Accelerated random fatigue testing by means of a tri-axial electrodynamic shaker: solutions for combing the multiple test specifications

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Abstract

Industrial products are subjected to extreme conditions throughout their entire life that could compromise its durability. Therefore, the environmental effects must be taken into account from the early stages of development. Typically, in laboratory tests the specimen is subjected to more severe conditions that speed up the testing process. A powerful tool to asses durability of components is random fatigue testing, where the combination of the vibration control technique and the shaker system become critical to reproduce vibration environments. This work presents a test campaign where a 3-DoF shaker table is used in combination with Multi-Input Multi- Output control techniques to carry out accelerated random fatigue tests on specially designed specimens. The paper investigates the effects of simultaneous multiple random excitations on the fatigue-life of the specimens. In particular, the paper reviews and compares theoretically and experimentally the currently available solutions for combining test specifications, providing a critical analysis of the issue.

1 Introduction

A key aspect in the product development process is to assess the capabilities of components to withstand severe environmental condition, by means of vibration and fatigue damage. In order to do so it is impossible to conduct full life tests in the actual working conditions due to cost and time consumption, therefore preliminary laboratory tests must be carried out. In this context a critical aspect is to replicate the same damaging process experienced in the real working environments while reducing the time to failure of the specimen under test. The most widely used technique to correlate the laboratory condition to the in-field one, is the so-called inverse power law [1] . Typically reference data for this application are provided by Standards [2, 3], which suggest the test tailoring practice to synthesize a vibration profile from experimental measurements in operational environment. Nowadays the most accredited tailoring procedure is the one proposed by Lalanne in [4], which employs the Extreme Response Spectrum (ERS) and the Fatigue Damage Spectrum (FDS) as metrics to compare the synthesized vibration profile with the experimentally obtained one. Basically, the excitation applied to test specimen should be capable to reproduce the same level of damage which the specimen would undergo in its operational life-cycle. The accuracy of the inverse power law has been investigated in several works, since its relevance is pivotal to synthesize reliable test specifications [5]. The theoretical foundations were addressed by Allegri and Zhang [6] that proposed an extended formulation of the law, capable of dealing with multi-axial stress states. Despite the inverse power law is widely used in common practice, the methodology is affected by limiting hypothesis, the strongest being that the unit under test is to be approximated to a single degree of freedom system. Such assumption neglects the possibility of multi-axis excitation testing. Therefore, the current standard methodologies of accelerated dynamic tests are intended to be implemented only on single-axis shakers. In the recent past, several works emphasised the the benefits of adopting Multi-Input Multi-Output (MIMO) control strategies in environmental vibration testing [7]. Indeed, as documented in [8, 9], multi-axis vibration testing allows to replicate the operational condition of real vibration environments, enhancing the realism of the test [10]. Within this context, the main objective of this manuscript is to investigate the impact of multi-axial excitations by means of fatigue life and damage of the specimen. More specifically, this research activity points out the relevance of the MIMO target generation technique when performing multi-axis accelerated random fatigue tests.The definition of the control strategies is still an open discussion, as highlighted in the US Military Standard [11]. When multiple axes needs to be controlled simultaneously, the test specification for each control channel (Power Spectral Density) is not enough,and the cross-correlation between each pair of control channels must be defined (Cross Spectral Density). The control target is thus a Spectral Density Matrix (SDM)[12]. Unfortunately, the test specification are provided in terms of PSDs only and CSD data are often missing. When no information about the CSDs are available, the CSD terms must be computed starting from the respective reference PSDs by specifying proper profile of phase and coherence between the reference channels as a function of frequency [13]. De facto, the solution of the reference SDM with fixed PSD terms is not unique. Nowadays, the most commonly practised methodology to fill in the reference SDM is the Independent References Method (IRM): the reference SDM is specified as a diagonal matrix by simply setting low coherence for all the cross terms. This MIMO target generation procedure assumes that the multiple vibration requirements are independent from each other in the entire frequency band. Even if this hypothesis cannot be considered truthful for all the real vibration environments, the IRM is very simple and user-friendly. For these reasons it is the procedure suggested by the Standards when no information about the CSDs are available [11]. Other MIMO target generation procedure have been recently formalised: the Independent Drives Method (IDM) [14], the Minimum Drives Method (MDM) [15] and the Minimum Single Drives Method (MSDM) [16]. These methodologies are all focused on the drives power minimization, with the aim of preserving the excitation system. In the literature, at the authors knowledge, there is no MIMO target generation technique focused on the fatigue-life of the test specimen.Therefore, the major novelty proposed in this paper is an innovative methodology, namely Extreme Dynamic Response Method (EDRM), which combines the test specifications in order to get out the maximum dynamic response from the specimen under test. The method guarantees to excite the specimen with the most adverse and extreme multi-axis vibration conditions, thus assuring the shortest Time to Failure (TtF).

2 Proposed Methodology : Extreme Dynamic Response Method

The basic concept of the Extreme Dynamic Response Method (EDRM) is the same of the Minum Drives Method (MDM) proposed in [15]. Despite the similarities between the MDM and the EDRM their purpose are completley different. While the MDM aims to minimize the drives' power, safe guarding the excitation system, the objective of the EDRM is to enable the full potential of the test specifications. The EDMR correlates the test specifications in order to maximize the response from the specimen dynamics, for example accelerations or stress/strain at crack location. In the following the formulation of the EDRM is presented for the acceleration application, however it could be adapted to any response by changing the initial transfer function relationship. The MIMO transfer function relationship between the reference SDM and the acceleration SDM of a generic location on the specimen is [17] :

$$S_{xx} = \widehat{H_{aa}} S_{yy}^{ref} \widehat{H_{aa}^{H}} \tag{1}$$

The EDRM aims to find the proper values of coherences and phases to be set in reference SDM (S_{yy}^{ref}) in order to maximize/minimize the trace of the acceleration SDM (S_{xx}) at the measuring point, which means to maximisze/minimize the PSD of all response channels.

$$P = Tr[S_{xx}] = \sum_{i=1}^{m}(\sum_{j=1}^{l}\sum_{k=1}^{l} \widehat{H_{aa,ij}} S_{yy}^{ref} \widehat{H'_{aa,ik}}) \tag{2}$$

By introducing the Hermitian matrix $W = \widehat{H_{aa}{}^{H}}\widehat{H_{aa}}$, Equation 2 can be rewritten in terms of reference coherences and phases between the pairs of control channels :

$$P = \sum_{j=1}^{l} S_{yy,jj}{}^{ref} W_{jj} + 2 \sum_{j=1}^{l-1} \sum_{k=j+1}^{l} \sqrt{\gamma_{jk}{}^2 S_{yy,jj}{}^{ref} S_{yy,kk}{}^{ref}} |W_{jk}| \cos(\phi_{jk} - \xi jk) \tag{3}$$

where

$$S_{jk}^{ref} = |S_{jk}| e^{i\phi_{jk}} = \sqrt{\gamma_{jk}{}^2 S_{yy,jj}{}^{ref} S_{yy,kk}{}^{ref}} e^{i\phi_{jk}} \tag{4}$$

$$W_{jk} = |W_{jk}| * e^{i\xi_{jk}} \tag{5}$$

Equation 2 is very practical, as the only unknowns are the reference phases ϕ_{jk} and the reference coherences $\gamma_{jk}{}^2$, the other terms are known quantities : $S_{yy,jj}{}^{ref}$ are the test specifications and W can be estimated during the system identification pre-test phase, when the control algorithm estimates the FRFs matrix between the drive inputs and the control outputs. Therefore, the response trace (Equation 2) has a maximum (minimum) when the reference coherences are all unitary and all cosines equal to 1 (-1). This observation leads to the definition of the $Extreme\ Dynamic\ Conditions$:

$$P \text{ is maximum} \iff \begin{cases} \gamma_{jk}{}^2 = 1 \\ \phi_{jk} = \xi_{jk} \end{cases} \quad \forall j,k = 1:l, j \neq k \tag{6}$$

$$P \text{ is minimum} \iff \begin{cases} \gamma_{jk}{}^2 = 1 \\ \phi_{jk} = \xi_{jk} + \pi \end{cases} \quad \forall j,k = 1:l, j \neq k \tag{7}$$

As explained in [15] and [16], setting fully coherent control responses means to establish strong physical relations between the phases of the control channels. The Extreme Dynamic Condition of Equation 6 defines all the phase terms of the control target,neglecting the phase dependencies that mutually link the control channels pairs. Equation 6,as is, does not guarantee the resulting SDM to be positive (semi)definite, a fundamental algebraic constraint needed to ensure the physical feasibility of the MIMO control target during the test [18]. To overcome this limitation, the EDRM can be completed with the Phase Pivoting Principle, proposed in [15]. Referring to Equation 2, the Phase Pivoting Principle applies the Extreme Dynamic Condition for the $(l-1)$ independent terms with the biggest amplitude $|W_{JK}|\sqrt{S_{yy,jj}{}^{ref} S_{yy,kk}{}^{ref}}$, which are the CSD terms that contribute the most to the maximization of the specimen response. The remaining terms are determined by the existing phase dependencies ($\phi_{jk} = \phi_{ik} - \phi_{ij}$).The integration of the Phase Pivoting Principle inside the EDRM algorithm provides always physically realizable and controllable MIMO targets.

3 Experimental validation

The experimental activity consist of a fatigue-testing campaign, in which 48 specimens are tested until complete rupture and the test specifications are combined according to different target generation techniques.The objective of the test campaign is to experimentally establish the S-N curves in order to compare the effect of the target generation techniques in terms of fatigue damage and time to failure.

3.1 Test set up

The experimental campaign is carried out by exploiting the three-axial electrodynamic shaker available at the University of Ferrara, namely Dongling 3ES-10-HF-500, shown in Figure 1 .This avant-garde excitation system is a 3-DoF shaker table capable of exciting test specimens in three orthogonal directions simultaneously.A cantilever beam with rectangular section is chosen as suitable geometry for the test specimen.

Figure 1: Experimental setup: 3-DoF shaker table Dongling 3ES-10-HF-500(left); top views of the specimen (right-top); side view of the specimen (right-middle); detail of strain gauge rosette applied on the notch (right-bottom).

Figure 2 reports the details of the geometry. U-shaped notches, placed near the location of maximum bending moment, identify the zones of prevalent damage accumulation. The specimen is milled from a bar made of aluminium alloy EN AW 6082, selected for its wide commercial availability. Its mechanical properties were inferred from literature [19]. A lumped mass of 0.47 kg is fixed at the specimen's free-end and used to accurately tune the resonant frequencies under the maximum working frequency of the shaker.

The specimen is mounted on the head expander by means of an ad-hoc designed fixture, as shown in Figure

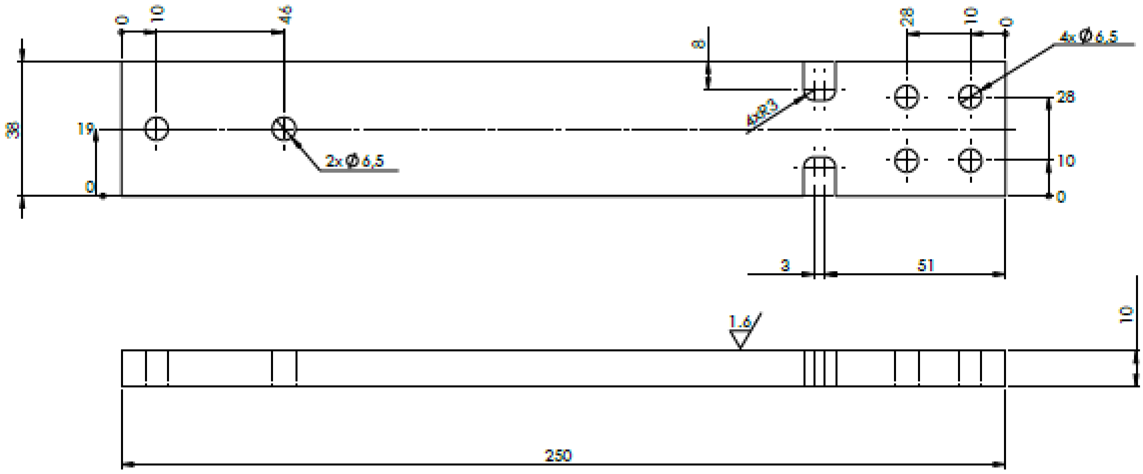

Figure 2: Detailed geometry of the test specimen (dimensions in mm).

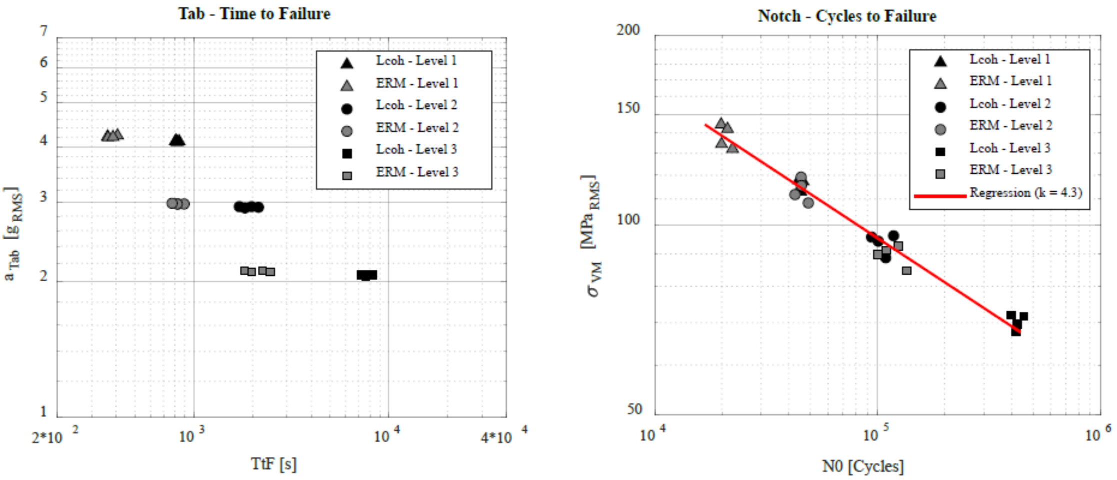

Figure 3: Fatigue-testing campaign: single-mode excitation results.

1. Labels Tab and Tip denote the locations of the two triaxial accelerometers utilized during the entire test campaign: Tab is the control accelerometer and it is fixed on the fixture close to the specimen mounting point; Tip is the measuring accelerometer and it is mounted at the specimen's free-end on top of the mass

3.2 Fatigue test campaign

To validate the EDRM, a series of multi-axial random control tests has been carried out until total breakage of the test specimens. During the test campaign the EDRM is applied for the maximization of the tenison at the notch of the specimen, with the objective of establishing the S-N curve under different MIMO target generation techniques. In particular, this analysis compares the novel proposed EDRM-max and the 0ph-Lcoh Method, that is the most commonly used procedure in standard practice. For each specimen the stress SDM and the corresponding von Mises PSD are calculated using the TFs matrix estimated during a pre-test phase exploiting a strain gauge rosette positioned on the maximum stress location at the notch. In accordance with the standard guidelines [20], 12 specimens have been chosen as total number of samples to be tested at 3 different stress levels. Statistically, this sample size should guarantee to correctly approximate the S-N curve by a straight line in the high-cycles fatigue region [21]. The test specifications are flat PSDs with the same RMS value for all three control direcrions (Tab accelerometer). The levels are 1.2, 1.7 and 2.4 gRMS. The considered MIMO target techniques are compared in two distinct exitation bandwidth :

- Single Mode Excitation : the excitation bandwidth is [10 - 85] Hz, used for exciting the first bending mode of the specimen (Mode I at 56 Hz)
- Multi Mode Exitation : the excitation bandwidth is [10 - 200] Hz, used for exciting the first two bending modes of the specimen (Mode I at 56 Hz and Mode II at 118 Hz)

The bandwidth is set in order to excite the desired modes even after the decay of natural frequency due to crack propagation [22].

3.2.1 Single mode excitation

The results of the Single Mode Excitation campaign are shown in figure 3. Note that in the "Tab-Time to failure" graph (left) the RMS of the table acceleration is calculated as the quare root of the squares sum of the RMS values of each control axis :

$$a_{Tab} = \sqrt{{a_{Tab,x}}^2 + {a_{Tab,y}}^2 {a_{Tab,z}}^2} \tag{8}$$

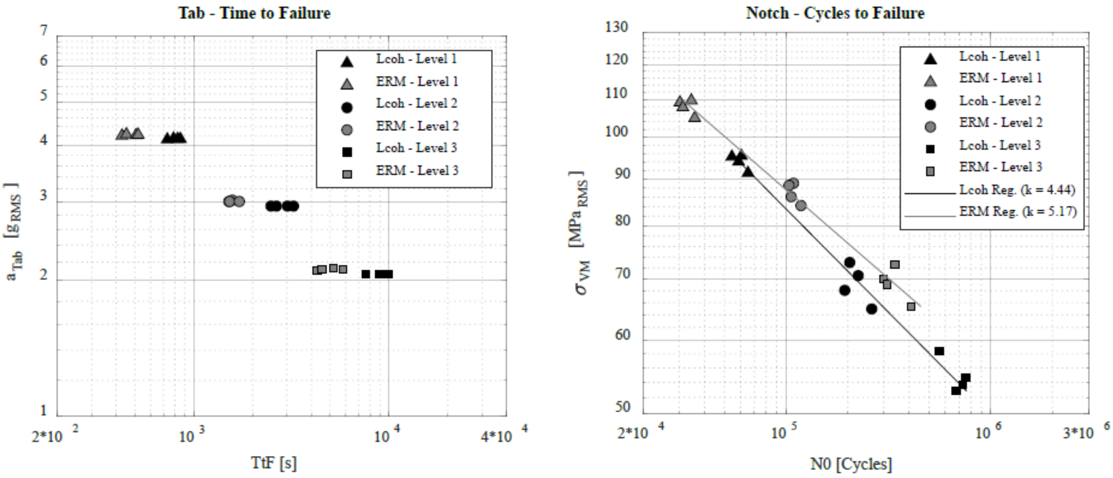

Figure 4: Fatigue-testing campaign: multi-mode excitation results.

The diagram highlights the potential of the proposed MIMO target generation technique (EDRM-max), which is capable of exploiting the full potential of the test specifications, yielding an higher response of the specimen and a much shorter time to failure. Moreover the test results highlight that the data distribution is approximated by the same line (red depicted line) with a slope k = 4.3 for both methods. Therefore it can be concluded that the two procedures cause the same fatigue damage and the same crack mechanism of the specimen. The results is in agreement with the assumption taken by Allegri and Zhang in [6] : if only one natural mode is excited, the resonse SDMs resulting from the EDRM and the 0ph-Lcoh method are simply scaled with one-another. It can be concluded that the EDRM-max can accelerate fatigue tasting with respect to the standard 0ph-Lcoh method, causing the same fatigue damage in a shorter time.

3.2.2 Multi Mode Excitation

The results of the fatigue tests conducted in multi-mode excitation are depicted in Figure 4. Also in this case, the figure presents two different diagrams: on the left, the overall RMS value of the table accelerations (a_{Tab}) is plotted against the time to failure in seconds; on the right, the RMS value of the equivalent von Mises stress (σ_{vM}) is represented against the total number of cycles (N0). Once again the results shows how the EDRM-max method yields shorter time to failure, with a reduction of the time until total breakage of the specimen between 39% and 45% for the three levels of excitation. On the other hand Figure 4 shows two different regression lines for the test data, displaying that, in multi-mode excitation, EDRM-max and 0ph-Lcoh method produce different fatigue damage accumulation and different fatigue behaviour. This is due to the fact that when multiple modes are concurrently excited, the response SDMs from different MIMO target techniques are diversely scaled at distinct natural frequencies.

3.2.3 Fracture surfaces analysis

As last comparison between the effects of fatigue testing with different MIMO target techniques, a fracture analysis of the broken specimens ah been carried out. The aim of the analysis is to determine the overall appearance of the fractured surface and to compare the macro-effects (beach marks, ratchet lines, surface flatness, etc.). The typical fracture surfaces of the specimens are shown in Figure 5

First of all, it is worth noting that all the fracture surfaces are flat, which indicates the absence of plastic deformation during the fatigue process [23]. The fracture surfaces produced by the 0ph-Lcoh Method and by the EDRM-max after testing with single-mode excitation (Figure 5, top) are very similar and comparable. All the various stages of the fracture appear to be practically identical. The section corners are the four major crack initiation sites. Two evident ratchet marks on the long sides of the section reveal where the

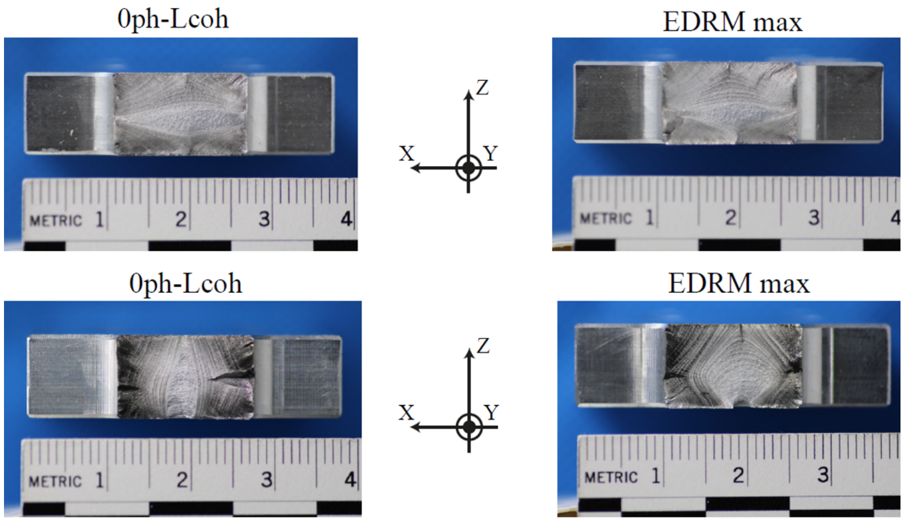

Figure 5: Fatigue-testing campaign: fracture surfaces obtained in single-mode excitation (top) and multi-mode excitation (bottom).

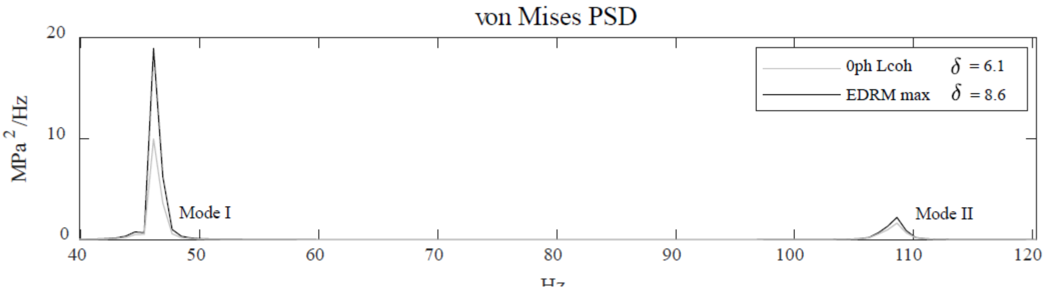

Figure 6: Fatigue-testing campaign: von Mises PSD obtained in multi-mode excitation.

cracks have been coalesced.The identical fracture surfaces are in agreement with the previous result shown in Figure 3: the experimental data-set of the two techniques are approximated by the same linear regression line, therefore the same crack mechanism occurs when using different MIMO target techniques. In case of multi-mode excitation (Figure 5, bottom), the two fracture surfaces exhibit some differences. By going side to side in the cross section, the beach markers define different arch-shaped lines, which point out a different direction of the crack propagation [24]. Furthermore, the shapes of the final fracture regions are different. In particular, the final fracture of the EDRM-max is more stretched along the X-direction than the one of the 0ph-Lcoh Method, suggesting a greater contribution of the first bending mode. This last assumption is confirmed by the resulting von Mises PSD calculated at the notch of the specimen, shown in Figure 6: the δ values are the ratio between the amplitude of the PSD at the two natural frequencies. The EDRM-max yields a higher δ, confirming the prevalence of Mode I. Obtaining two different fracture surfaces is in accordance with the results shown in Figure 4 (bottom) : the experimental data set of the two techniques are approximated by two different linear regressions, therefore different MIMO target techniques generate different crack propagation mechanism.

4 Concluding remarks

This manuscript investigates the effects produced by the use of different MIMO control target techniques on the fatigue-life of components subjected to environmental vibration control testing. Nowadays, the standard guidelines for durability test with simultaneous multi-axis random excitation suggest to combine the test specifications following the 0ph-Lcoh Method, i.e. defining the control SDM with decorrelated test specifications. The original contribution of this work is to provide a novel technique for the definition of the MIMO control target which is capable of maximizing the resulting dynamic response of specimen. The effectiveness of the EDRM has been proven during a fatigue-testing campaign, where 48 specimens have been tested until rupture. The EDRM has been compared with the 0ph-Lcoh Method in terms of fatigue damage, duration and failure modes of the notched specimens. Two different excitation bandwidths have been used: [10 - 85] Hz for exciting the first natural frequency and [10 - 200] Hz for simultaneously exciting the first two natural frequencies. For both the excitation bandwidths, the EDRM-max has provided shorter time-to-failure. Moreover, in single-mode excitation, the experimental data-set deriving from the two MIMO target techniques have been approximated by the same straight line, meaning that the specimens exhibit the same fatigue behaviour regardless of the adopted MIMO target technique. The result has been confirmed by the analysis on the fracture surfaces, which shows identical features. In this context, the EDRM-max can be considered as a methodology to accelerate the fatigue test, because it produces the same fatigue damage in a shorter time. On the pther hand in multi-mode excitation, the two techniques cause distinct S-N curves and fracture surfaces. It can be concluded that, when multiple natural modes are simultaneously excited, different combinations of the test specifications produce different fatigue behaviours of the specimen under test. Therefore, when the operational CSDs data are not available, there may be the risk of underestimating the fatigue damage produced during laboratory testing. The EDRM-max overcomes the problem exciting the specimen with the most damaging vibration environment and always guaranteeing the shortest time-to-failure. All the drawn observations have been obtained using ad-hoc machined specimens. However, despite the simple geometry involved, it is plausible that the outcomes of this work maintain validity also for real mechanical components.

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