defined setting, COMSOL eigenfrequency study is performed to achieve the band structure. To validate these findings, a frequency response study on the finite length model is also conducted to envisage wave attenuation inside the BG frequencies. For both wave dispersion and frequency response studies, fine quadratic elements, tetrahedral meshes are applied on the polymeric skeleton and COMSOL built-in normal tetrahedral mesh is applied on the cylindrical steel inclusion.

Table 1: Material properties

	Young Modulus E (GPa)	Mass density ρ (kg/m^3)	Poisson ratio ν
Photosensitive Polymer	0.96	1300	0.35
Steel	210	7850	0.33

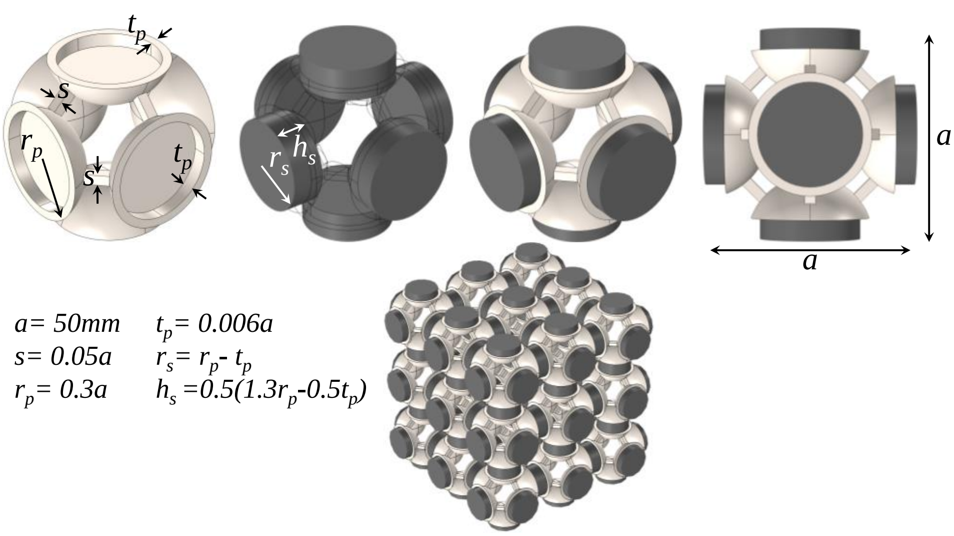

Figure 1: Composite metastructure design morphology with supercell lattice. The geometric entities are listed at the inset of figure.

3 Results and Discussions

According to the concept of mode separation/modal masses involvement, the composite metastructures are designed in such a way that the difference between the vibration modes that are responsible for the opening and closing of BG can be maximized [35-37]. This concept implies that the separation of vibrational energy between different parts of the unit cell structure can be increased for a certain design configuration of a periodic metamaterial structure to open ultrawide BG. The *global* and *local resonant modes* are two types of vibration modes that play a key role in the opening and closing of the BG. For example, a meta-atomic topology of composite metastructure formed by heavy cylindrical inclusion supported by thin elastic assembly is characterized by passbands with vibrational energy confined either in the complete meta-atom including masses and supporting slender beam/frame assembly (*global mode*) or vibrational energy locally confined in the ligaments (*local mode*). It has been proven in the prior studies [8-10] that these global and local modes are responsible for the opening and closing of BGs. The oscillations of heavy cylindrical masses and connecting beam assembly at a very low frequency regime characterize the eigenmode associated to BG opening. The closing bounding edge is characterised by vibrational energy localised at connecting beam without the oscillations of heavy masses, which shift the eigenmode to a far higher frequency, as opposed to the global eigenmode frequency. The authors [35-37] maximised this vibrational energy difference to open ultrawide BG by analytical modelling, numerical simulations and experiments. The inclusion selected are intended to be of larger mass density to obtain low frequency eigenmodes and thickness of connecting beam is minimised in order to maximise the discrepancy of global and local vibration energy (in terms of the eigenfrequencies) to achieve ultrawide BG. Although such metastructures can exhibit ultrawide BGs for low frequency vibration and noise control, practical applications (including manufacturing of prototypes and operation) pose significant challenges. Since the heavy masses are carried by lightweight ligaments, such structural elements cannot provide sufficient support to heavy masses and the probability of manufacturing or operation failure is notably high.

The principle of mode separation where global and local resonant modes are optimized to achieve ultrawide BG is an interesting approach for designing and manufacturing metastructures for low frequency wave manipulation and control. In supplement to the metastructure designs reported to date [35-37], this study proposes a novel composite metastructure design approach where both rigid inclusion/masses and supporting ligaments sizes are optimized to achieve ultrawide low frequency BG. Without compromising the size of supporting ligament (thinner ligaments/frame assembly are prone to failure during manufacturing and operation), low frequency BG can be achieved from the proposed designs.

A numerical wave dispersion study based on modal analysis provides important information on the composite metastructure's eigenfrequencies and resonant modes when the wavenumber is swept over the irreducible Brillouin zone's boundaries. The wavenumber with a definite frequency solution is termed as passband while the wavenumbers without any definite, real frequency solution is called bandgap where wave propagation is prohibited in this frequency region. Figure 2 shows the band structure with bandgap for the proposed composite metastructure. The modal analysis results revealed presence of low frequency ultrawide bandgap starting from 1938.6 Hz to 35231 Hz. The *global* and *local* resonant modes are presented at the side of wave dispersion curve. We observed opening of the BG with relative bandwidth 179%.

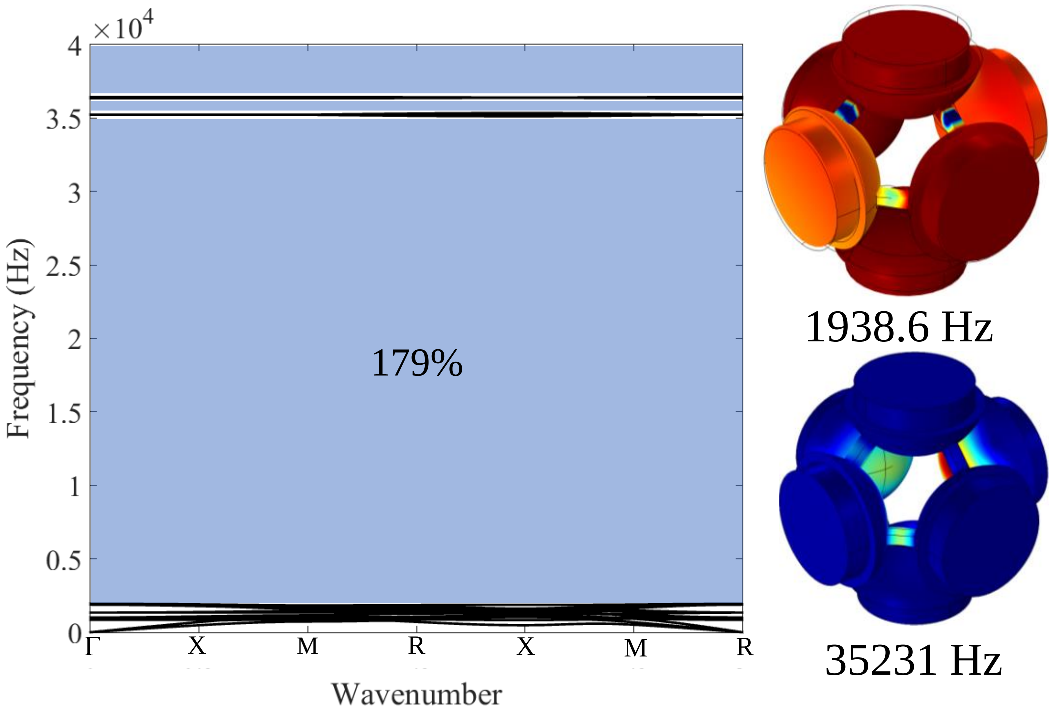

Figure 2. Band structure with BG for the proposed composite metastructure. The global and local resonant modes are shown aside the band diagram.

A finite array of supercell structures consisting of 3x3x1 unit cell structures has been created to simulate wave attenuation inside the BG frequency range by the proposed composite metastructures. The schematic is presented at the inset of Figure 3. The harmonic excitation is applied at the center left unit cell structure and response in the form of displacement field is captured at the center right unit cell structure. As shown in Figure 3, we observed robust attenuation of elastic wave inside the BG frequency region. The expression for wave transmission is $T = 20\log_{10}\left(u_{\text{out}}/u_{\text{in}}\right)$ where u_{out}, u_{in} are displacement field at the output end and reference input end respectively.

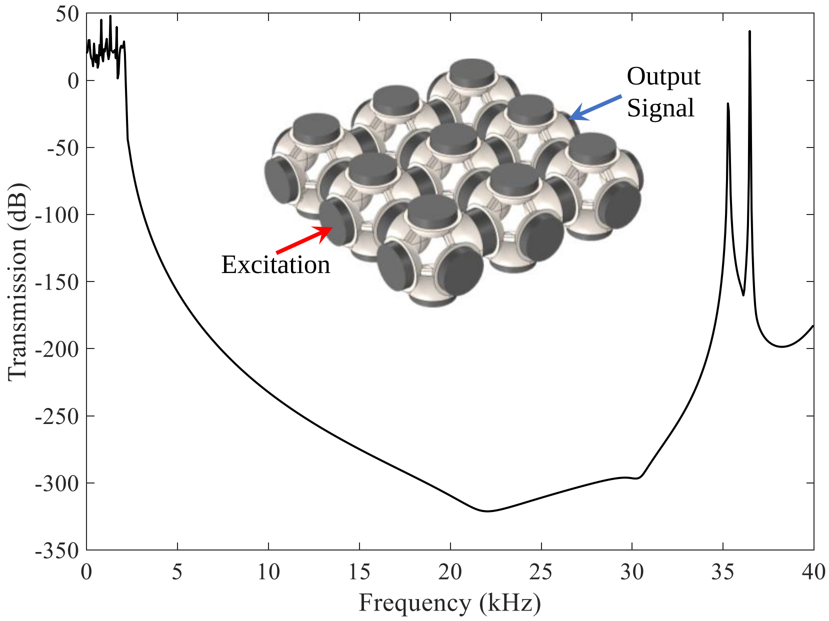

Figure 3: Frequency response spectrum for 3x3x1 unit cell array of composite metastructure. The wave attenuation inside the BG frequency region can be seen.

4 Conclusion

The current research provides new periodic composite metastructure designs for controlling vibration and noise throughout an ultrawide low frequency range. The metastructures are made up of a polymeric skeleton with cylindrical steel inclusion inserted inside. The inflexible steel masses increased the resonant system's effective mass density, resulting in low frequency ultrawide BGs distributed over a large frequency range. The study is based on numerical modelling and simulation works. A numerical wave dispersion study has been performed to determine the band structures and to highlight the ultrawide low frequency BG regions. The global and local resonant modes responsible for the opening and closing bounding edges of BG, respectively, are discussed to explain the BG generation mechanism. Vibration attenuation by the proposed metastructures is demonstrated by performing a frequency response study on the periodic array of supercell lattice. Both modal analysis result and frequency response spectrum matches well. The proposed composite metastructure designs can be potentially applicable in the vibration and noise control facilities and elastic waves manipulation, where broadband vibration and noise control over an ultrawide frequency range is conducive.

Acknowledgement

The work described in this paper is supported by Irish Research Council-Enterprise Partnership Scheme Postdoctoral Fellowship Scheme (Project No. 211705.16976-EPSPD/2021/108).

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