Dynamic model reduction of a flexible three-axis milling machine

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Abstract

The present paper discusses the numerical condensation of a finite element model of a three-axis high speed milling machine to a low-order state-space model suitable for control system design. The proposed reduction method forms an essential part of a Virtual Engineering environment for the design of a large class of mechatronic systems. In order to analyse the dynamic characteristics of the machine within the whole working domain, the dynamics of the machine in different spatial configurations should be investigated without the time consuming complete recalculation of the machine dynamics. The paper shows that a multi-stage component mode reduction and synthesis procedure provides a solution to that problem, because it allows a cheap analysis of different configurations. Substructures with known dynamic characteristics are described by their respective component modes. They are combined in the appropriate relative positions, and the system is solved through a reduced set of equations for each configuration. A well-considered selection of the types of component modes improves the quality of the reduced model.

1. Introduction

Mechatronics can be defined as the engineering science of motion control [1]. Essentially, motion control comprises the control of desired motion, or tracking control, and the control of undesired motion, or vibration cancellation. Nowadays, performance levels, imposed on mechatronic systems in terms of tracking error and residual vibration level, require the use of high bandwidth drive systems and complex control algorithms in order to improve the deficiencies in the dynamic behaviour of the mechanical subsystem. The classical rule that the control system bandwidth should be much lower than the lowest eigenfrequency of the mechanical structure, must necessarily be relaxed. As a consequence, the high bandwidth control systems interact with the structural dynamics of the mechanical subsystem. An optimal design can only be achieved by taking into account this interaction in an early design phase of a mechatronic system. This explains the necessity of developing a Virtual Concurrent Engineering or a Virtual Prototyping environment for mechatronic systems design [2], where the mechanical and the control design problem are tackled simultaneously.

An adequate description of the dynamic behaviour of a complex mechanical structure is based on a finite element model, usually with a very large number of degrees of freedom. The reduction of this finite element model into a model of much smaller dimensions, suitable for control system design, forms a necessary step in the development of such a Virtual Engineering environment.

An important class of mechatronic systems, like machine tools and robots, have a large number of different operation positions. Their dynamic behaviour, described by their eigenfrequencies and mode shapes, will change with changing working configuration. This will inevitably affect the performance and the robust stability of the control system [3], and should therefore be taken into account during controller design.

The present study focuses on the design of a three-axis high speed milling machine for the super-finishing of dies and moulds, as a representative example of a complex mechatronic system. As the milling operation is very time consuming, a faster machine, that works at higher speeds, while keeping the required accuracy, would yield a significant reduction in production costs.

This paper shows that the component mode synthesis (CMS) techniques [4] provide an

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appropriate solution for the dynamic analysis of the machine in different working configurations and for the conversion of the machine model into a reduced model for control system design. Moreover, the method developed in this study accurately simulates the tracking behaviour of the flexible machine in its entire working domain, because the machine model can be cheaply assembled in any desired position. This type of analysis was formerly not possible for machines with multiple flexible bodies with moving contact points.
The machine analysed here is represented by its finite element model in figure 1. This model has over 30000 degrees of freedom. The motion along the X-axis of the machine is the lateral motion of the tool, the motion along the Y-axis of the machine is the frontal motion of the tool, and the motion along the Z-axis of the machine is the vertical motion of the tool. This machine has been developed in the frame of the Brite-Euram project KERNEL II [5]. The model that is used here, represents an intermediate design of the final machine tool with substantial differences in eigenfrequencies and static stiffness.

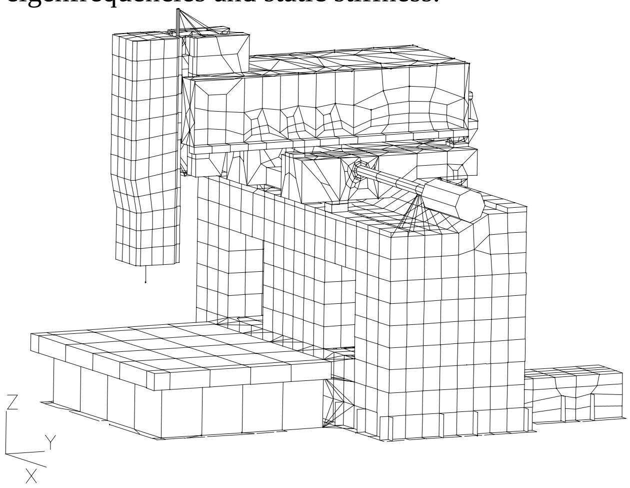

Figure 1 : Finite element model of the machine tool

The paper is divided in six sections. After this introduction, which motivates the development of the proposed reduction method, section 2 gives an overview of the different steps in the method. Section 3 explains how the finite element model of the milling machine is split up in components and how standard finite elements are used to allow an easy assembly of the machine model in any desired spatial configuration. Section 4 discusses a first reduction step of the model using a standard CMS procedure. It mainly focuses on the best choice of the type of component modes. Section 5 then deals with the conversion of the reduced finite element model into a state-space model that is suited for control system design. Finally, the main conclusions are stated in section 6.

2. Elaboration of the proposed reduction method

The discussion in this section focuses on the three-axis milling machine, but the methodology can be readily applied to other (flexible) multibody structures.
The proposed reduction method makes use of two standard engineering software packages. All finite element calculations are performed within MSC/Nastran [6]. The Matlab environment [7] is used to coordinate the assembly of the machine in the desired spatial configuration, to convert the modal machine model, obtained from MSC/Nastran, into a state space model, and to perform the subsequent control system synthesis and analysis.
Figure 2 shows the different steps in the method.
The construction of the three-axis milling machine allows a natural definition of the components as the ram (machine Z-axis), the Y-axis slide, the X-axis slide, and the base (see section 3). These components are represented in figure 2 by their finite element models in the upper part of the grey box. The modal models of the four components of the milling machine are obtained from a standard component mode reduction (see section 4), and are stored on hard disk as four separate MSC/Nastran databases. They are represented in the lower part of the grey box in figure 2 by their respective component modal stiffness and mass matrices, k_i and m_i, and by their component mode vectors Ψ_i.
The residual structure, on the third level in figure 2, is the finite element machine model after numerical condensation of the components. In the assembly phase, the reduced component models are attached to the residual structure as “external superelements” [6], each defined in a different relative coordinate system. These relative coordinate systems allow the assembly of the components in the appropriate relative positions. The locations of these relative coordinate systems, which actually determine the location of the tool tip in the operation space of the milling machine, are defined from within Matlab.
The component modal models should be independent of the spatial configuration of the milling machine, and may, therefore, not contain any element of the drive chains. These drive chains for the three slides, represented on the right of the grey box, belong to the residual structure as their

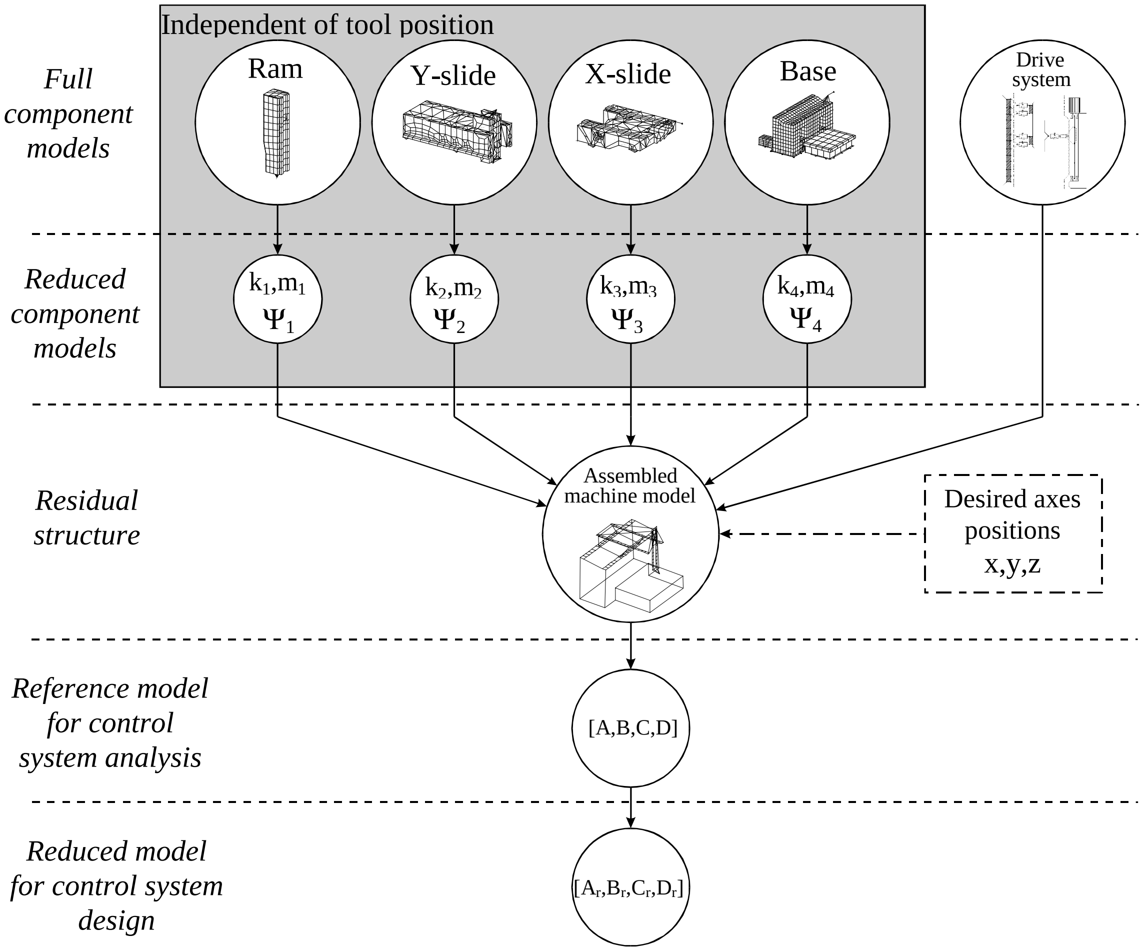

Figure 2 : Overview of the proposed reduction method

dynamic properties depend on the relative positions of the two slides to which they are connected. For example, when considering a ballscrew drive, the dynamics of the drive depend on the position of the nut on the screw.

The assembled residual structure of the machine, as depicted in figure 2, is obtained from MSC/Nastran by defining dedicated “plot elements” between the external nodes of the four components and a few extra nodes on the base (coincident with the actual corner nodes of the base, but without any physical meaning).

This assembled residual structure is then further considered in the second reduction step as a single component. It has only four boundary nodes, being the rotors of the drive motors of the three machine axes, and the tool. The component mode reduction is performed with a dedicated MSC/Nastran DMAP-alter that writes the reduced mass, stiffness and damping matrix and the modal transformation matrices to an ASCII-file (“alter1g.v70” [8], a DMAP-alter to the normal modes solution sequence). These matrices are then converted into a state-space model in Matlab (see section 5). This state-space model, represented by the matrices [A,B,C,D] in figure 2, forms a reference model for the design of a control system for the motors of the machine axes. A major part of this second reduction step is based on the work reported in reference [2].

The order of the reference model [A,B,C,D] is still considerably high. Besides some static deformation vectors, it generally contains component normal modes with eigenfrequencies up to twice the highest frequency of interest. In the case of the milling machine, this reference model is obtained from a component modal model with about 50 modal degrees of freedom, resulting in a state space representation with about 100 states. Prior to the control system synthesis, this reference model is again reduced to a state-space model of much lower order, represented by $[A_r,B_r,C_r,D_r]$ in figure 2. Different methods, including, for example, the balanced truncation method or the Hankel norm approximation method [9] are available for performing this third reduction step. This third reduction step is typically related to the control system design. Both this model reduction and the controller design are beyond the scope of the paper. They are treated in detail in reference [3].

Eventually, the procedure described above results in a state-space description of the milling machine in any desired position, combined with a control system for the drives of the three machine axes. This integrated simulation model, containing the dynamic characteristics of both the mechanical structure and the control system, allows to asses the quality of the virtual machine prototype in terms of tracking behaviour. This tracking behaviour can be accurately evaluated in the entire operation volume of the machine tool by integrating the time-dependent equation of motion (in state space form) over the desired trajectory :

$$\begin{aligned}\dot{x}(t) &= A_c(t)\,x(t) + B_c(t)\,u(t)\\ y(t) &= C_c(t)\,x(t) + D_c(t)\,u(t)\end{aligned}. \qquad (1)$$

The time-dependent state-space model $[A_c(t),B_c(t),C_c(t),D_c(t)]$ contains the controller model and the machine model. The reference trajectory input is u(t), the critical output y(t) is the tool position, and the state vector x(t) contains both the machine model and the controller states. The machine model is assembled in the relative positions of the axes at the time step being considered. This means that, theoretically, the machine model must be reassembled at each time step. As the component modal models are independent of the relative slide positions, this assembly takes only a fraction of the time required to assemble the entire finite element model. Depending on the accuracy required in a practical simulation, the reduced machine model can be assembled only once in a few time steps, or each time the desired tool position moves over a certain threshold distance.

This overview shows that the proposed method smoothens the way for the development of a Virtual Prototyping environment for the design of multibody mechatronic systems. The next sections go into more detail in the most important aspects of this method.

3. Division of the original FE model into components

The quality of the reduced finite element model is described by its accuracy in comparison with the original model and by the computation time required to perform an analysis of the dynamic behaviour of the model. This quality is mainly determined by two factors : the definition of the individual components and the definition of the modal models that are used to describe the dynamics of the components in the assembled structure. The second factor is extensively discussed in the next section 4. The first issue has already been brought up in section 2, and is now further elaborated and applied to the finite element model of the milling machine.

The residual structure contains the exterior nodes of the components, the component generalised coordinates, and the elements of the drive chains and the guideways of the three axes. These elements enable the relative motion of the slides. The relative motion between the components is defined by the component relative coordinate systems. This is explained for the relative motion between the ram and the Y-axis slide in the figures 3 and 4. Figure 3 shows the guideways of the machine Z-axis as represented in finite elements. All nodes on the left of the dash-dotted line in figure 3 are defined in the coordinate system of the ram. The location of the origin of this coordinate system (CS_{ram}) is expressed in the coordinate system of the Y-axis slide (CS_{ysl}) :

$$\begin{aligned} x^0_{ram} &= x^0_{Y-slide}\\ y^0_{ram} &= y^0_{Y-slide} \qquad (2)\\ z^0_{ram} &= z^0_{Y-slide} + \Delta z_{ram}\end{aligned}$$

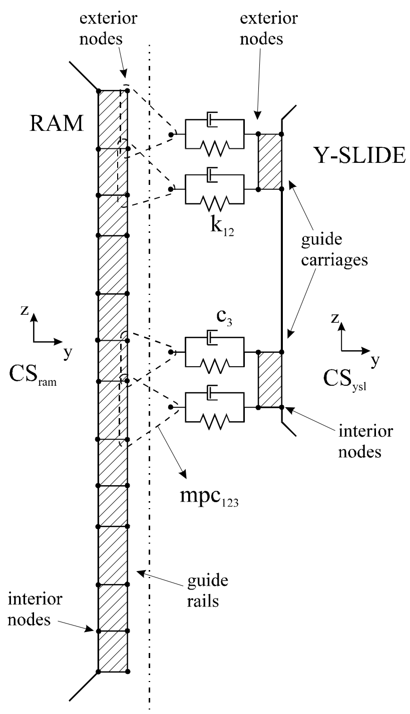

Figure 3 : Mechanism of relative motion between the ram and the Y-axis slide for the guideways.

The offset Δz_{ram} defines the vertical position of the ram with respect to its reference position. The stiffness of the guideways and the carriages perpendicular to the axis of motion is represented by the spring elements k_{12}, where the subscripts 1 and 2 refer to the x- and y-translation degrees of freedom. The viscous damping in the carriage bearings is represented by the damping elements c_3, where the subscript 3 refers to the z-translation degree-of-freedom along which relative motion is allowed. The x-, y- and z-displacement continuity between the exterior nodes on the ram guideways and the end nodes of the spring-damper combinations, which belong entirely to the residual structure, is provided by multi-point constraints, indicated by mpc_{123} in figure 3 (in dashed lines). Multi-point constraints express one degree-of-freedom as a linear combination of other degrees of freedom. The weighting coefficients in that linear combination are in this case determined by the z-position of the ram, given by Δz_{ram}. The selection of the appropriate exterior nodes on the guideways and of the weighting coefficients, for a certain offset Δz_{ram}, and the generation of these multi-point constraints in MSC/Nastran format is done by Matlab, outside the finite element software.
Figure 4 shows the elements of the ram drive chain. Here again, all nodes on the left-hand side of the dash-dotted line are defined in the ram coordinate system, while all nodes on the right-hand side of this line are defined in the Y-axis slide coordinate system. The stiffness of the screw-nut connection in z-direction is represented by the spring element k_3. The stiffness in other directions is assumed to be infinite, and is modelled by rigid-body elements which are not shown in figure 4. The rotational motion of the screw (degree of freedom 6) is converted in the nut to a translational motion of the ram (degree of freedom 3). This is represented by the multi-point constraint $mpc_{6\text{-}3}$, whose weighting coefficients are determined by the transmission ratio of the complete ballscrew. The contact points between the nut and the screw are represented by one node, that belongs to the screw. The location of this contact node on the screw depends on the position of the ram. The displacement continuity between this contact node and the other nodes on the screw is enforced by the multi-point constraints mpc_{123456}. The selection of the appropriate nodes on the screw and of the weighting coefficients for a certain offset Δz_{ram}, and the generation of these multi-point constraints in MSC/Nastran format is again done by Matlab.
Similar constructions for the other axes allow the milling machine model to be assembled in any spatial configuration. The assembly process is controlled from within the Matlab environment. The component mode reduction, with component modal models which are independent of the component relative positions, cuts down the computation time for an analysis in one configuration to a small fraction of the time for an analysis of the full model.
Standard flexible multi-body dynamic analysis software packages, like DADS [10], use a similar component mode representation of structural flexibility in specific bodies of a mechanical structure. However, experiences with DADS indicated that their methods are limited to flexible bodies with geometrically fixed contact points (like, for example, in antropomorphous robots). Such an approach does not provide an appropriate solution in the case of the milling machine, because the locations of the contact points between two slides change as the tool moves from one position to another.

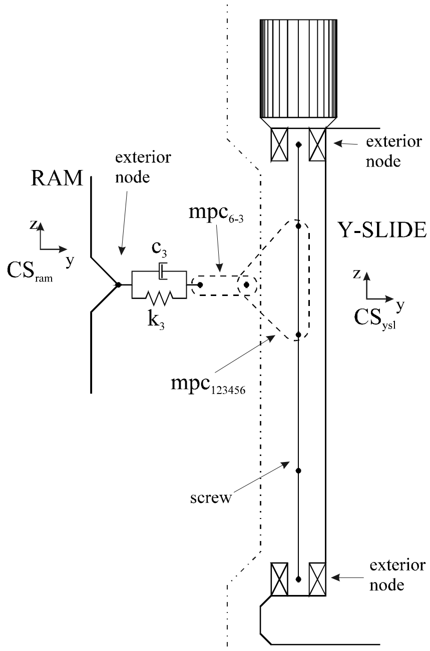

Figure 4 : Mechanism of relative motion between the ram and the Y-axis slide for the drive chain.

4. A comparison of CMS methods for the undamped finite element model

Several sets of modes can be used to form the component models. The choice of these sets has a

	ram	Y-axis slide	X-axis slide	base
interior dofs	1714	3496	3436	19738
exterior dofs	324	408	138	462
total number of dofs	2038	3904	3574	20200

Table 1 : Division of the degrees of freedom (dofs) in interior and exterior degrees of freedom for the four components

great impact on the accuracy of the reduced model. The time required to perform an analysis of the machine in one configuration, is mainly determined by the number of modes that are taken up in the component modal models. This number of modes is strongly influenced by the number of external degrees of freedom of the components, because MSC/Nastran always takes up all constraint modes (which are identical to the Guyan reduction vectors) in the modal base of a component. The numbers of internal and external degrees of freedom of each component are shown in table 1. The entire model has 30598 nodal degrees of freedom, of which 882 belong to the residual structure (the guideways and the drive chains) and are therefore not mentioned in table 1.

The performances of different CMS methods are evaluated in this section in terms of accuracy of the calculated eigenmodes. Three correlation measures are used to compare the eigenmodes, calculated with the CMS methods, with the eigenmodes, calculated with the full model with 30598 degrees of freedom : the relative eigenfrequency difference for the ten lowest modes of the full structure, the MAC-value [11] between the lowest ten corresponding eigenmodes, and the COMAC-values [11] for all degrees of freedom of the full model.

Three CMS methods have been tested in this study. The performances of these three methods will be discussed hereafter, and the obtained mode shapes (after expansion to the omitted degrees of freedom) will be compared to the '*exact*' mode shapes obtained from the full model. A more detailed discussion of these methods, together with the mathematical formulations of the component modes and the implementation of the methods in MSC/Nastran, is given in reference [12].

In the first method, ten fixed-interface normal modes are used in each component modal transformation matrix, together with the constraint and the rigid-body modes (if any). This method is the so-called Craig-Bampton method [4]. The left-hand column of figure 5 summarises the performance of this method as estimated by the three above mentioned correlation measures for the first ten eigenmodes of the finite element model of the high speed milling machine. These results indicate that the correlation between the actual mode shapes and those calculated with the Craig-Bampton method is sometimes not so good. As appearing from the lower left-hand part of the figure, there are a considerable number of degrees of freedom whose COMAC-value is far from unity.

In the second method, ten free-interface normal modes are used in each component modal transformation matrix, together with the constraint and the rigid-body modes (if any). The central column of figure 5 shows the performance of this method as estimated by the three above mentioned correlation measures for the first ten eigenmodes of the finite element model of the high speed milling machine. The results indicate that the correlation between the actual mode shapes and those calculated with this second CMS method is already somewhat better than for the Craig-Bampton method. The reason for this improvement is that a major part of the boundary nodes of the slides in the assembled model are free, while only the boundary conditions of the nodes to which an adjacent slide is connected, are closer to a clamping. Of course, this distinction in boundary conditions between those two types of nodes cannot be made in the proposed method, because different nodes are connected to adjacent components for different spatial configurations of the machine, and the component modal models must be independent from the spatial configuration in which the machine is assembled.

In the last method, four free-interface normal modes and six inertia-relief modes are combined with the constraint and the rigid-body modes (if any). Inertia-relief modes describe the deformation of the component, loaded by d'Alembert inertial forces due to rigid-body motion, and supported such that the stiffness matrix is not singular. This method is comparable to Hintz' method of constraint modes [13], supplemented with a small number of free-interface normal modes. The total number of component generalised degrees of freedom is again the same as in the previous methods, and amounts to

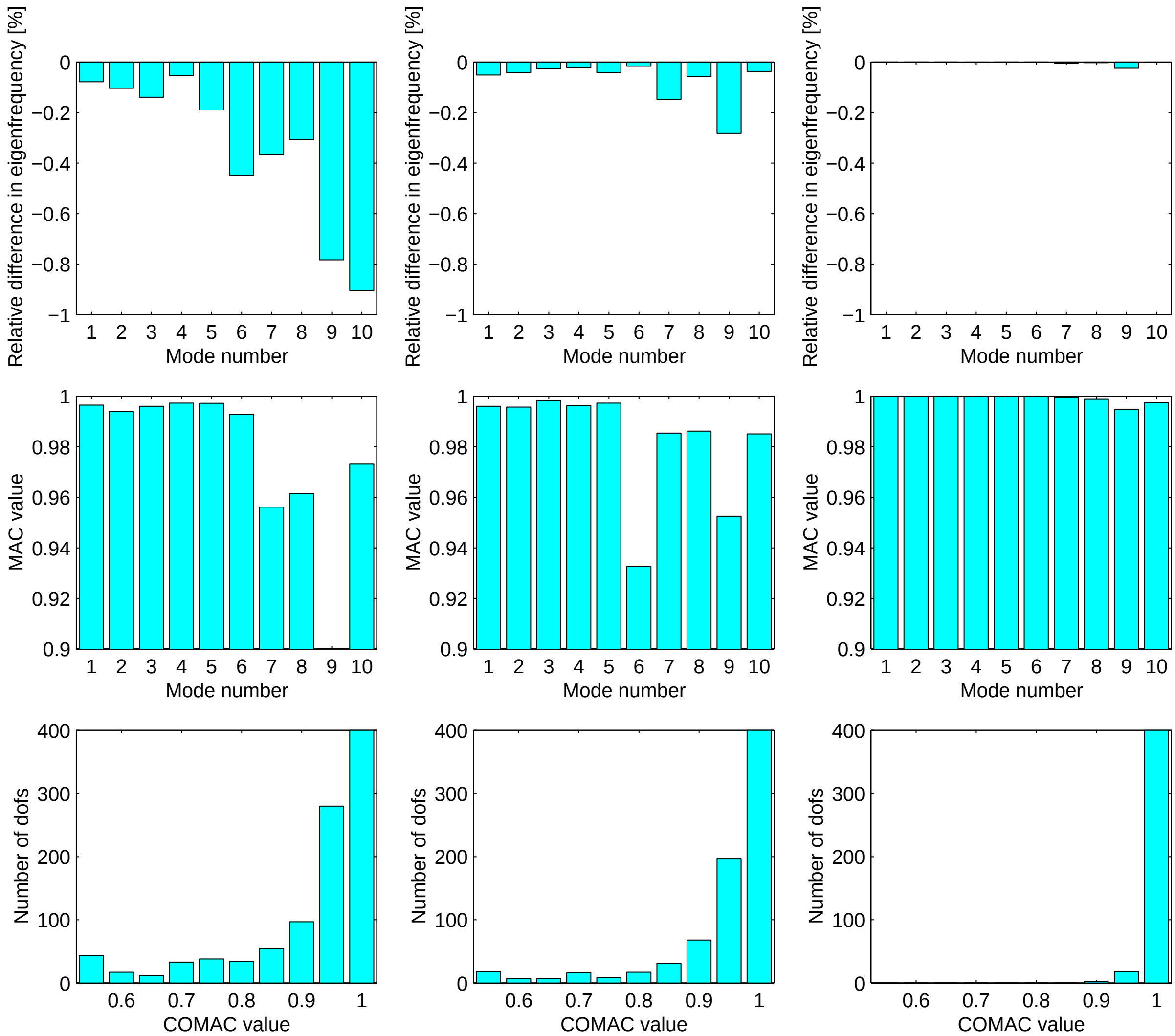

Figure 5 : Performance evaluation and comparison of the three CMS methods

334 for the ram, 418 for the Y-axis slide, 148 for the X-axis slide and 472 for the base.

The right-hand column of figure 5 shows the performance of this method as estimated by the three above mentioned correlation measures, also for the first ten eigenmodes. The results indicate that the correlation between the actual mode shapes and those calculated with this last CMS method is almost perfect. The relative eigenfrequency difference has been reduced by a factor 30 compared to the first method, and by a factor 10 compared to the second method, and amounts now to no more than 0.025 % ! The same holds for the spatial correlation of the mode shapes : not more than 20 degrees of freedom, out of more than 30000, have a COMAC-value below 97.5%. The time required for a modal analysis of the reduced model is a factor 12 smaller than for the analysis of the full model. The reason for this nearly perfect correlation is the fact that the inertia-relief modes account for the residual effects of the omitted high-frequency normal modes at zero frequency. This means that this third component mode set can represent exactly the static deformation of the component, loaded by the inertial forces due to a uniform acceleration. Analogously, the static deformation due to concentrated forces in the exterior nodes, is exactly represented by the constraint modes [13].

In the results presented above, the improvements between the different methods are from small to very small frequency error, or from high to very high mode shape correlation, and may therefore appear to be of academic interest only. However, in other cases, similar improvements may be of great practical value. This is especially true in the computation of strain which is much more sensitive to truncation (see e.g. [14]).

5. Conversion of the reduced FE model into a state-space model

The residual structure of the milling machine, assembled in any desired position, still contains a considerable number of degrees of freedom (794 in the example being considered). The second reduction step converts this assembled residual finite element model into a state-space model of much smaller dimensions (see figure 2). A component mode reduction procedure, similar to the one described in reference [2], projects the degrees of freedom of the residual structure on a modal base consisting of constraint modes, normal modes and inertia-relief modes.

The assembled residual machine model has only nine external degrees of freedom. These are, on the one hand, the rotations around the three motor axes, on which the torque input for generating the desired trajectory is applied, and, on the other hand, the six degrees of freedom of the tool, on which possible external disturbance forces, originating from the cutting process or from a spindle unbalance, apply.

An analysis similar to that presented in the previous section, indicates that a few fixed-interface normal modes more closely represent the dynamic behaviour of the milling machine than an equal number of free-interface normal modes.

Next to the type, also the number of component modes determines the quality of the reduced model. The accuracy of three modal models with different numbers of modes have been compared. Table 2 gives the composition of these three models.

	constraint modes	normal modes	inertia-relief modes
model 1	9	36	6
model 2	9	18	6
model 3	9	18	0

Table 2 : Composition of the three modal bases.

The modal bases always contain nine constraint modes, corresponding to the nine external degrees of freedom. The rigid-body modes are obtained as linear combinations of the constraint modes (see e.g. reference [4]).

The accuracy of the modal models is evaluated by the Frequency Domain Assurance Criterion [15] (FDAC), which is a correlation coefficient similar to the MAC :

$$\mathrm{FDAC}(\omega)=\frac{\left|\{H_{dir}(\omega)\}^H\{H_j(\omega)\}\right|^2}{\left(\{H_{dir}(\omega)\}^H\{H_{dir}(\omega)\}\right)\left(\{H_j(\omega)\}^H\{H_j(\omega)\}\right)} \qquad (3)$$

The vectors $H_{dir}(\omega)$ and $H_j(\omega)$ are frequency responses in 36 degrees of freedom for a unit input torque, successively applied at the X-axis motor, at the Y-axis motor and at the Z-axis motor. The subscript “j” refers to a modal superposition calculation scheme using each of the three modal bases, and the subscript “dir” to a direct calculation on the assembled model. The 36 response degrees of freedom are selected as possible locations on the machine tool where position or acceleration feedback sensors for the control system can be installed. For that reason, they are the most critical response locations on the machine tool.

Figure 6 compares the quality of the modal bases for two different positions of the tool. In the left-hand part of figure 6, the machine is in its original configuration (also depicted in figure 1). All three proposed modal models are very accurate, as the FDAC never drops below 97 %. The incorporation of six inertia-relief modes improves the quality of the modal model in the upper part of the frequency range being considered. The highest order model provides a perfect correlation over almost the entire frequency range. Based on this comparison, one could eventually retain the second model for use as the “full” model in the control design stage, because it yields a good accuracy and its order is much lower than that of the third model. However, a similar comparison of the three modal models for the tool in a different position strongly contradicts this preliminary conclusion. This is shown in the right-hand part of figure 6 (on a different scale as the left-hand part !) for the tool moved 0.7 m in x-direction, 0.6 m in y-direction and 0.5 m in z-direction. In this case only the highest order model provides a good approximation for the dynamic behaviour of the assembled finite model.

To illustrate this, figure 7 shows the computed frequency response from a unit torque input at the X-axis motor to the displacement of the tool in x-direction. The curve obtained from the assembled residual model (the reference) is plotted in thick fainted line, that from the first model in dashed line, that from the second model in solid line, and that from the third model in dash-dotted line. The left-hand part and the right-hand part of figure 7 show the response for the machine in the same configurations as in the left-hand part and,

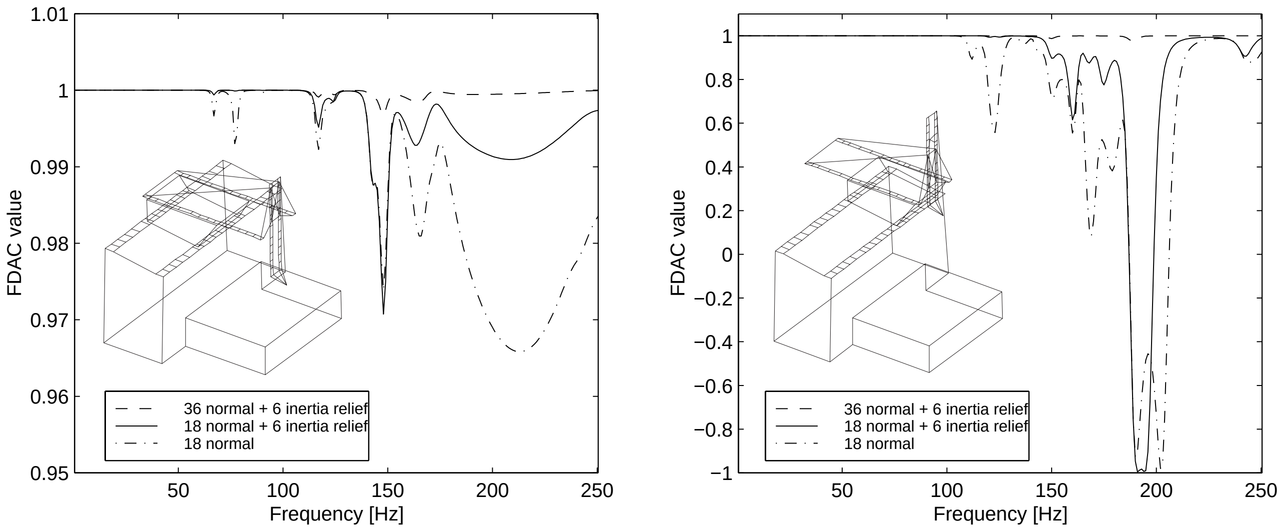

Figure 6 : Comparison of the quality of three modal models for the milling machine with the tool in two different positions

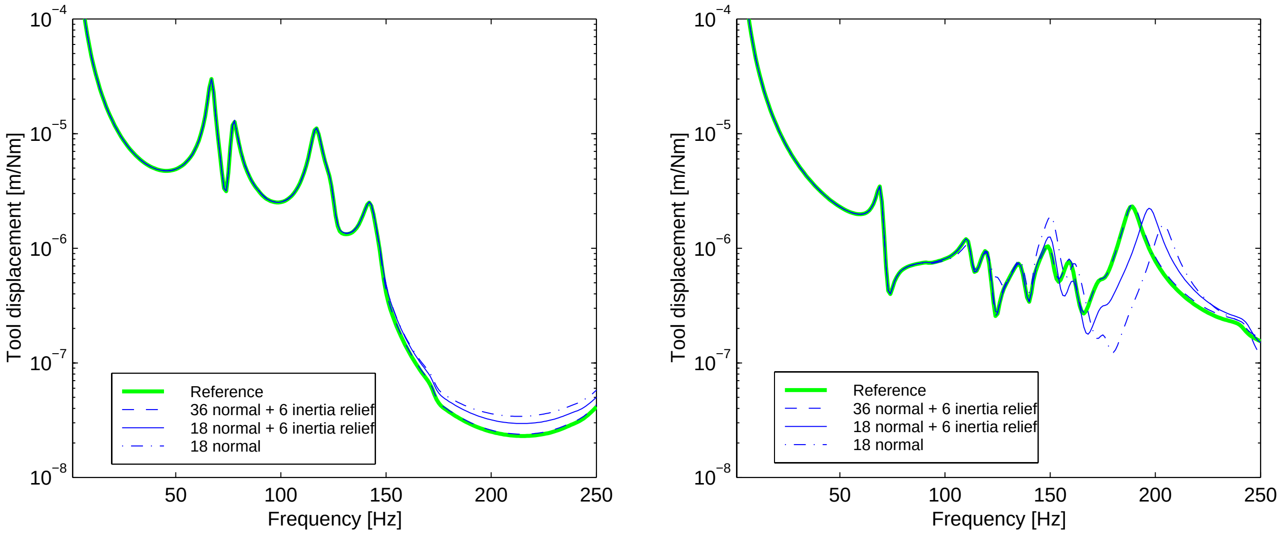

Figure 7 : Comparison of a frequency response function from the three models and the assembled residual structure (reference) for the milling machine with the tool in two different positions

respectively, in the right-hand part of figure 6. The right-hand part of figure 7 clearly indicates that the models with too little modes (model 2 and 3) overestimate the stiffness of the machine in the frequency range around 200 Hz. The model with 36 normal modes (model 1) achieves an almost perfect correlation with the reference model over the entire frequency band of interest.

This analysis clearly illustrates that great care should be taken of a judicious selection of the number of component modes used for reducing the assembled finite element model to a state-space model. In an automated reduction procedure, for example during the simulation of the tracking behaviour of the machine on a large trajectory, this number of component modes is fixed for all spatial configurations of the machine. Therefore, the accuracy of the state-space model should be verified in advance for at least a few different spatial configurations.

A comparison of the left- and the right hand parts of figure 7, showing a frequency response between the same in- and output, clearly illustrate that the dynamic behaviour of the milling machine changes drastically from one position to another.

The next challenge in this research work is the design of a control system that is stable and performs well for all configurations of the machine tool.

6. Conclusions

This paper proposes a dynamic model reduction scheme which enables an accurate and computationally efficient study of the dynamic characteristics of flexible mechatronic systems in multiple spatial configurations. Such a dynamic analysis module forms an essential part of a Virtual Prototyping environment for mechatronic systems.

The proposed reduction scheme essentially consists of a multistage component modal reduction and synthesis procedure. In each step the accuracy of the reduced model can be preserved, in comparison with the full original finite element model, by a judicious selection of the type and number of component modes in the modal transformation matrix. It appears in this study that inertia-relief modes are always beneficial in that sense.

Also the partition of the full finite element model into individual components plays an important role in the proposed method. A definition of the components independently from their relative positions allows a quick dynamic analysis of the machine in different spatial configurations, or an accurate evaluation of the tracking behaviour of the machine over a large trajectory.

The next step in this research work is the development of a tracking control system for the three machine axes. This control system should be robust against the inevitable model inaccuracy. One important type of model inaccuracy is due to the changing machine configuration, which varies with the desired working position of the tool. The analysis of the machine dynamics in different configurations allows to derive a more accurate estimate of the machine model uncertainty used for control system design.

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