Material and component based simplified vibroacoustic analysis approach for double wall application

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Abstract
Sandwich structures are widely used for aircraft secondary structure such as aircraft cabin side wall, ceiling and luggage bins. The need for quieter cabin is ever growing and aircraft manufactures are striving for multifunctional cabin structures. However, for a reliable prediction of cabin noise at the early design stage, it is paramount to have a strong confidence in modelling and simulation approaches for vibroacoustic analysis of sandwich structures used in double wall configuration. The aim of this paper is to address vibroacoustic numerical analysis of sandwich panel where finite element based studies at component level is supported by structural dynamic measurements at material level. The sound transmission loss (STL) under acoustic diffuse field excitation is used as a metric to validate the numerical studies with the measurements. The agreement for STL obtained using measurement and numerical studies on flat sandwich panel is found to be within ±1.0 dB. For curved sandwich panel, the agreement for STL is found to be within ±3.0 dB.

1 Introduction

The secondary structures such as aircraft cabin side walls, ceiling panels, floor panels and overhead storage compartments are normally manufactured using honeycomb core sandwiched between two face sheets [1]. The honeycomb core is glued to the face sheets using an adhesive interface layer between the core and the face sheet. Due to the complexities involved in the manufacturing of sandwich panels, it is very challenging and expensive for the manufacturer to provide the effective material properties of manufactured panels supplied to original equipment manufacturers (OEMs) of aircrafts. On the other hand, OEMs always need the effective material properties for the vibroacoustic numerical studies to access the cockpit and cabin noise levels during the early design stage of an aircraft with advanced future propulsion systems.

In this work, we have attempted to develop an approach where the vibroacoustic numerical studies on component levels are supported by the structural dynamic measurements performed at the material level where material properties are obtained using 1-D wave propagation model. A similar approach was used by Kurtze and Watters [2] where they proposed a new wall design for high sound transmission loss. Multifunctional aspect in the pioneering work of Kurtze and Watters was achieved by having a wall construction with significantly high ($\approx$1000) ratio of static to dynamic stiffness. Dym and Lang [3] corrected some of the published intermediate research and analysed the symmetric sandwich plate using variational formulation including symmetric as well as antisymmetric motion of the skin of the sandwich panel. The transmission loss calculation of Dym and Lang uses plane wave impedance where both symmetric and antisymmetric wave impedances are considered. Dym and Lang also reported a reasonable agreement of their analytic transmission loss calculation to the experimental work reported in Smolenski and Krokoskys [4] as well as in Ford, Lord, and Walker [5].

The overarching goal of this work can be graphically represented in Figure 1. As we improve the technology readiness levels (TRLs) moving from material level test either on sandwich beam or in impedance tube to Flight test, the size, complexity, reliability and cost of measurements increases significantly. Therefore, the idea of the developed approach discussed in this paper is to increase the starting TRL level without moving to next level (shown on the X-axis of Figure 1 with light blue text). The rest of the paper is organized into four sections. Following introduction, section 2 addresses the measurement method used to calculate sandwich material properties using roving impact test on sandwich beams. Section 3.1 discusses the validation of finite element model using the shear modulus derived from the measurements on sandwich beams. Section 3.2 addresses the sound transmission loss study performed using validated finite element model of flat sandwich panel. The sound transmission loss calculated using finite element approach is validated using measured sound transmission loss of the same panel at the transmission loss facility of AIRBUS in Hamburg, Germany. Section 3.3 discusses the sound transmission loss of curved sandwich panel together with comparison to the measured sound transmission loss results of AIRBUS. At the end, paper is summarized with conclusions.

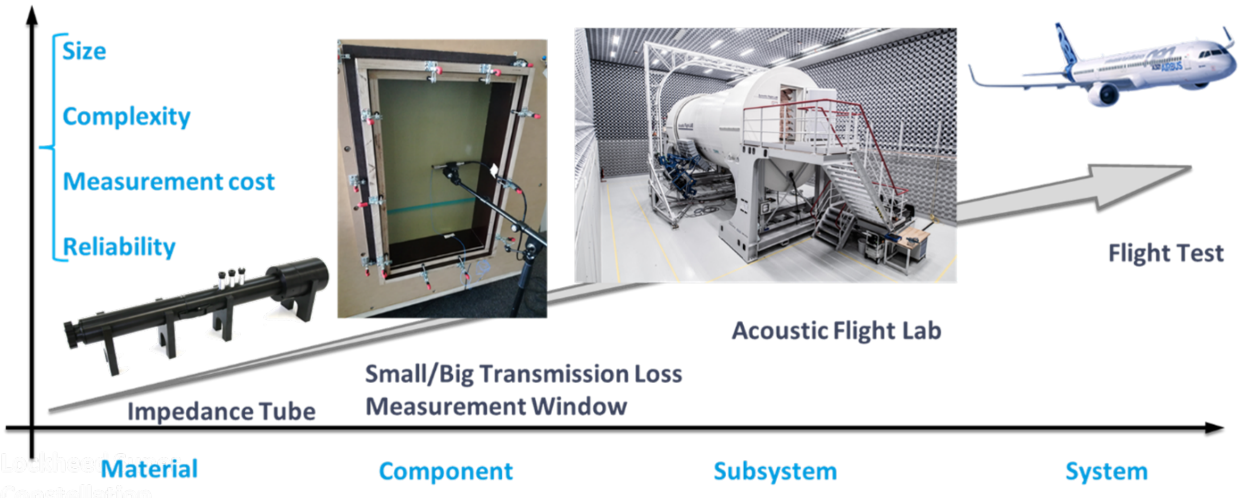


Figure 1: Multi-level approach for vibroacoustic analysis of aircraft or cabin noise-control development at various design stages

2 Measurement based method for finding effective material properties

2.1 Wave propagation model: 1-D

The structural dynamic response of a sandwich beam (as shown in Figure 2a) is dominated by bending as well as shear in the core. The equation of motion for a one dimensional sandwich beam without external force is given by a 6^{th} order partial differential equation and can be written as [6]:

$$\begin{aligned} & -2D_1D_2\frac{\partial^6\zeta}{\partial x^6}+2D_2I_\zeta\frac{\partial^6\zeta}{\partial x^4\partial t^2}+G_cSD_1\frac{\partial^4\zeta}{\partial x^4}-\left[\left(D_1+2D_2\right)m+G_cSI_\zeta\right]\frac{\partial^4 w}{\partial x^2\partial t^2}+ \\ & G_cSm\frac{\partial^2\zeta}{\partial t^2}+mI_\zeta\frac{\partial^4\zeta}{\partial t^4}=0 \end{aligned} \tag{1}$$

Considering plane wave propagation, $\zeta(x,t)=\zeta_0\cdot\exp\left[i\left(\omega t-k_x x\right)\right]$, in an infinite beam, we arrive at the following governing equation of motion in frequency domain:

$$\begin{aligned} & 2D_1D_2k_x^6-2D_2I_\zeta k_x^4\omega^2-\left[m\left(D_1+2D_2\right)+I_\zeta G_cS\right]k_x^2\omega^2+ \\ & G_cS\left[D_1k_x^4-m\omega^2\right]+mI_\zeta\omega^4=0 \end{aligned} \tag{2}$$

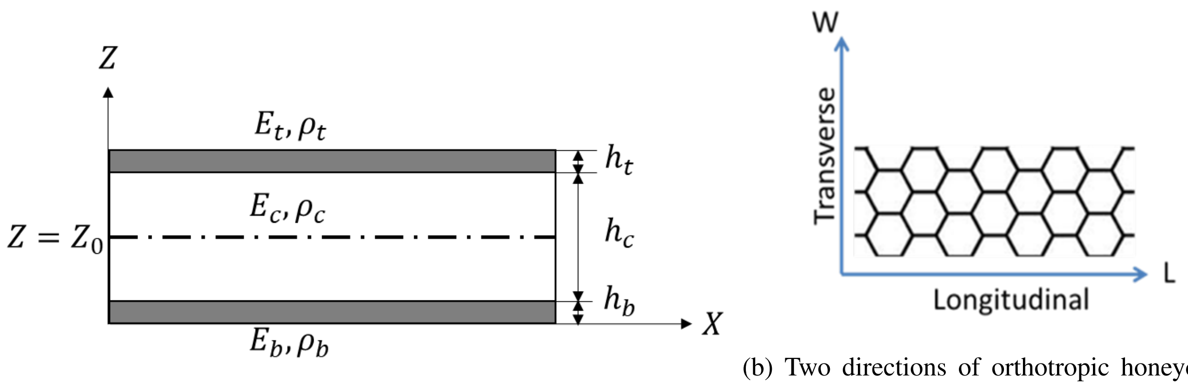
where, $D_1=b\left[\frac{E_ch_c^3}{12}+E_t\left(\frac{h_c^2h_t}{2}+h_ch_t^2+2\frac{h_b^3}{3}\right)\right]$ is the equivalent bending stiffness of the sandwich panel,

$D_2 = b\frac{E_t h_t^3}{12}$ is the face sheet bending stiffness, G_c is the effective shear modulus of the core, E_t or E_b is the modulus of elasticity for the face sheet, ω is the angular frequency in rad/sec, k_x is the wavenumber of the 1-D beam in x-direction, m is the mass density per unit length, I_ζ is the mass moment of inertia and $S = b.H$ is the area of the crossection of the core. The six solutions of the Equation 2 are given as: $k_x = \pm k_1, \pm ik_2$ and $\pm ik_3$. With the help of the equivalent bending stiffness approximation (i.e., $k_1^4 = m\omega^2/D_a$), we can write apparent bending stiffness (D_a) as a function of frequency ($f = \frac{\omega}{2\pi}$):

$$\frac{A}{f}D_a^{3/2} - \frac{B}{f}D_a^{1/2} + D_a - C = 0 \tag{3}$$

In Equation 3, A, B and C are constants and can be written in terms of global bending stiffness and bending stiffness of face sheet as follows:

$$A = \frac{G_c S}{\sqrt{m}2\pi D_1}; \quad B = \frac{G_c S}{\sqrt{m}2\pi}; \quad C = 2D_2 \tag{4}$$



(a) Symmetric sandwich beam/panel geometric parameters

(b) Two directions of orthotropic honeycomb sandwich panel material

Figure 2: Symmetric sandwich panel

The following subsection addresses the structural dynamic measurements for the calculation of apparent bending stiffness (D_a) as a function of frequency (f).

2.2 Measurement on sandwich beam

The sandwich panels are generally considered as orthotropic in nature. The two directions which define the orthotropy in sandwich panel are due to the configuration of the sandwiched honeycomb cell orientation. These two orthotropic directions are shown in Figure 2b. To obtain the material properties in two orthotropic material directions, two beams are cut from the sandwich panel and roving impact tests are performed on the beams. The test set-up for the impact test on a free hung sandwich beam can be seen in Figure 3. The eigen frequencies and corresponding mode orders obtained using impact tests are used to get the apparent bending stiffness of the sandwich beam in L and W directions. The obtained frequency dependent apparent bending stiffness is then used to fit a curve defined in Equation 3. The least-square-fit of apparent bending stiffness to measurement points is shown in Figure 4. Such a least square fit gives the values of the coefficients. A, B and C, respectively. Coefficient B gives the shear modulus of the core for the beam in L direction. A correct value of shear modulus is necessary for building the confidence in vibroacoustic numerical studies on honeycomb sandwich structures in the mid frequency region. Similarly, one can repeat the above procedure to get the shear modulus of the sandwich beam in W direction. Once the shear modulus is obtained, the next step is to use these parameter to develop the finite element based simulation model for vibroacoustic numerical studies.

Figure 3: Free-hung sandwich beam and a simplified impact test set-up

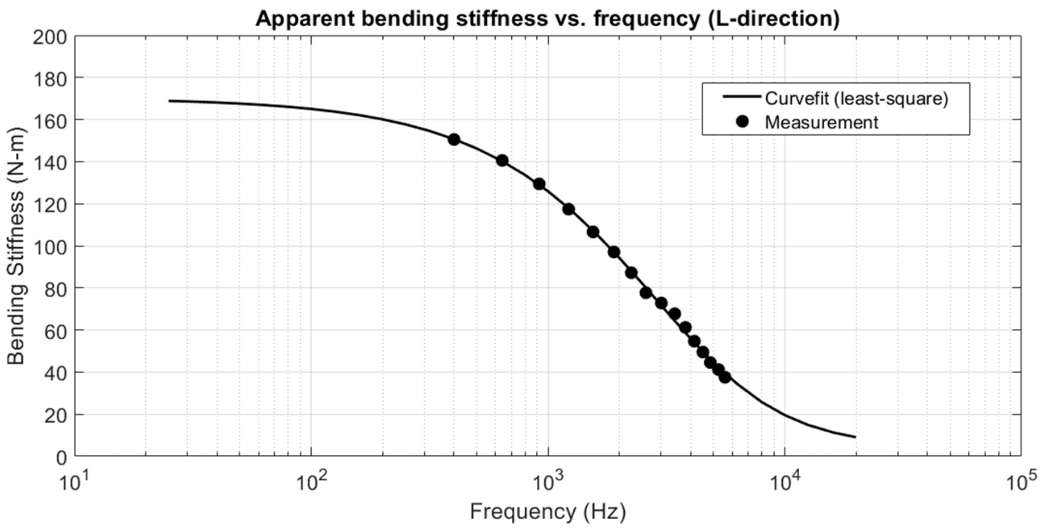
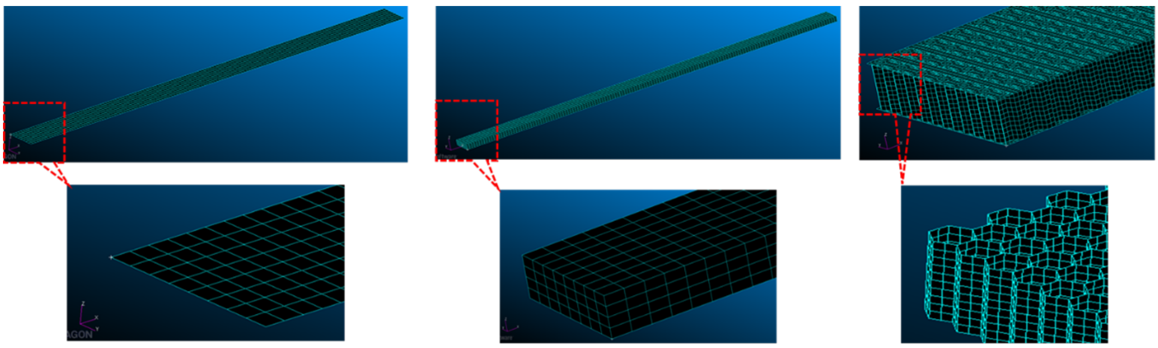


Figure 4: Apparent bending stiffness (least square curve fit and measured) of a symmetric sandwich beam

3 Numerical model development for vibroacoustic analysis

3.1 Finite element method based numerical model

This section addresses the finite element model development for the selected honeycomb sandwich beam/panel. Three approaches for finite element model generation are investigated. These three approaches are depicted in Figure 5. The FEM approach - I treats the sandwich panel core as well as top and bottom face sheets as three layers in the laminate using PCOMP material card in NASTRAN [7]. In FEM approach - II, the core is modelled as solid finite elements with shell element for the face sheets. Whereas, FEM approach - III uses shell elements for the modelling of hexagonal honeycomb core as well as the face sheets. Based on computational efficiency and FEM data handling, it is found out that the FEM approach – I is more practical when vibroacoustic numerical studies are performed on an industrial scale models (e.g., full scale aircraft cabin). Therefore, we selected FEM approach – I for vibroacoustic numerical studies at the component level. However, it is also found out that the FEM approaches -II and -III could be extremely useful for numerical studies at material level (e.g., impedance tube probes, small beams, etc.) where number of finite element degrees of freedom are still possible for FEM based matrix calculations.



(a): FEM approach-I (lamina approach)

(b): FEM approach-II (equivalent solid approach)

(c): FEM approach-III (meso-scale modelling approach)

Figure 5: Three approaches for finite element modelling of honeycomb sandwich beam/panel

Once FEM approach - I is selected for further numerical studies, we validated the finite element model of the beam with material properties of the core obtained from the measurement discussed in section 2.2. Figure 6 shows the result of the validation test where, measured acceleration in sandwich beam due to point load is compared with the acceleration obtained using FEM approach - I. A very good match between measurement and simulation validates the applicability of FEM approach - I with derived material properties from the impact test.

3.2 Vibroacoustic study at component level: Flat sandwich panel

For vibroacoustic studies at component level, a flat panel made out of material that is used for an aircraft side wall (i.e., honeycomb sandwich core with composite face sheets) is selected for vibroacoustic numerical as well as measurement based studies. At first, a modal test is performed on a free hung sandwich panel. Following paragraphs discuss the modal test and compares the corresponding numerical results obtained using finite element method. A free hung sandwich panel used for the modal test is shown in Figure 7. In total, 247 impact points were used while performing a roving impact test. Measured mode shapes and eigenvalues are then compared with the finite element based modal analysis results.

The comparison of numerical and measured mode shapes and eigenvalues is shown in Figure 8. These first eight modes of sandwich panel show reasonably good match with FEM based mode shapes and eigenvalues. The outcome of this modal test led us to investigate the vibroacoustic response of the sandwich panel subjected to random acoustic diffuse field.

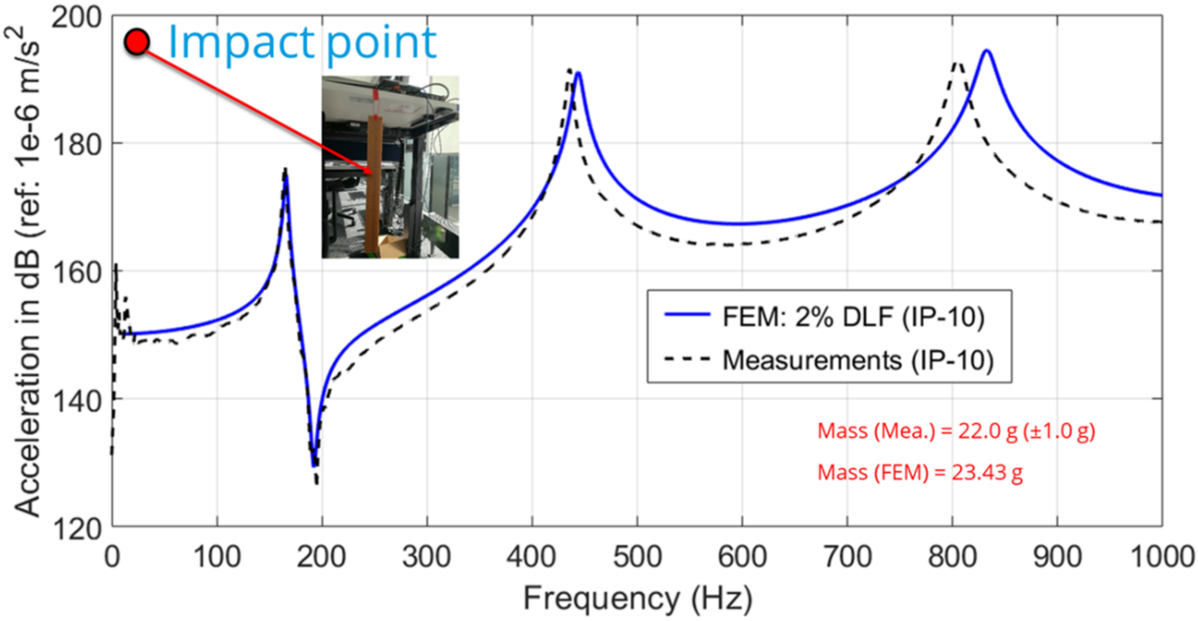


Figure 6: Frequency response function of a symmetric sandwich beam subjected to unit point harmonic load (IP-10: impact point number 10, DLF: damping loss factor)

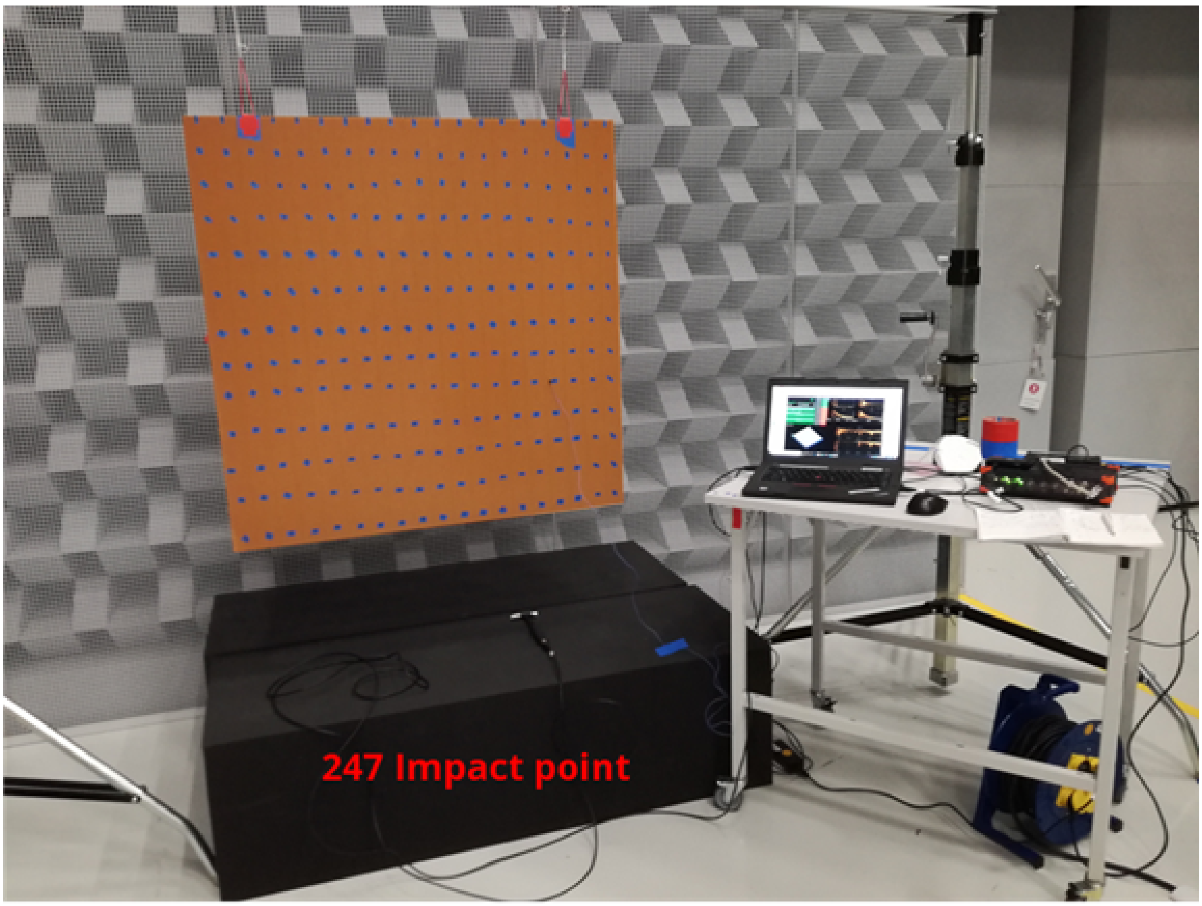


Figure 7: Freely hung flat sandwich panel for modal test

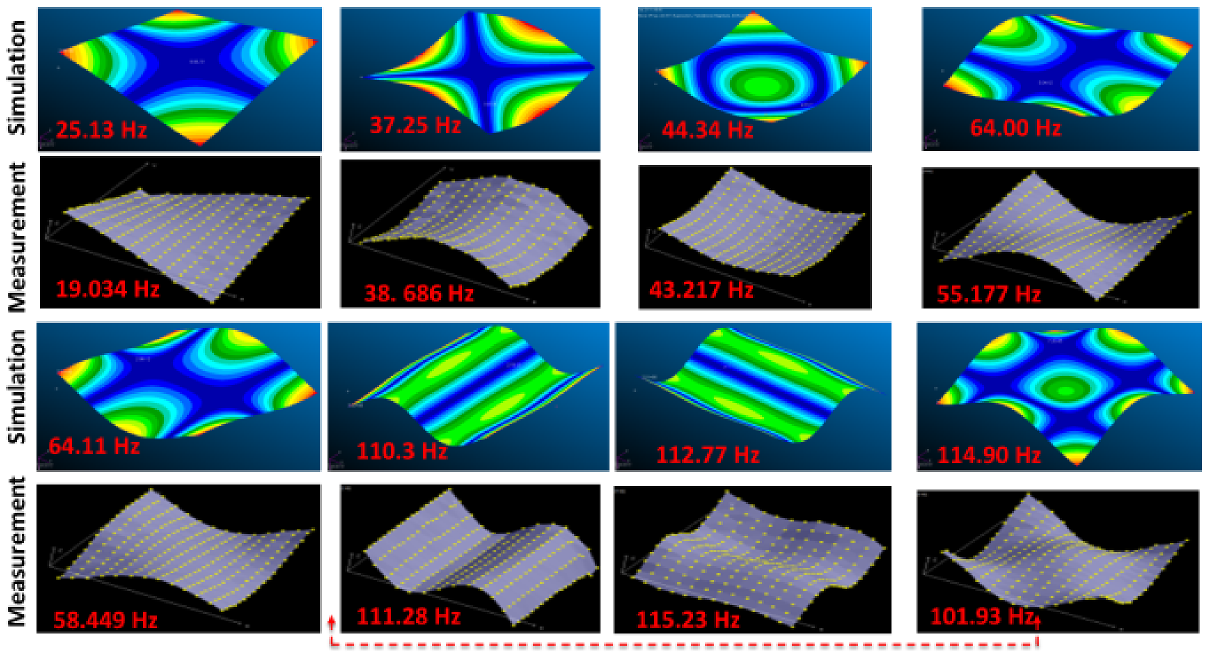


Figure 8: Measured (modal test) and numerical (FEM based) mode shapes and corresponding eigenvalues

A detailed description of the mathematics to generate acoustic diffuse field for vibroacoustic numerical studies is beyond the scope of this manuscript and readers are referred to earlier work of author [8, 9] as well as to Fahy [10] and Pierce [11]. However, for the completeness of this manuscript, the cross power spectral density [11] of acoustic diffuse field is briefly discussed here and can be written as:

$$S(r,\omega) = S_0(\omega)S_{sc}(r,\omega) \tag{5}$$

where $S_0(\omega)$ is the auto power spectral density and $S_{sc}(r,\omega)$ is the spatial correlation of the applied random load. The distance r is used to represent radial location in a spherical coordinate system. For random acoustic diffuse field, $S_{sc}(r,\omega)$ is known to have a spatial distribution given by the following equation:

$$S_{sc}(r,\omega) = \sin(kr\sin(\theta_{cr})/kr \tag{6}$$

In Equation 6, $k = 2\pi/\lambda$ is the wavenumber with λ being the wavelength of the plane propagating wave used for generating random acoustic diffuse field and θ_{cr} is the limiting maximum angle of incidence of the plane propagating wave. In this work, θ_{cr} is 78^0 and avoids the grazing incidence problem when θ_{cr} tends to 90^0. Random excitations such as acoustic diffuse field is assumed to be stationary random process and can be simulated using sampling techniques. Therefore, a total of 30 plane waves impinging on panel from all random directions are considered for generating such random acoustic diffuse field. For such random excitation, STL is used as a metric for comparing vibroacoustic response of sandwich panel with measurement result discussed in this paper. The following equation is used to calculate STL in dB:

$$STL(dB) = -10\,log(\tau) \tag{7}$$

where, τ is known as sound transmission coefficient and it is defined as the ratio of radiated sound power and input power. The STL measurements on flat sandwich panel was performed by AIRBUS at their full scale STL test facility. To capture the uncertainty in the values of damping, two structural loss factors, 2% and 5%, respectively are used in the numerical model in ACTRAN [12]. The measured and numerical results for STL of flat sandwich panel are compared in Figure 9. For a frequency band of $200.0Hz$–$1000.0Hz$, a good match is observed between measured and numerical values of STL. A ± 1.0 dB difference between measured and numerical (i.e., 5% loss factor) STL is considered to be in line with inherent uncertainties in both numerical and measured STL. However, below 200.0 Hz, we observe a strong discrepancy between measured and FEM results. We suspect that such discrepancy might be because of the difference between theoretical boundary condition in numerical calculation and practical boundary condition in the STL test set-up. It is however open for future investigation. A strong agreement of STL for 200.0 Hz – 1000.0 Hz band (as shown in Figure 9) validates the finite element model and corresponding input parameters to be used for further studies on curved sandwich panel addressed in the following subsection.

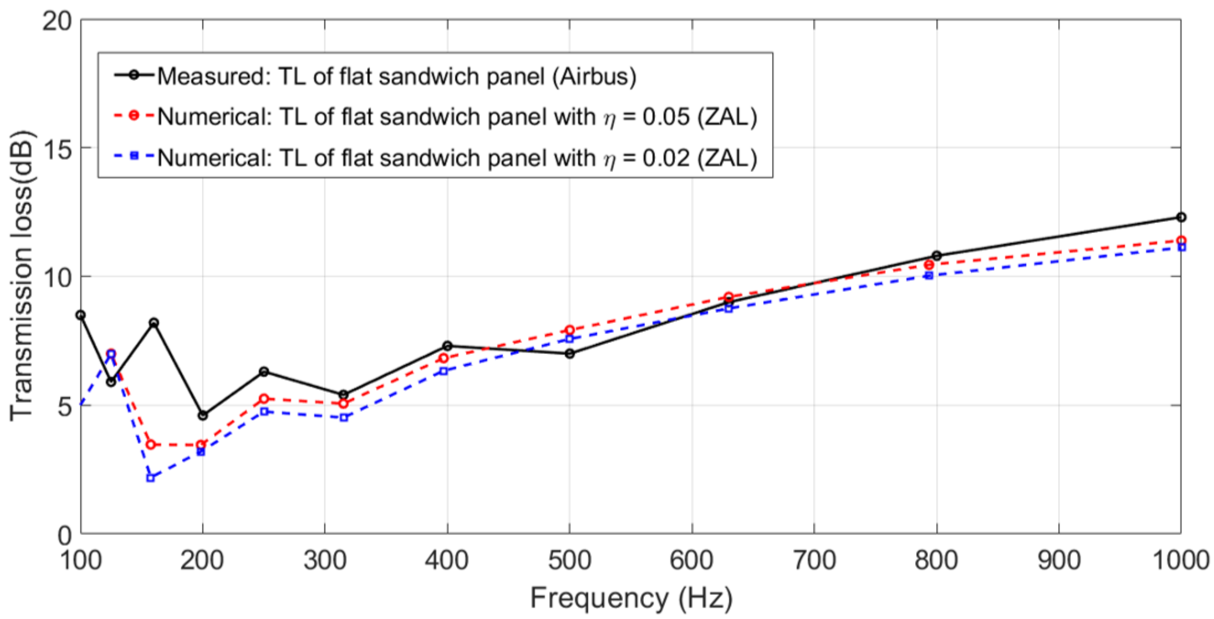


Figure 9: Measured and numerical STL of flat honeycomb sandwich panel

3.3 Vibroacoustic study at component level: Curved sandwich panel

Typically aircraft secondary structures such as cabin sidewall have curved geometry. Therefore, the proposed measurement based approach for numerical vibroacoustic studies is used to access the STL of curved honeycomb sandwich panel as shown in Figure 10. For flat panel, Rayleigh integral approach implemented in commercial code, ACTRAN is used. Rayleigh integral approach works well for planar structure. However, for a curved panel, either boundary element method or finite element method with infinite element approach or perfectly matched layer is necessary. In this work, we have used finite - infinite element based approach. Though, an exhaustive discussion is not the scope of this manuscript, a brief summary of the approach is discussed in the following paragraph.

As a starting point for the calculation of sound radiation/propagation in linear acoustics, the following Helmholtz equation for an unbounded domain is solved.

$$\nabla^2 p(\boldsymbol{x}) + k^2 p(\boldsymbol{x}) = 0, \boldsymbol{x} \in \boldsymbol{\Omega_f} \subset \mathbb{R}^d, \quad d = 3 \tag{8}$$

The steady-state solution of Equation 8 is obtained using appropriate boundary conditions and the solution is given as $\tilde{p}(\boldsymbol{x}, t) = \Re\left\{p(\boldsymbol{x})e^{i\omega t}\right\}$, with $\tilde{p}(\boldsymbol{x}, t)$ representing the time-harmonic sound pressure and $p(x)$ giving the sound pressure amplitude. The wavenumber is provided by $k = \omega/c$ and the speed of sound in the acoustic fluid is given by c. The circular frequency is given as $\omega = 2\pi f$ with f being the frequency of the problem in Hz. The exterior or unbounded acoustic problems (e.g., free field sound radiation from planar structures) are solved in a fluid domain of an infinite extent. For the application of finite element method for such problems, the unbounded fluid domain is divided by an interface Γ_c into a finite fluid domain Ω_f and an exterior domain (Ω_e) which is modelled using infinite element. A simplified representation of various domains (i.e., finite (interior), infinite (exterior), interface) of the coupled fluid-structure-interaction problem is shown in Figure 11. For the original concept of infinite elements for wave propagation, readers can refer

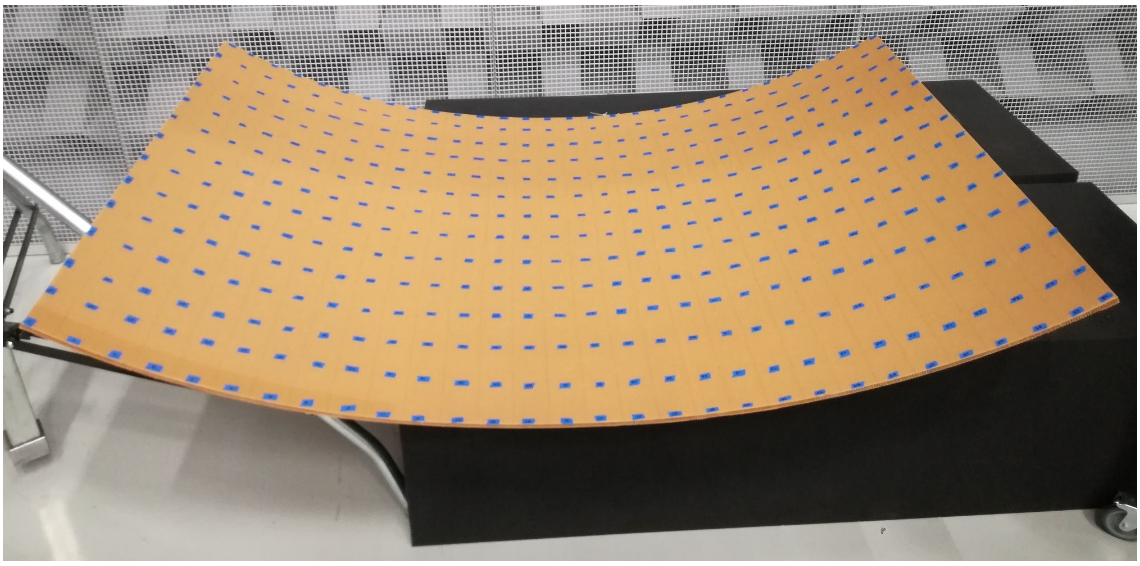


Figure 10: Curved honeycomb sandwich panel

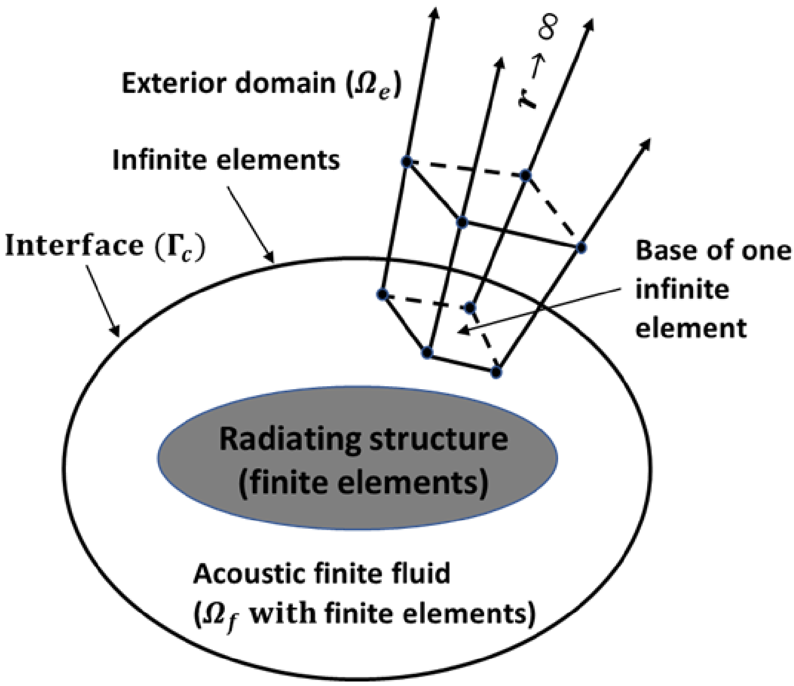


Figure 11: A simplified representation of sound radiation analysis using finite element and infinite element approach using FEM

to the work of Zienkiewicz and Bettess [13, 14]. Furthermore, research articles by Astley [15] also provide a review of formulations and an assessment of the accuracy of infinite elements. Like finite element analysis, the coupled finite - infinite element based problem formulation can be given by the following set of equations written in matrix form:

$$\left\{ \begin{bmatrix} \boldsymbol{K}_f & \boldsymbol{K}_{fe} \\ \boldsymbol{K}_{ef} & \boldsymbol{K}_e \end{bmatrix} - i\omega \begin{bmatrix} \boldsymbol{D}_f & \boldsymbol{D}_{fe} \\ \boldsymbol{D}_{ef} & \boldsymbol{D}_e \end{bmatrix} - \omega^2 \begin{bmatrix} \boldsymbol{M}_f & \boldsymbol{M}_{fe} \\ \boldsymbol{M}_{ef} & \boldsymbol{M}_e \end{bmatrix} \right\} \begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{p}_e \end{pmatrix} = \boldsymbol{F} \tag{9}$$

In Equation 9, the index e is used to denote quantities belonging to the infinite domain. The matrices K_f, D_f, and M_f are finite element matrices of the fluid domain with the contribution of the finite domain. The coupling matrices are denoted by K_{fe}, D_{fe}, M_{fe} matrices with switched indices. For further information, readers can refer to Felix et al. [16] where a 2-D problem using such combination of finite-infinite element approach with Petrov-Galerkin scheme is discussed thoroughly. In this research work, we have used the available finite-infinite element approach in commercial software ACTRAN.

As a next step, finite element analysis is performed to solve coupled fluid-structure-interaction problem in ACTRAN with applied boundary condition to satisfy the Sommerfeld radiation condition using available infinite element in the commercial solver. The geometry of radiating structure (i.e., curved sandwich panel) and corresponding fluid domain are shown in the middle and right of the Figure 12, respectively. Five acoustic fluid surface boundaries not facing the curved sandwich structure are discretized using acoustic infinite elements and represent the unbounded (or infinite) domain as shown in the left of Figure 12.

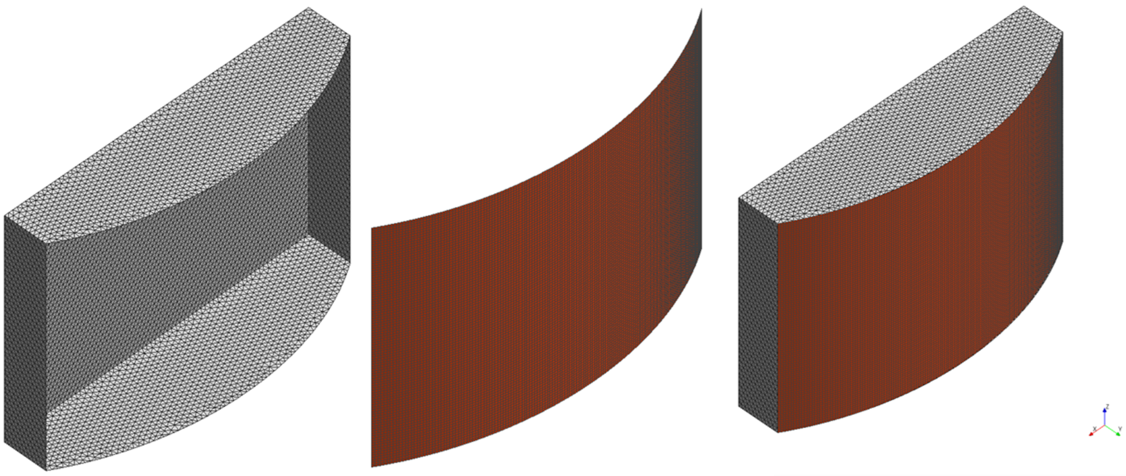


Figure 12: Curved sandwich panel for vibroacoustic analysis using finite element and infinite element approach

An infinite domain for the problem addressed in this manuscript is defined by three parameters. The first parameter is the material of the domain (i.e., air). The second parameter is the radial interpolation order and the third parameter is the reference ellipsoidal coordinate system which facilitates the truncated multipole expansion as described in ACTRAN user's guide [17]. It is to be noted here that the finite element mesh for structure and fluid is non-matching. Therefore, to perform coupled fluid-structure-interaction analysis, fluid and structure coupling surfaces are coupled through a defined interface while setting-up the analysis in the solver. A frequency range of 100 to 1000 Hz with a 1.0 Hz step is defined for vibroacoustic response under acoustic diffuse field. Figure 13 compares the STL calculated using measurement and numerical simulation. Three cases of damping (2.0% and 5.0% loss factor for structure with 0.0% fluid damping and 5.0% loss factor for structure with 0.1% fluid damping) are considered while calculating transmission loss using FEM. Based on STL comparison in Figure 13, it can be observed that the STL for curved sandwich panel is lower than the measured STL and lies within ± 3.0 dB for a frequency band from 200.0 Hz to 1000.0 Hz. For below 200.0 Hz, similar conclusion can be drawn as observed for the flat sandwich panel and will be addressed in future.

4 Conclusion

An approach for multi-level vibroacoustic analysis is discussed. Each stages of analysis (i.e., numerical and measurement based) are utilized in increasing the fidelity of the results. Sandwich beams are used to obtain shear modulus of honeycomb sandwich panel in two directions of orthotropy. 1-D plane wave propagation model is applied to both sandwich beams to calculate apparent bending stiffness using roving impact test.

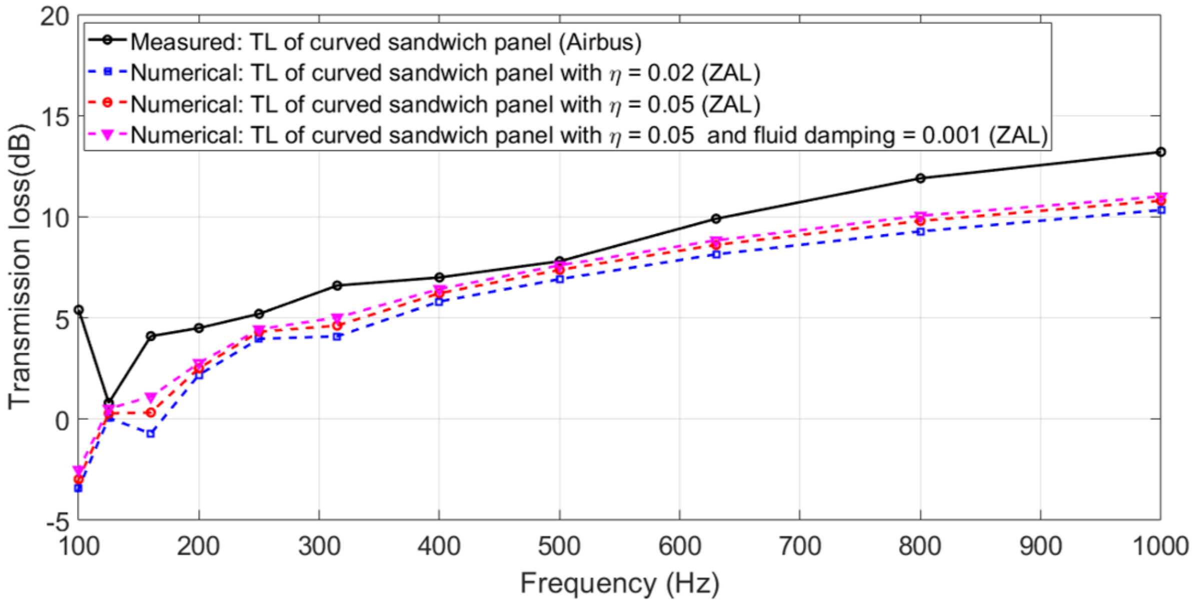


Figure 13: Measured and numerical sound transmission loss of curved honeycomb sandwich panel

A least-square-fit to apparent bending stiffness curve gives the shear modulus of the core in two directions. These material parameters are then used to validate the finite element model of beam and flat sandwich panel using modal test. The validated finite element model of the sandwich panel is then used to predict random acoustic diffuse field STL. The predicted STL is afterward validated with measured STL at a full scale STL test facility of AIRBUS. The predicted STL of flat sandwich panel using finite element method shows strong agreement for a frequency from 200.0 Hz to 1000.0 Hz. Thus, the established approach for STL prediction using finite element method supported by impact test on beams of same material only, can be used at the design stage when small test samples (i.e., beams) of material are available. Similar results are reported for STL of curved sandwich panel. Quantitatively, for flat sandwich panel, numerical STL results are within ±1.0 dB when compared to measured STL. Whereas, for curved sandwich panel, numerical STL results are within ±3.0 dB when compared to measured STL. The discrepancy between numerical and measured STL for both (i.e., flat and curved sandwich panels) below 200.0 Hz is most likely due to difficulty in exact simulation of the test set-up. However, it is open for future investigation.

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