Multiple input reconstruction using an impulse response filter: Experimental investigation

L.M. Zapata [1,2], **T. Tuerlinckx** [3], **W. Desmet** [1,2], **F. Naets** [1,2]
[1] KU Leuven, Department of Mechanical Engineering,
Celestijnenlaan 300, B-3001, Heverlee, Belgium

[2] DMMS Core lab, Flanders Make, Belgium

[3] Flanders Make, MotionS,
Gaston Geenslaan 8, B-3001 Heverlee, Belgium

Abstract

Unlike single input reconstruction, the inverse problem of determining several input time histories poses the challenge of retrieving a unique solution that accurately describes the real input allocation in the system. In many instances, different combinations of inaccurate force functions might satisfy the system of equations of the estimator being used. In this work, an experimental impulse response matrix deconvolution is adopted in order to estimate simultaneously: 1) the force applied by a hammer impact and 2) a constant zero force at two different locations of a vehicle sub-frame. The data driven nature of an impulse response filter carries the benefit of robustness against complex boundary conditions and avoids the need of calibration of a numerical model. Additionally, the deconvolution implementation relies purely on affordable sensor data commonly gathered during experimental modal analysis. The proposed validation case presents the effects induced by different types of sensors in the estimation of multiple forcing functions.

1 Introduction

System input estimation has been traditionally known as an ill-posed problem that typically requires the inversion of a linear system of equations that may not guarantee a unique solution. The system of equations relates the initial conditions of the structure, the unknown inputs and the measurements. Typically, such mathematical representation is achieved by finite element (FE) modelling. However, the effect of the time elapsed between the system excitation and the observed responses of the system is commonly not considered. This is due to the fact that such models assume that the transfer of information from the input location to the output location takes place instantly [1], which means that they cannot capture dead-time. Equivalent non-parametric representations can also be built purely from measured data [2]. With the advantage that any measured signal intrinsically contains the dead-time information.

FE-based estimators have been widely discussed in literature [3, 4, 5, 6]. Many of these schemes bring advantages in terms of stability and identifiability. However, the updated models used in such estimators have to accurately describe the response of the system in order to obtain satisfactory input estimates. Therefore, accurate input estimation becomes challenging when the structure considered is difficult-to-model (e.g. complex boundary conditions, high modal density, complex damping phenomena, etc.) and only a rough approximation of the system response is achievable. This work explores the use of an experimentally identified impulse response filter (IRF) as a non-parametric alternative to circumvent the issues of dead-time and modelling errors common in FE models. Particularly, to estimate multiple inputs in the setup of a vehicle subframe.

Ill-conditioned behaviour arising from noisy measurements is commonly addressed with Tikhonov regularization methods [7]. Classically, the regularized input estimates are obtained by minimizing a weighted sum of the norms of the residual and the solution, and the weighting constant is referred to as the regularization parameter. In some instances, regularization schemes can be avoided when the data collected contains relatively low levels of noise or after performing low-pass filtering of the noisy observations and the estimated inputs. In this work, the signals acquired lie mostly in the first category (i.e. low noise levels) avoiding the need of regularization.

The objective of this paper is to explore the benefits of the IRF and investigate the effects of using different types of sensors to retrieve estimates of forces acting simultaneously. The mechanical system considered is complex enough such that a model-based approach would be prohibitive to retrieve accurate input estimates. The IRF is run with information acquired from sensors commonly used for experimental modal analysis such as accelerometers and strain gauges. Moreover, the sensors are installed in a non-collocated layout which implies asynchronous observations are collected. Some studies have particularly addressed non-collocated input estimation [8], [9] with mixed results. The paper is divided in three main sections. Section 1 covers the theoretical formulation of the impulse response filter. In section 2 a brief description of the study subject is presented along with the setup used for measurement acquisition. Lastly, the results of the experimental work and the IRF estimator are shown and discussed in section 3.

2 Impulse response matrix deconvolution

The inverse problem of force reconstruction lies in the identification of a set of unknown excitation forces $\boldsymbol{f}$, given an impulse response matrix $\boldsymbol{G}$ and a vector of observations y. For a linear, time invariant system, the relationship between the input and the output observations at any instant in time is given by the following convolution operation [2]:

$$\boldsymbol{y}(t) = \int_0^t \boldsymbol{G}(t-\tau)\boldsymbol{f}(\tau)d\tau \tag{1}$$

Where the matrix $\boldsymbol{G}$ can be interpreted as the response of the system when such system is excited by a unit impulse (i.e. a Dirac $\boldsymbol{\delta}$ function). Typically, the unit impulse responses are obtained by means of numerical simulation, however, the numerical representation of an infinitely short impulse requires workarounds that might be cumbersome. Moreover, if the model available is only a rough approximation of the real system, the input estimation errors may become staggering. Generally, Eq. (1) is solved using measured sampled data. Thus, it can be discretized for n_s time-steps (Δt) as:

$$\begin{bmatrix} \begin{Bmatrix} y_1 \\ \vdots \\ y_i \end{Bmatrix}_0 \\ \vdots \\ \begin{Bmatrix} y_1 \\ \vdots \\ y_i \end{Bmatrix}_{n_s} \end{bmatrix} = \Delta t \begin{bmatrix} \boldsymbol{G}_0 & 0 & \dots & 0 \\ \boldsymbol{G}_1 & \boldsymbol{G}_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{G}_{n_s} & \boldsymbol{G}_{n_s-1} & \dots & \boldsymbol{G}_0 \end{bmatrix} \begin{bmatrix} \begin{Bmatrix} f_1 \\ \vdots \\ f_j \end{Bmatrix}_0 \\ \vdots \\ \begin{Bmatrix} f_1 \\ \vdots \\ f_j \end{Bmatrix}_{n_s} \end{bmatrix} \tag{2}$$

Where each entry of the impulse response matrix

$$\boldsymbol{G}_t = \begin{bmatrix} G_{11} & \dots & G_{1j} \\ \vdots & \dots & \vdots \\ G_{i1} & \dots & G_{ij} \end{bmatrix}_t$$

is the unit impulse response at a particular time instant $t = 0 \dots n_s$ for all input-output combinations, y_i is the i-th component of the vector of output measurements and f_j is the j-th component of the vector of input forces. The indices i and j represent the number of output and input signals, respectively. In this work both input and output are assumed to have the same number of temporal samples n_s.

Alternatively, the $\boldsymbol{G}$ matrix can be identified experimentally by measuring the responses of the system to a hammer excitation. The hammer excitation is also referred to as *quasi-impulse* (QI). Even though the quasi-impulse is not a perfect Dirac impulse, the resulting estimated quasi-forces can still be transformed into impulsive terms. In this work $\boldsymbol{G}$ is obtained experimentally in order to avoid the inherent issue of model accuracy of a model-based estimation. Based on Eq. (2), it is possible to retrieve the forces by taking the pseudo-inverse of the impulse response matrix and multiplying by the vector of output measurements as follows:

$$\boldsymbol{f} = (\Delta t \boldsymbol{G})^{\dagger} \boldsymbol{y} \tag{3}$$

Computing the pseudo-inverse requires a determined system of equations. Ideally, an over-determined system of equations will provide additional noise reduction, therefore the amount of inputs to be determined should not exceed the amount of measurement sensors available. For the applications considered in this work, the length of the time signals considered does not constitute a prohibitive factor to compute the pseudo-inverse for the full time series. An important advantage of constructing the impulse response matrix using measured data, as is proposed in this work, is that the dead time of the transfer of information between the input and output is embedded into the acquired signal data. This contrasts with the use of a finite element model to build the G matrix that inherently does not account for this dead time [1].

2.1 Transformation of quasi-impulses into impulses

In this technique, hammer excitations are practical representations of a Dirac impulse, however, the measured responses obtained are product of a quasi-impulse and not of a perfect Dirac impulse. Therefore, an additional transformation is necessary. The objective is to compute the equivalent impulsive terms from the quasi-impulsive terms obtained from Eq. (3), this operation is given by the following convolution:

$$\hat{\boldsymbol{f}}(t) = \int_0^t \boldsymbol{f}^{QI}(t-\tau)\boldsymbol{f}^*(\tau)d\tau \tag{4}$$

Similar to Eq. (2), this expression can also be discretized as:

$$\begin{bmatrix} \begin{Bmatrix} \hat{f}_1 \\ \vdots \\ \hat{f}_j \end{Bmatrix}_0 \\ \vdots \\ \begin{Bmatrix} \hat{f}_1 \\ \vdots \\ \hat{f}_j \end{Bmatrix}_{n_s} \end{bmatrix} = \Delta t \begin{bmatrix} \boldsymbol{f}_0^{QI} & 0 & \dots & 0 \\ \boldsymbol{f}_1^{QI} & \boldsymbol{f}_0^{QI} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{f}_{n_s}^{QI} & \boldsymbol{f}_{n_s-1}^{QI} & \dots & \boldsymbol{f}_0^{QI} \end{bmatrix} \begin{bmatrix} \begin{Bmatrix} f_1^* \\ \vdots \\ f_j^* \end{Bmatrix}_0 \\ \vdots \\ \begin{Bmatrix} f_1^* \\ \vdots \\ f_j^* \end{Bmatrix}_{n_s} \end{bmatrix} \tag{5}$$

Eq. (5) allows the computation of the actual input force estimates $\hat{\boldsymbol{f}}$ by means of the convolution of the quasi-impulses measured experimentally

$$\boldsymbol{f}_t^{QI} = \begin{bmatrix} f_1^{QI} & 0 & \dots & 0 \\ 0 & f_2^{QI} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & f_j^{QI} \end{bmatrix}_t$$

and the quasi-impulsive inputs computed from Eq. (3) that from now on we will refer to as $\boldsymbol{f}^*$.

$$\hat{\boldsymbol{f}} = \Delta t \boldsymbol{f}^{QI} \boldsymbol{f}^* \tag{6}$$

The convolution described by Eq. (6) acts as a low pass filter that reduces the contribution of high frequency components into the estimates. Even though, the measured quasi-impulses $\boldsymbol{f}^{QI}$ are forces in newtons [N], in order to make the convolution consistent units-wise, $\boldsymbol{f}^{QI}$ is considered unit-less; and $\boldsymbol{f}^{*}$ have force units consistent with the hammer measurement settings. No normalization is required to perform the convolution. In general, the implementation of an experimental impulse response filter has the advantage of not requiring extended model updating. It is worth mentioning that even though the impulse response filter does not require tuning additional parameters, regularization schemes may still be necessary in some cases to treat the inherent ill-conditioned behavior of the problem in Eq. (2).

3 Study case: Vehicle sub-frame

A test rig of vehicle sub-frame connected to a supporting frame (see Fig. 1) is used in this study to estimate the forces induced in the system. The forces act in the structure simultaneously at the locations shown in Fig. 1(c). Moreover, a total of seven sensors were glued at different positions over the surface of the sub-frame. The type of sensors selected consists of three uni-axial lightweight accelerometers, two tri-axial accelerometers and two strain gauges.

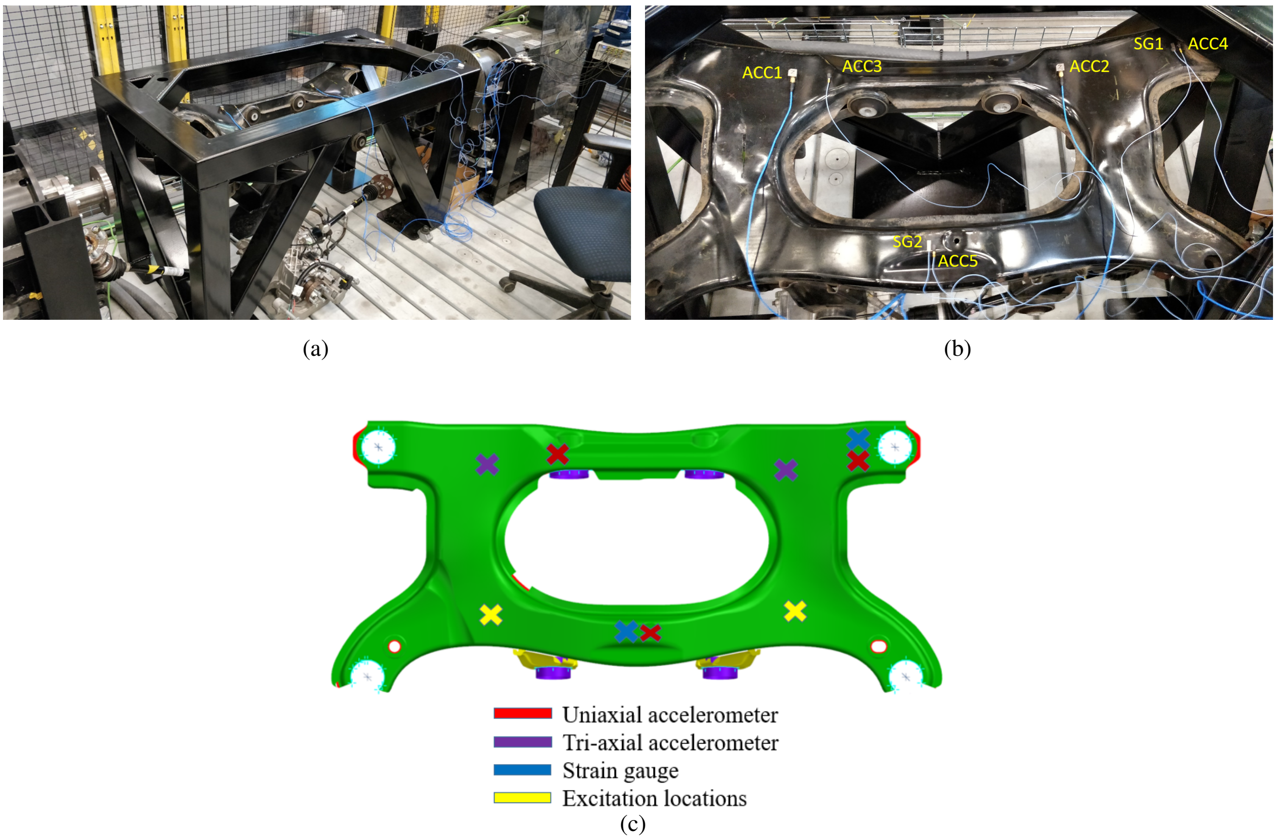


Figure 1: (a) Frame and sub-frame assembly. (b) Sub-frame close-up and sensor deployment. (c) Sensor layout (not to scale) including excitation locations where the forces are estimated.

3.1 Multiple input estimation

The objective of this validation case is to estimate two force functions at two different locations of the assembly. The first function corresponds to a hammer impact that excites all the system's frequencies up to 300Hz, while in the second location a constant zero force is to be retrieved. The non-parametric model built herein assumes that the system behavior is linear, and ideally have limited amount of DOFs. This alleviates the computational time needed and the amount of measurements necessary to retrieve correct force estimates.

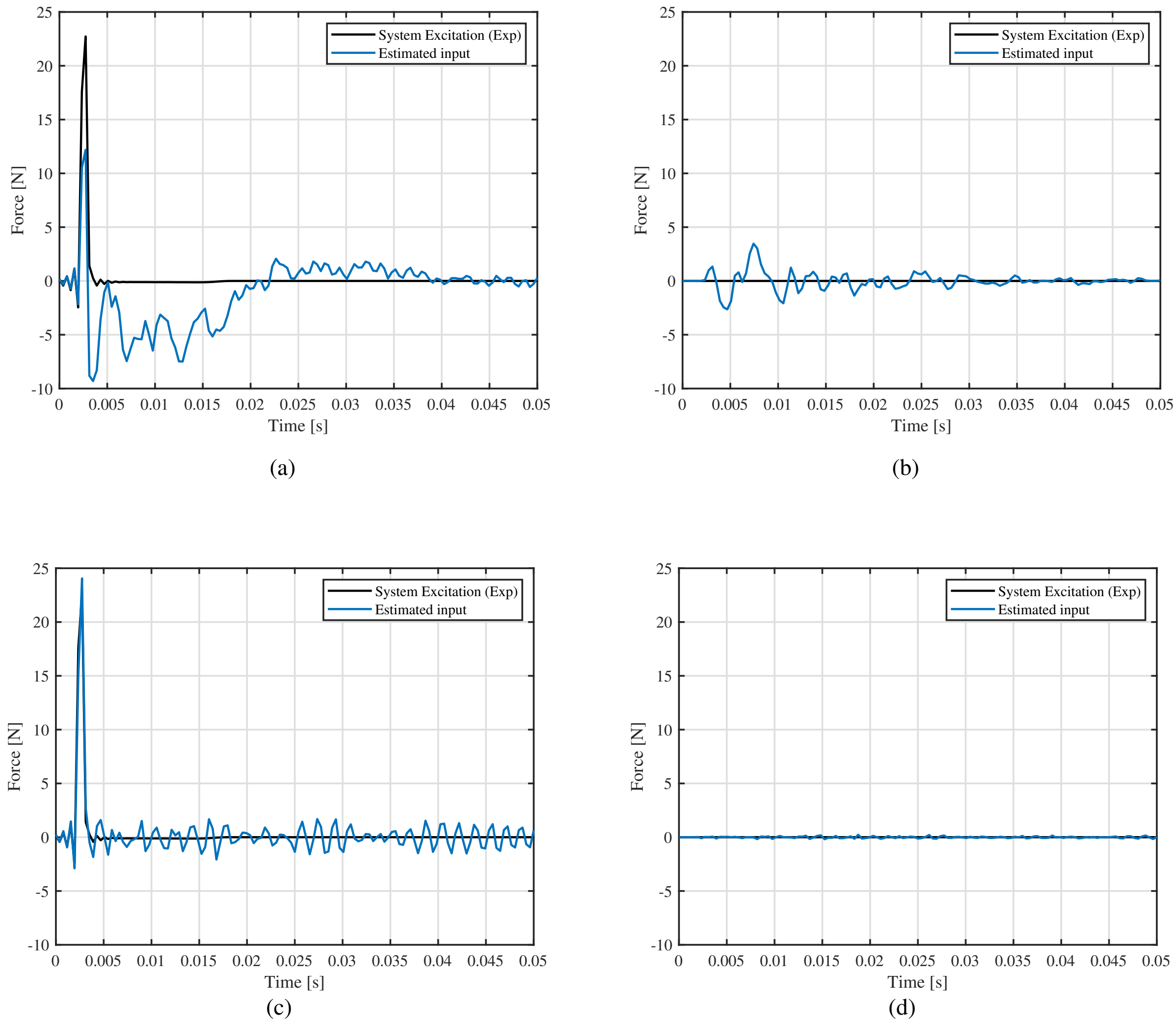
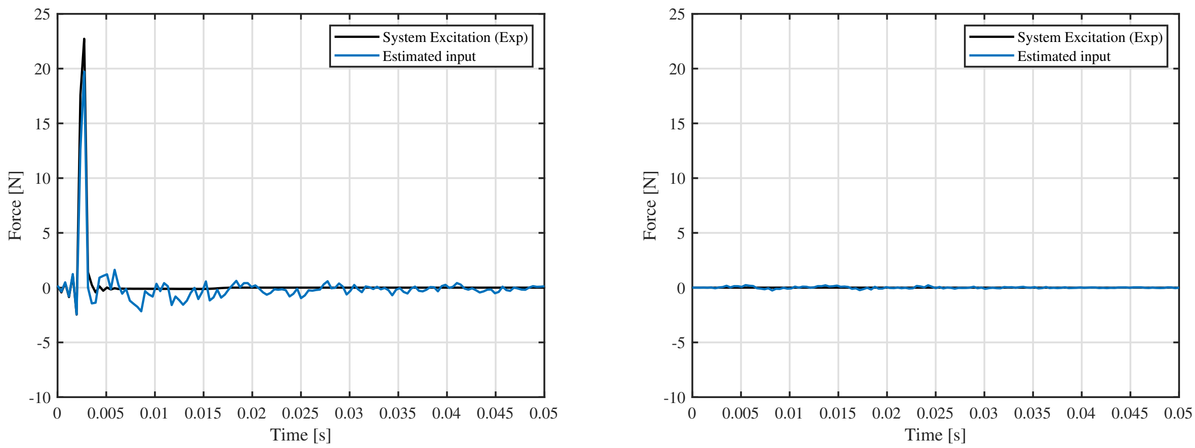


Figure 2: Comparison of the estimated and experimentally measured forces at two locations using (a, b) acceleration signals and (c, d) strain signals

Reference and impulse response measurements are obtained experimentally to construct the IRF non-parametric model. The measurements obtained at the DOFs of interest contain relatively low levels of sensor noise. The amplitudes of the noise are particularly low for the strain gauges that would typically be represented with a noise covariance of roughly $10^{-16}\varepsilon^2$. The time-step is 0.39ms and the total estimation time is 3.2s. However, the force estimates are plotted for a snapshot of time of 0.05s when the peak impact force is acting. Three estimation cases using different sensor data are discussed in the following:

- Acceleration-only measurements.
- Strain-only measurements and
- Combined acceleration and strain measurements.

When only acceleration signals were used in the estimator, Figs 2(a) and 2(b) show that the reconstructed estimated forces are significantly deviated from the measured forces in terms of peak amplitude and additional transients emerging in the estimated force signal. In contrast, the reconstructions made involving only strain measurements (Figs 2(c) and 2(d)) contain less transients and the peak amplitude of the impact is slightly over-estimated. These observations are consistent with other techniques explored in literature that indicate problems of drift and conditioning when only accelerations are used [4, 6]. However, in this particular case, the lack of accuracy in the acceleration-based estimation is attributed mainly to the accumulation of errors in the deconvolution process product of the noise embedded in the signals which reduces the conditioning of the problem. The noise associated with the strain sensors is relatively lower which represents an improved estimate. Finally, if both strains and accelerations are combined with an appropriate scaling of the units of any of the two types of sensors used to avoid numerical instabilities. Then, it results in the reconstruction shown in Figs 3(a) and 3(b), the estimated force has a diminished presence of transients with lower amplitudes, and the peak impact force is almost in line with the measured one. In general, appropriate combination of information coming from different sensor types improves the conditioning of the problem and may balance out the noise contribution in case of sensors with less expected quality in the data.



(a) (b)

Figure 3: Input estimation using a combination of acceleration and strain signals (a) Impact force. (b) Constant zero force.

4 Conclusions

In this study, an impulse response filter process has been explored by means of experimental identification to provide a reliable and accurate force identification scheme for dynamic structures. Different types of sensors were incorporated into the IRF and the method has been validated using a vehicle sub-frame subjected to complex boundary conditions. The experimental IRF approach showed reliability and accuracy for dynamic force estimation under difficult-to-model conditions. When only acceleration measurements, which are cheap and easy-to-obtain in practice, were used, the IRF was found to be lackluster due to the measurement noises and low frequency components of the force. Use of strain measurements resolves the stability issues. Furthermore, strain measurements are equivalently easy to deploy and practical in many cases. As a remedy to the inaccuracy and instability with the force estimation problem, a combination of both acceleration and strain measurements was studied providing satisfactory input reconstruction. Additionally, it was observed that using only strain measurements for IRF can estimate the low-varying components of excitation with good accuracy. This compensates the performance when using accelerations alone.

Acknowledgements

Internal Funds KU Leuven are gratefully acknowledged for their support. This research was also partially supported by Flanders Make, the strategic research centre for the manufacturing industry and VLAIO (Flanders Innovation & Entrepreneurship Agency).

References

[1] D. Bernal and A. Ussia, "Sequential deconvolution input reconstruction," *Mechanical Systems and Signal Processing*, vol. 50, pp. 41–55, 2015.

[2] L. Jankowski, *Dynamic load identification for structural health monitoring*. IPPT Reports on Fundamental Technological Research, 2013.

[3] E. Lourens, E. Reynders, G. De Roeck, G. Degrande, and G. Lombaert, "An augmented kalman filter for force identification in structural dynamics," *Mechanical Systems and Signal Processing*, vol. 27, pp. 446–460, 2012.

[4] F. Naets, J. Cuadrado, and W. Desmet, "Stable force identification in structural dynamics using Kalman filtering and dummy-measurements," *Mechanical Systems and Signal Processing*, 2014.

[5] K. Maes, S. Gillijns, and G. Lombaert, "A smoothing algorithm for joint input-state estimation in structural dynamics," *Mechanical Systems and Signal Processing*, vol. 98, pp. 292–309, 2018.

[6] S. E. Azam, E. Chatzi, and C. Papadimitriou, "A dual kalman filter approach for state estimation via output-only acceleration measurements," *Mechanical Systems and Signal Processing*, vol. 60, pp. 866–886, 2015.

[7] A. N. Tikhonov, A. Goncharsky, V. Stepanov, and A. G. Yagola, *Numerical methods for the solution of ill-posed problems*. Springer Science & Business Media, 1995, vol. 328.

[8] L. J. Nordström and T. P. Nordberg, "A time delay method to solve non-collocated input estimation problems," *Mechanical Systems and Signal Processing*, vol. 18, no. 6, pp. 1469–1483, 2004.

[9] M. S. Allen and T. G. Carne, "Delayed, multi-step inverse structural filter for robust force identification," *Mechanical Systems and Signal Processing*, vol. 22, no. 5, pp. 1036–1054, 2008.