Performance and robustness of a 3D metabsorber

E. Sadoulet-Reboul, E. Bachy, K. Jaboviste, G. Chevallier
Université Bourgogne Franche-Comté, Department of Applied Mechanics, FEMTO-ST Institute
24, chemin de l'Epitaphe, 25000 Besancon, France
e-mail : **emeline.sadoulet-reboul@univ-fcomte.fr**

Abstract

Multiple configurations of tuned mass dampers are recognized as strategies to enhance the performance of such vibratory control systems, and robustify them. This study focuses on the numerical and experimental robustness of a 3D metabsorber consisting in a set of beam-like absorbers with varying lengths, designed to control the third bending mode of an aircraft model. Optimal frequency distribution for different configurations with increasing number of absorbers are determined using Finite Element modeling and energy computations. The robustness of these designs to a lack-of-knowledge on the frequency of the structure to control is quantified using the Info-Gap Theory which is a non-probabilistic decision-support tool in uncertain context. Experimentations are performed to validate the results, and the effective robustness of the different metabsorbers is estimated integrating a modal expansion approach. Results are in good agreement and show that for the configuration studied, performance and robustness can be ensured with a limited number of absorbers in the metabsorber.

1 Introduction

Tuned Mass Dampers (TMDs) are vibration control strategies known for their effectiveness, but lacking robustness when vibration conditions change. One solution in this case is to use multiple solutions obtained by associating in a network absorbers tuned to different frequencies. The optimal distribution of these frequencies is classically determined in the literature by considering an association of discrete mass/spring/damper systems whose dynamic properties are determined by solving an optimization problem that considers the structural displacement as the performance measure [1, 2]. This approach has the advantage of being efficient, but it does not take into account the real topology of the absorbers, nor their spatial location for example. Recent works [3] have thus studied the impact of location on a structure of spatially distributed TMDs. The work presented here aims to develop a methodology for the design of a 3D network of absorbers, called metabsorber. In order to reduce the computation time linked to this modeling, the approach is based on a modal decomposition and the calculation of global energy quantities. The methodology is applied to design a network aiming at controlling a bending mode of an aircraft model. Several configurations of metabsorbers are studied, they differ by the number of specific frequencies in the network. A first objective is to compare the vibration attenuation performances of all these configurations. A second objective is to compare the robustness of these configurations in the presence of a modification of the natural frequency of the mode to be controlled. The design of TMDS by integrating the uncertainties related to the material or mechanical data, to the dimensions has been the subject of various studies in the literature in order to propose a robust optimal design : quite conventionally, uncertainties are introduced through probabilistic quantities to quantify the mean and the variance of the outputs of interest of the problem [4, 5]. The problem addressed in this work is the case of a detuning of the natural frequency of the structure to be controlled, which corresponds to a lack-of-knowledge that cannot be described in a probabilistic way. The proposed approach is based on the Info-Gap method [6] which is a decision support tool adapted in an uncertain non-probabilistic context, and which is used here to quantify the robustness of the metabsorbers. It has already been applied for identification of an appropriate model in presence of lack-of-knowledge [7], to study the collapse resistance of structures under uncertain loads [8], or to quantify the damping performances of viscoelastic materials in an uncertain temperature environment [9]. The design approach is implemented on an aircraft model, which has also been built to compare numerical and experimental results.

2 Optimal design of a 3D metabsorber

2.1 Presentation of the study case

The main structure studied in this paper is shown in Figure 1.

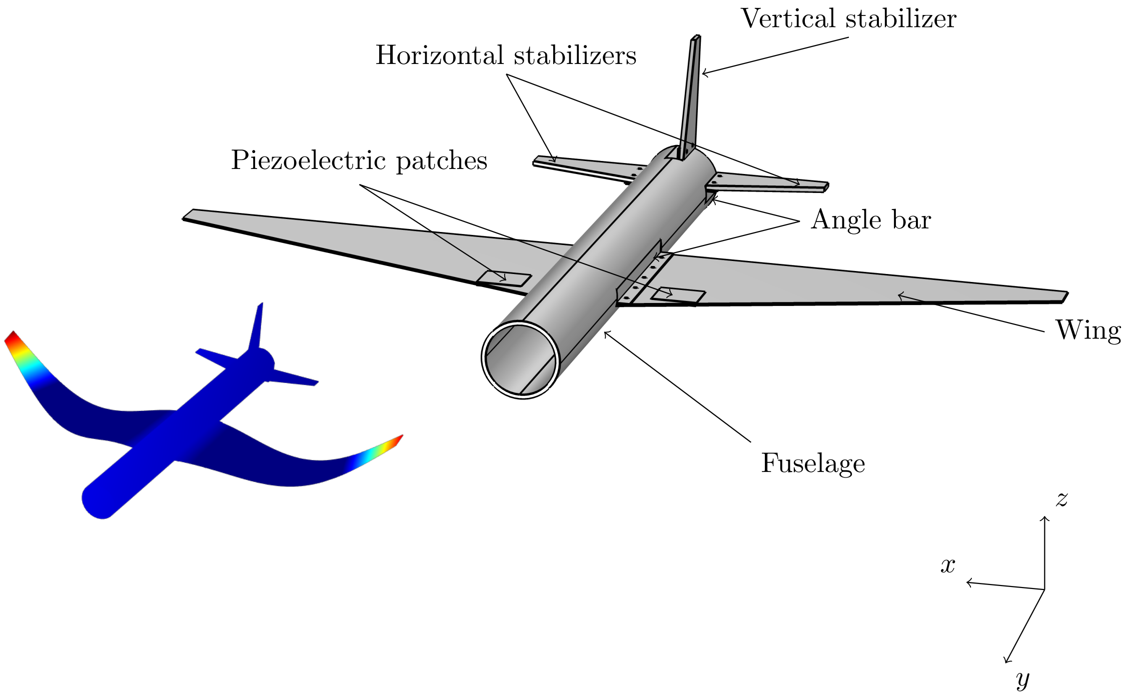


Figure 1: The study case concerns the design of a metabsorber to control the third bending mode (here shown in colors) of an aircraft model.

The metabsorber consists of N beam absorbers, which can have different properties in order to modify the frequency distribution of the control solution (see Figure 2). Six different configurations are studied to evaluate their respective performances and robustness, all absorbers are identical for the first configuration, and different for the last one. The intermediate configurations correspond to distributions with 2, 3, 6 and 9 different natural frequencies in the metabsorber. The dynamic behavior of the structure equipped with the metabsorber is studied by the Finite Element method in order to take into account the exact topology of the components, as well as the location of the metabsorber on the main structure.

2.2 Numerical optimization of the metabsorbers

To reduce computational costs, the vibratory response of the structure $\hat{\mathbf{U}}$ is projected into the basis $\mathbf{\Phi}$ of the real modes such that,

$$\hat{\mathbf{U}} = \mathbf{\Phi}\hat{\mathbf{q}}. \tag{1}$$

The dynamic equations for the reduced model are thus written as,

$$\left(\mathbf{k}^* - \omega^2\mathbf{m}\right)\hat{\mathbf{q}} = \hat{\mathbf{f}}, \tag{2}$$

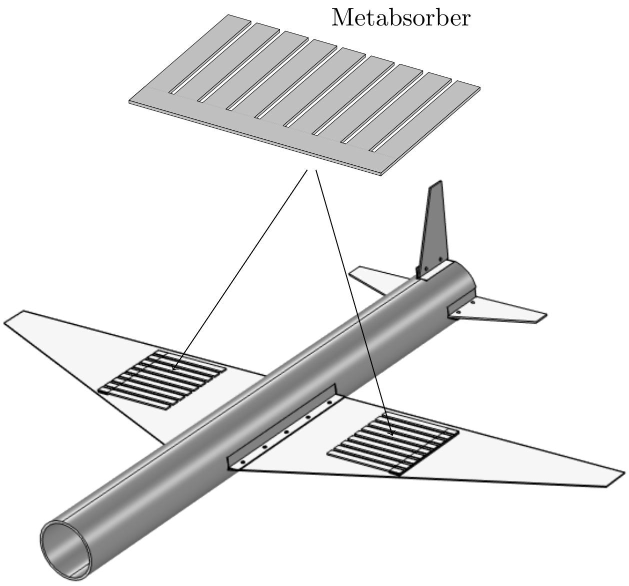


Figure 2: The metabsorber consists in a set of beam-like absorbers attached on the wing of the airplane to control: the configuration is here presented for a uniform distribution such that all the beams have the same length. Computations are done on the wing surface opposite to the metabsorber.

where $\mathbf{m} = \mathbf{\Phi^T M \Phi}$, $\mathbf{k}^* = \mathbf{\Phi^T K^* \Phi}$ and $\hat{\mathbf{f}} = \mathbf{\Phi^T}\hat{\mathbf{F}}$ are the projected operators. $\hat{\mathbf{q}} = \mathbf{\Phi^T}\hat{\mathbf{U}}$ are the generalized coordinates. $\mathbf{k}^*$ is the sum of $\mathbf{k}^*_\mathbf{s}$ that is the reduced matrix for the main structure, and of $\mathbf{k}^*_\mathbf{abs}$ that is the reduced matrix for the absorbers. A deterministic optimization is performed to determine the optimal frequency distribution in the different metabsorber configurations in order to reduce vibrations over a chosen frequency range between ω_{min} and ω_{max}. The chosen optimization variables are coefficients α_i ($1 \leq i \leq N$) introduced on the stiffness of each absorber of the metabsorbers : the case where all the α_i are equals corresponds to a uniform distribution and can be assimilated to an equivalent single TMD (1-TMD), the case where all the α_i are different corresponds to a N-TMD metabsorber. The metabsorbers are in PMMA, the added mass is almost 4.5% of the wing mass, and the structural damping is fixed at the value of 6.6% corresponding to the value previously identified from an experimental test using an Oberst approach.
The objective function chosen is the minimization of the elastic strain energy on the frequency band of interest, which has the advantage of being a global quantity ensuring that the metabsorber is efficient on the whole structure, and not only locally. Moreover, the computational time required to estimate this quantity is low when working with projected operators. For an harmonic excitation, the average strain energy over the vibration period is defined using the reduced operators as,

$$E(\omega, \alpha_i) = \frac{1}{4}\hat{q}(\omega, \alpha_i)^H \mathbf{k_s} \hat{q}(\omega, \alpha_i) \tag{3}$$

where H denotes the hermitian or conjugate transpose. The optimization problem can thus be written as,
The lower and upper bounds for the parameters are arbitrary defined as $\alpha_{min} = 0.5$ and $\alpha_{max} = 1.5$, and the

Given	$\omega_{min}, \omega_{max}, \alpha_{\min}, \alpha_{\max}$
Find	$\alpha_i, 1 \leq i \leq N$
Minimizing	$\int_{\omega_{min}}^{\omega_{max}} E(\omega, \alpha_i)\, d\omega$
Subject to	$\alpha_{\min} \leq \alpha_i \leq \alpha_{\max}$

parameters are initially fixed to 1. The optimization problem is solved using the fmincon function available in mathematical software package MATLAB. Once the the distribution of the α-coefficients is determined, the corresponding topology of the absorbers is determined by considering them as embedded-free beam.

2.3 Numerical results

Figure 3 presents the average of the vibratory frequency responses (receptances) obtained on the outer surface of the wing, for the different metabsorber configurations.

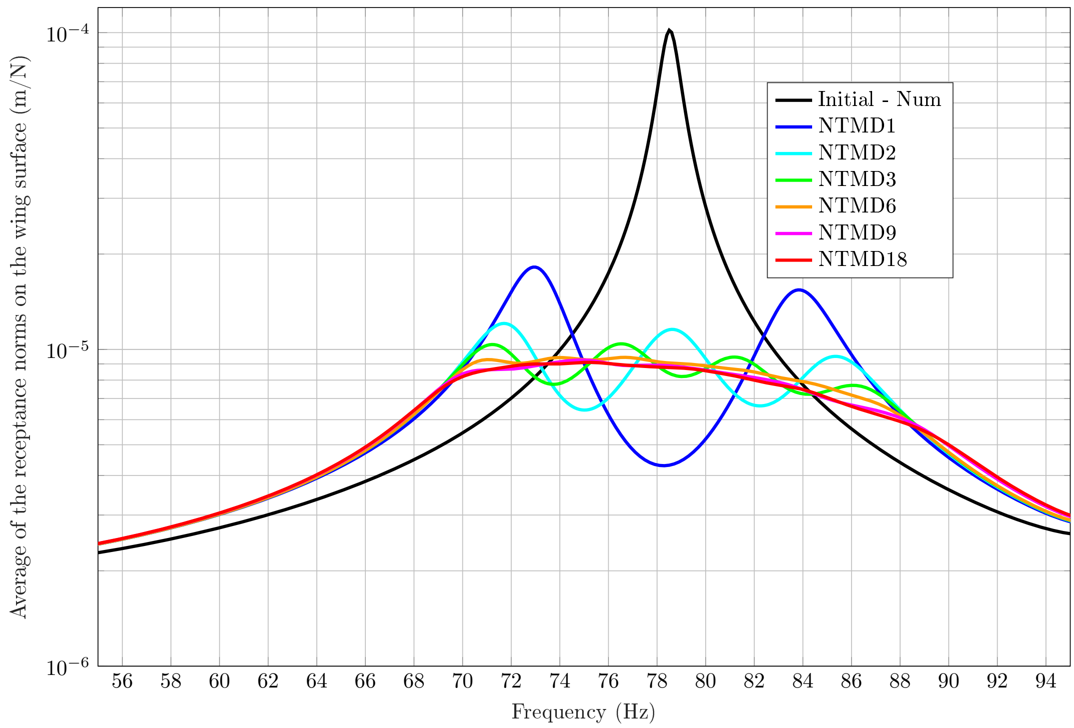


Figure 3: Average of the vibratory frequency responses (receptances) on the wing surface for the initial airplane and for the airplane with metabsorber NTMD1/2/3/6/9/18 (Legend color on the web version of the article)

It can be seen that the response obtained for a configuration that can be assimilated to a single absorber leads to the emergence of two secondary resonances, and that the complexification of the frequency distribution leads to an attenuation of these additional resonances such that the vibratory response is considerably attenuated. We can notice a stabilization effect as results obtained with configurations NTMD9 (9 different natural frequencies in the metabsorber), and NTMD18 (18 different frequencies) are quite similar, probably linked to the quite high damping of the metabsorbers.

3 Robustness analysis using the Ingo-Gap method

3.1 Presentation of the methodology

Lack-Of-Knowledge is assumed on the frequency of the structure to control. It is considered introducing a perturbation $\mathbf{dm}$ on the reduced mass matrix $\mathbf{m}$ of the master structure. The level of perturbation is defined through a coefficient γ_0 such that $\gamma_0 = \tilde{\gamma}_0 = 0$ corresponds to the nominal case. The perturbed dynamic problem to solve then becomes,

$$\left(\mathbf{k}^* - \omega^2 \left(\mathbf{m} + \gamma_0 \mathbf{dm}\right)\right) \hat{\mathbf{q}} = \hat{\mathbf{f}}. \tag{4}$$

Let define an horizon of uncertainty h on the nominal value $\tilde{\gamma}_0$, the envelope model U such that,

$$U(h, \tilde{\gamma}_0) = \{\gamma_0 : |\gamma_0 - \tilde{\gamma}_0| \leq h\}, h \geq 0 \tag{5}$$

contains all coefficients γ_0 whose distance from $\tilde{\gamma}_0$ is not greater than h. The performance of the metabsorber is estimated from the calculation of the ratio R_0^{MTMD} between the elastic strain energy on the frequency band of interest with and without the metabsorber,

$$R_0^{MTMD} = \frac{E^{MTMD}}{E^0}, \tag{6}$$

where E^{MTMD} is the strain energy of the master structure estimated when the metabsorber is attached to the master structure and E^0 is the energy without metabsorber. When the efficiency of the metabsorber decreases, the ratio R_0^{MTMD} increases and tends to 1. The robustness function $\hat{h}$ is thus defined as the greatest horizon of uncertainty h, that is to say the greatest variation in the frequency to control, such that the worst case ratio (max) remains inferior to the critical value R_0^c,

$$\widehat{h}(R_0^c) = max\left\{h : \left(\max_{\gamma_0 \in \mathcal{U}(h,\tilde{\gamma}_0)} R_0^{MTMD}\right) \leq R_0^c\right\} \tag{7}$$

3.2 Numerical results

Figure 4 presents the robustness computed with the Ifo-Gap method for the different metabsorber configuration depending on the uncertainty introduced on the mass of the master structure translated in terms of uncertainty on the natural frequency of the structure.

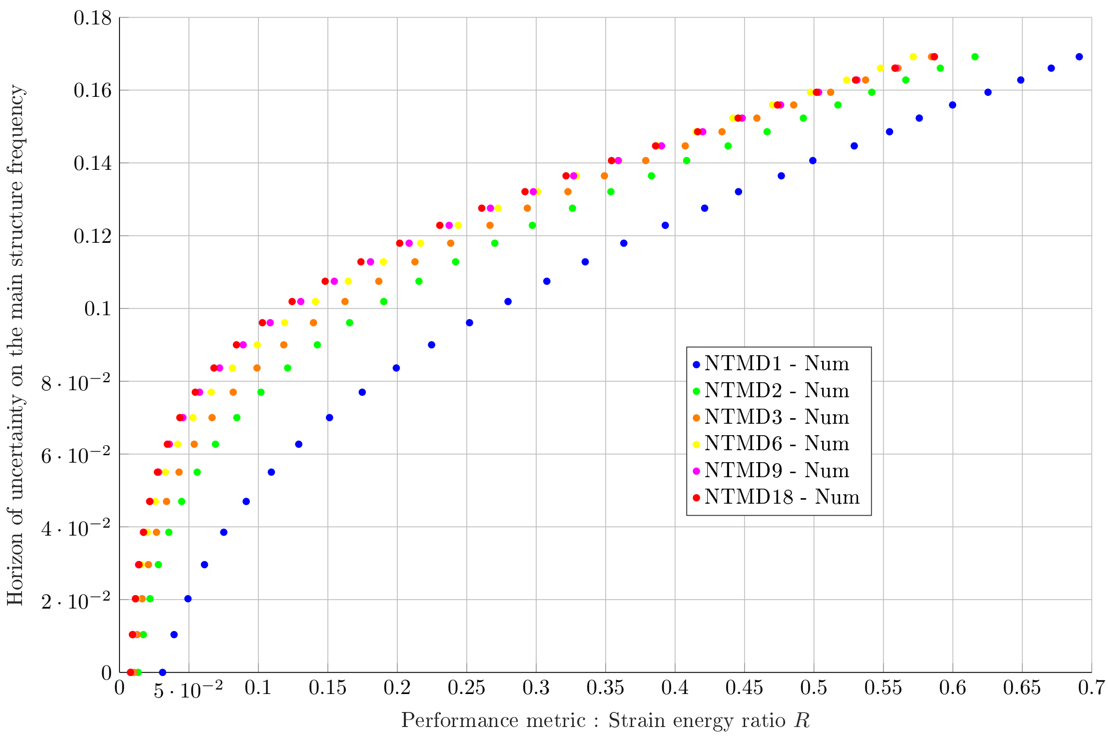


Figure 4: Robustness of the metabsorber NTMD1/2/3/6/9/18 estimated using the Info-Gap method (Legend color on the web version of the article)

The performance obtained for a zero uncertainty horizon corresponds to the solution resulting from the deterministic optimization: the configuration of the single absorber (NTMD1) is thus less efficient than the other configurations. As the uncertainty horizon increases, the different metabsorbers lose efficiency. The NTMD9 and NTMD18 solutions with the highest complexity in terms of frequency distribution are the most robust, thus confirming a result established in the literature of the robustness of multiple solutions.

4 Experimental application

Figure 5 presents the experimental setup used to validate the numerical results obtained. The vibratory field at the wing surface is measured for the aircraft model equipped with the different metabsorber configurations, NTMD1 to NTMD18.

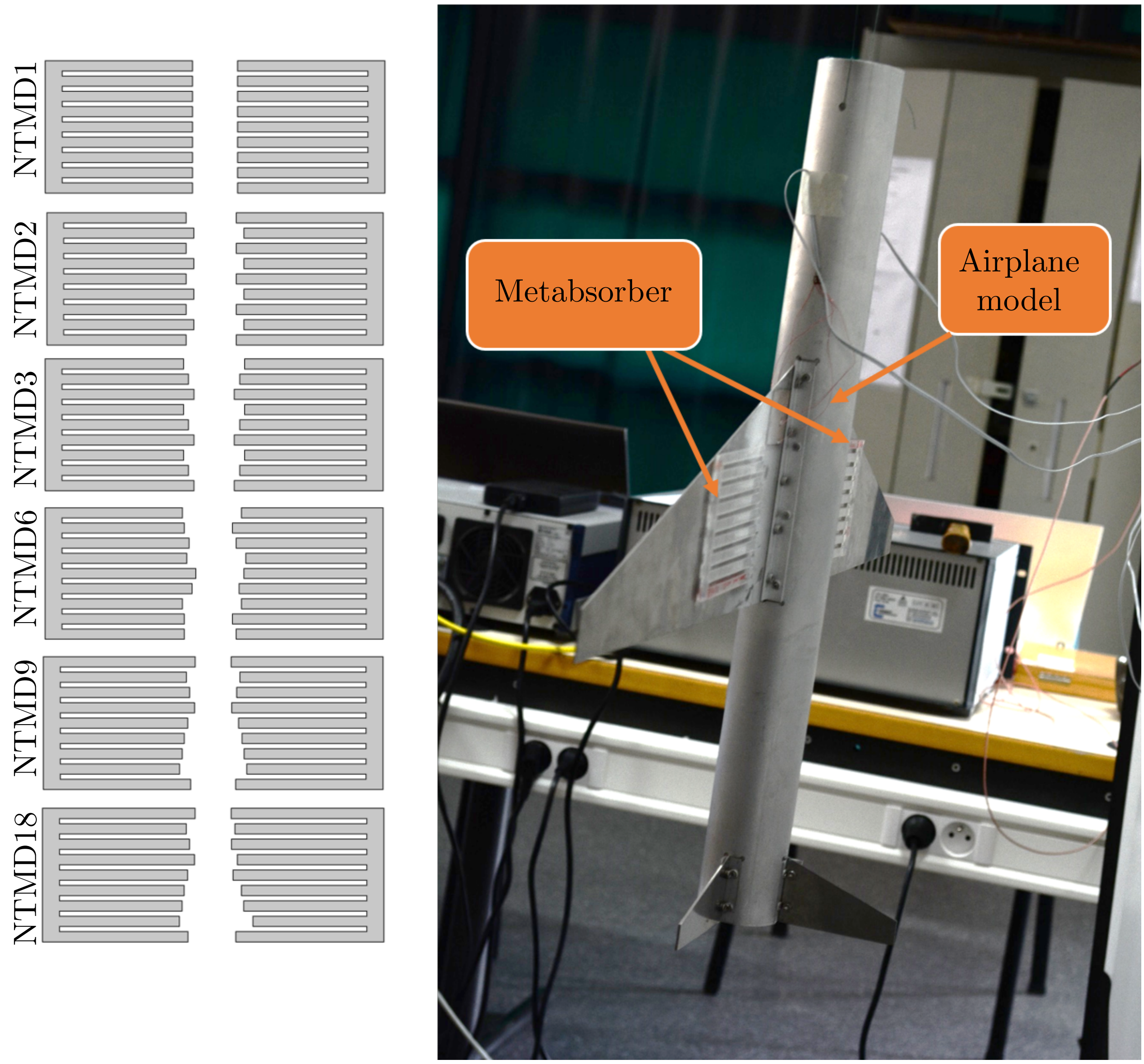


Figure 5: Experimental device: vibratory measurements are carried out by laser vibrometry on the surface of the wing for an aircraft model equipped with different configurations of metabsorber (NMD1 to NTMD18). The aircraft is suspended and excited by piezoelectric patches.

Figure 6 shows the average vibration responses on the wing surface for the different configurations of metabsorbers the trends obtained are in agreement with the numerical results presented in Figure 4. The frequency distribution allows to generate a flat level even if the performances obtained for the complex configuration (NTMD18) is of the same order as those obtained with the uniform configuration (NTMD1) which is here particularly optimized. This can be explained by the manufacturing variability, and by the high level of damping in the metabsorbers.

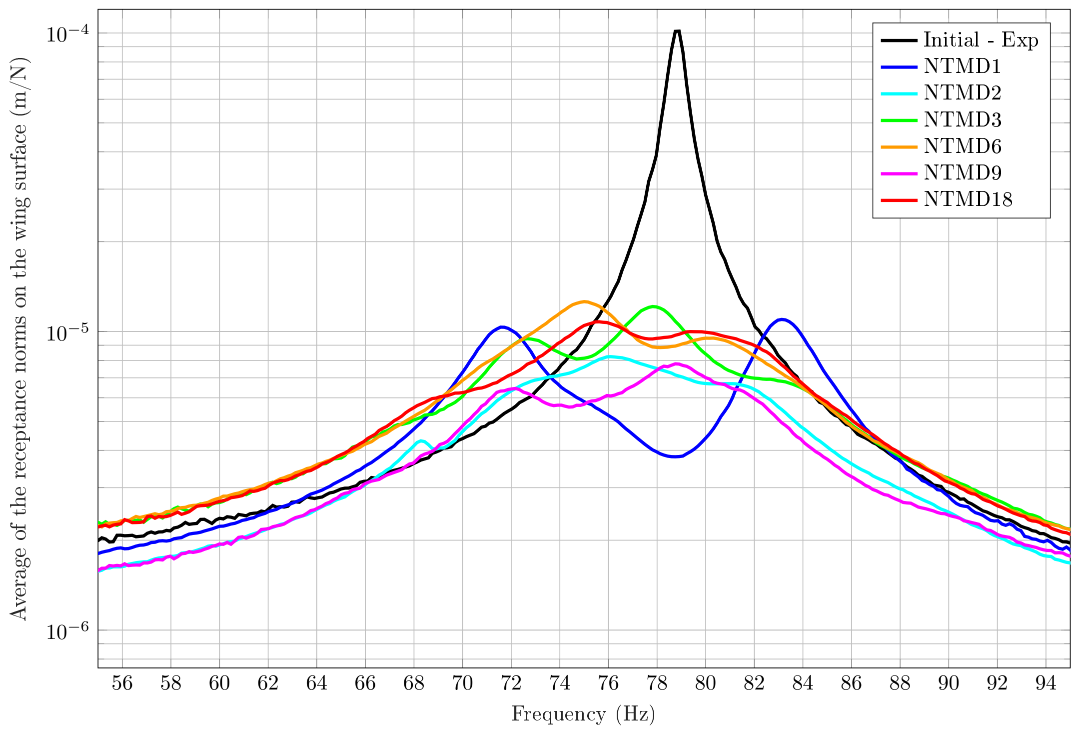
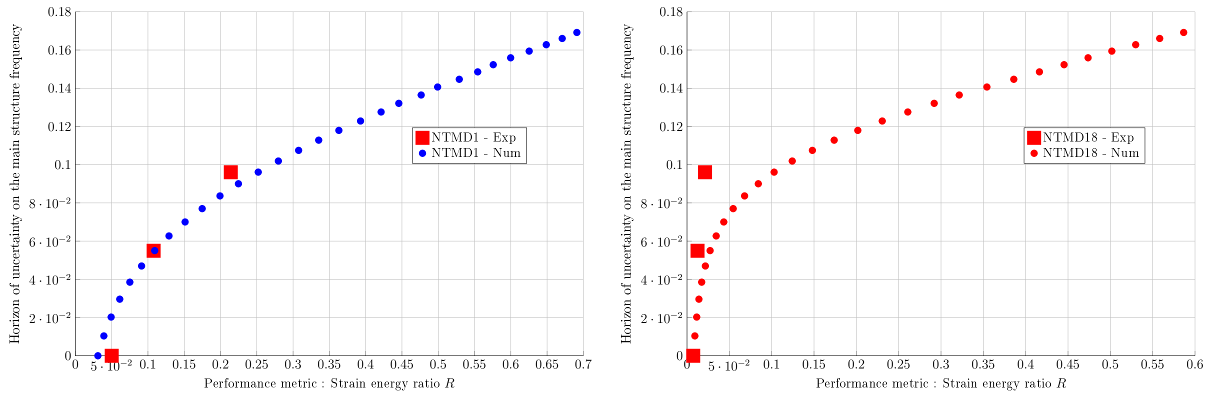


Figure 6: Average of the vibratory frequency responses (receptances) on the wing surface for the initial airplane and for the airplane with metabsorber NTMD1/2/3/6/9/18 (Legend color on the web version of the article)

A modal expansion technique is used to estimate experimental strain energies from receptance measurements, the methodology is described in [10]. These energies are determined for three configurations, the aircraft equipped with a metabsorber, then the aircraft equipped with magnets on the wingtips to generate a variation in mass and therefore natural frequency. These two configurations called shifted lead respectively to a natural frequency variation of the order of 5.5% and 9.6%. Figure 7 presents the comparison between the robustness estimated using the Info-Gap approach, and the robustness estimated using the experimental results, that is for a null, for a 5.5% and for a 9.6% horizon of uncertainty on the natural frequency of the main structure, and for metabsorbers NTMD1 and NTMD18. Results show that the robustness of the NTMD1 configuration is close to the worst predicted configuration with the Info-Gap solution, the robustness of the NTMD18 configuration is better: this tends to confirm the more robust behavior of a distributed configuration. More experimental results on this project can be found in [10].



(a) NTMD1 (b) NTMD18

Figure 7: Distribution of the numerical and experimental robustness for metabsorber NTMD1 and NTMD18

Acknowledgements

This work has been supported by the EUR EIPHI Project (contract ANR-17-EURE-0002), Bourgogne Franche-Comté Region and the ANRT (CIFRE). Authors are grateful to the french company THALES LAS France for their financial support.

References

[1] H. Yamaguchi and N. Harnpornchai, "Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations," *Earthquake Engineering & Structural Dynamics*, vol. 22, no. 1, pp. 51–62, 1993.

[2] B. Mokrani, Z. Tian, D. Alaluf, F. Meng, and A. Preumont, "Passive damping of suspension bridges using multi-degree of freedom tuned mass dampers," *Engineering structures*, vol. 153, pp. 749–756, 2017.

[3] M. M. A. da Costa, D. A. Castello, C. Magluta, and N. Roitman, "On the optimal design and robustness of spatially distributed tuned mass dampers," *Mechanical Systems and Signal Processing*, vol. 150, p. 107289, 2021.

[4] A. K. Rathi and A. Chakraborty, "Reliability-based performance optimization of tmd for vibration control of structures with uncertainty in parameters and excitation," *Structural Control and Health Monitoring*, vol. 24, no. 1, p. e1857, 2017.

[5] L. S. Vellar, S. P. Ontiveros-Pérez, L. F. F. Miguel, and L. F. Fadel Miguel, "Robust optimum design of multiple tuned mass dampers for vibration control in buildings subjected to seismic excitation," *Shock and Vibration*, vol. 2019, 2019.

[6] Y. Ben-Haim, *Info-gap decision theory: decisions under severe uncertainty*, 2nd ed. Amsterdam: Elsevier/Academic Press, 2006.

[7] K. L. V. Buren, S. Atamturktur, and F. M. Hemez, "Model selection through robustness and fidelity criteria: Modeling the dynamics of the CX-100 wind turbine blade," *Mechanical Systems and Signal Processing*, vol. 43, no. 1, pp. 246 – 259, 2014.

[8] D. Wu, W. Gao, G. Li, S. Tangaramvong, and F. Tin-Loi, "Robust assessment of collapse resistance of structures under uncertain loads based on info-gap model," *Computer Methods in Applied Mechanics and Engineering*, vol. 285, pp. 208–227, 2015.

[9] K. Jaboviste, E. Sadoulet-Reboul, N. Peyret, C. Arnould, E. Collard, and G. Chevallier, “On the compromise between performance and robustness for viscoelastic damped structures,” *Mechanical Systems and Signal Processing*, vol. 119, pp. 65–80, 2019.

[10] E. Bachy, K. Jaboviste, E. Sadoulet-Reboul, N. Peyret, G. Chevallier, C. Arnould, and E. Collard, “Investigations on the performance and the robustness of a metabsorber designed for structural vibration mitigation,” *Mechanical Systems and Signal Processing*, vol. 170, p. 108830, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0888327022000279