Application of the global/local model concept for the computation of stress frequency response functions

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Abstract

Fatigue life prediction of dynamically loaded structures employs stress frequency response functions (FRF's) in the critical area of the structure. The global/local model concept is used for the computation of the stress FRF's. The method combines the result of a dynamic finite element (FE) analysis applied to a coarse model of the total structure with the result of a static FE analysis applied to a fine model of a small region around the critical zone. This paper contains a global description of the global/local model concept. A special feature of the method, the internal force distribution, is explained in detail and the result of the global/local model concept is presented for a test case.

1. Introduction

The fatigue life of a dynamically loaded structure is determined largely by stress concentrations in its critical parts. Stress frequency response functions (FRF's) are being applied for the fatigue life prediction. Computation of the FRF's with the finite element (FE) method is expensive, since a fine FE mesh should contain geometrical details of the structure for the accurate modelling of stress concentrations. However, it is possible to use the computational power of the FE method more intelligently as is the case with the global/local model concept presented by De Langhe *et al.* [1]. The dynamical behaviour of the structure is almost independent of geometrical details. A dynamic analysis can be performed on a coarse FE model consisting of beam elements, plate elements, springs, lumped masses etcetera. The results are combined with the results of a linear static (LS) analysis on a fine FE mesh consisting of 2D and 3D solid elements of a small region of the structure containing the geometrical detail.

In section 2 the global/local model concept is described. Section 3 considers an important process in the global/local model method, the internal force distribution. In section 4 a numerical experiment is presented and finally the conclusions are stated in section 5.

2. Global/local model concept

The global/local model concept as proposed by De Langhe *et al.* [1] is described in this section. The method combines the result of a dynamic analysis applied to a coarse model or global model of the total structure with the result of a LS analysis applied to a fine model or local model of a region containing the critical zone. The FRF is computed from an excitation point somewhere on the structure to one or several components of stress in the critical zone (see figure 1).

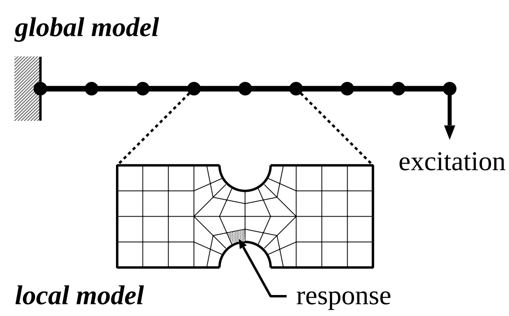

Figure 1: global/local model concept

2.1 Overview

Two separate FE models are used. The global model consisting of beam elements, springs, dampers etcetera describes the dynamical behaviour of the total structure. The second model is a local model of a region containing the geometrical detail. The local model can be considered as a separated part of the total structure. In order for the deformation in the separated part to be consistent to the deformation of the same region in the total structure internal forces and boundary conditions have to be applied. There are more nodes in the local model on the global/local interface than there are in the global model. Therefore, the internal force FRF's of the dynamic analysis of the global model have to be distributed over the boundary of the local model according to the classical beam theory (see figure 2). A linear static (LS) analysis is performed on the local model. The inertia of the separated part is not taken into account. To overcome this inconsistency the acceleration field of the global model is interpolated over the local model (see figure 3). Multiplication by the lumped masses yields an inertia force field that can be applied in the LS analysis. Finally, statically determined boundary conditions (see figure 4) are employed to ensure that the deformation in the separated part is similar to the deformation in the same region of the total structure.

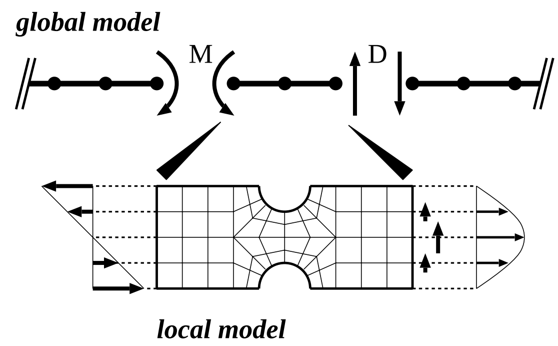

Figure 2: 2D example of internal force distribution

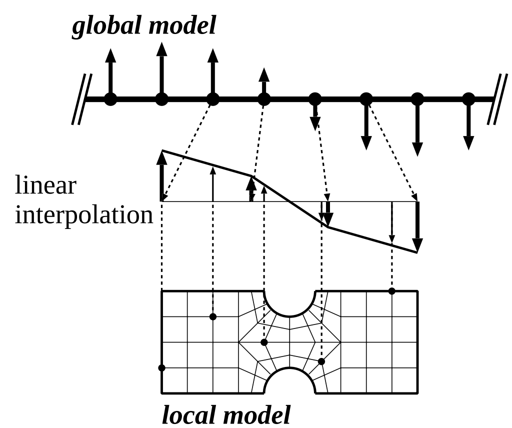

Figure 3: 2D example of acceleration interpolation

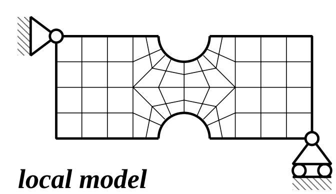

Figure 4: 2D example of statically determined boundary conditions

2.2 Superposition of load cases

The direct approach (see [1]) is considered here only. The internal force FRF's and the acceleration FRF's of the dynamic analysis are used instead of the modal internal force vectors and the modal acceleration vectors in case of the indirect approach. For each frequency f_k the stress $\sigma(f_k)$ is computed by a LS analysis with the corresponding distributed boundary forces $F^B(f_k)$ and inertia forces $F^I(f_k)$

$$\sigma(f_k) = H_t \cdot F_t(f_k),\ \forall\, k \in [1, n_f] \tag{1}$$

where H_t represents the total transfer function, $F_t(f_k)$ the union of $F^B(f_k)$ and $F^I(f_k)$ ($F_t(f_k) = F^B(f_k) \cup F^I(f_k)$) and n_f the number of frequency lines. The $F_t(f_k)$ is not used in one load case, but it is divided in various load cases. Two major classes are distinguished, namely one based on the distributed boundary forces and one based on the inertia forces. Since the FE analysis is a linear operation the superposition principle is valid so the results of the various load cases can be combined as follows

$$\begin{aligned} \sigma(f_k) &= \sum_{j=1}^{m_B} H_j^B \cdot F_j^B(f_k) + \\ &+ \sum_{j=1}^{m_I} H_j^I \cdot F_j^I(f_k) \\ &\qquad \forall\, k \in [1, n_f] \end{aligned} \tag{2}$$

where m_* represents the number of load cases with $* = B$ for the distributed boundary forces and $* = I$ for the inertia forces, H_j^* the transfer function for load case j and $F_j^*(f_k)$ the force FRF from the dynamic analysis. Both the transfer functions H_j^* and the shape of the force distributions per load case $F_j^*(f_k)$ are independent of the frequency. Therefore it is sufficient to compute a reference stress value per load case $\sigma_{j,unit}^*$ as a response to a unit force distribution $F_{j,unit}^*$ using one LS analysis per load case. The stress FRF per load case $\sigma_j^*(f_k)$ is obtained by scaling the reference stress values for each frequency. The procedure

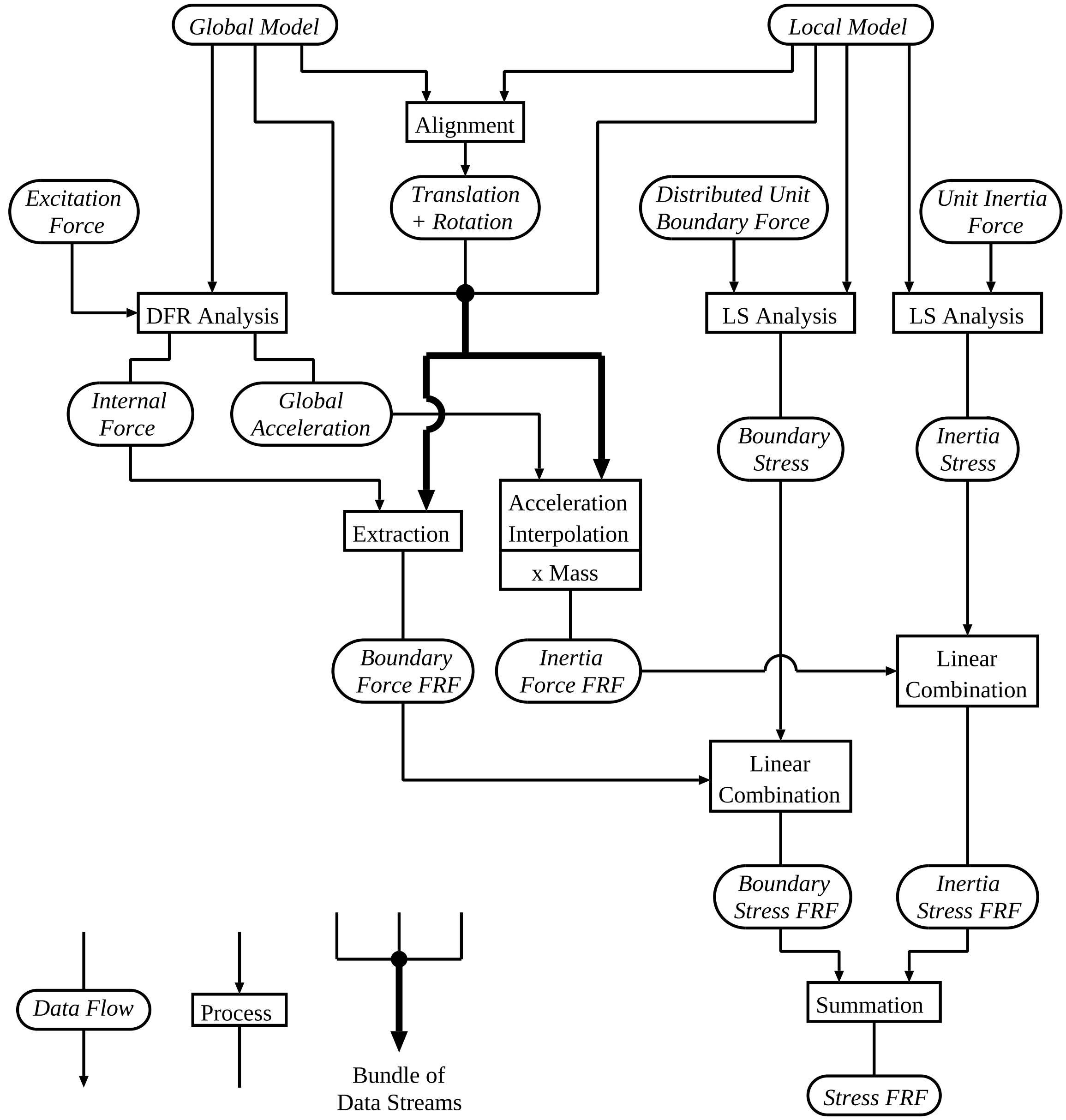

Figure 5: Flow chart

above is summarised as follows

$$\begin{aligned} \sigma(f_k) &= \dots \\ &= \sum_{j=1}^{m_B} \sigma^B_{j,unit} \cdot (F^B_j(f_k)/F^B_{j,unit}) + \\ &+ \sum_{j=1}^{m_I} \sigma^I_{j,unit} \cdot (F^I_j(f_k)/F^I_{j,unit}) \qquad (3) \\ &\qquad \forall\, k \in [1, n_f] \\ &\text{where } \sigma^*_{j,unit} = H^*_j \cdot F^*_{j,unit}. \end{aligned}$$

2.3 Implementation

The various processes in the global/local model concept e.g. the internal force distribution and the acceleration interpolation have to be automated in order for the new concept to be applied in practice. The processes and the data flows of the global/local model method are graphically represented by the flow chart in figure 5 and they are discussed below.

1. The global model and the local model are aligned resulting in a translation vector and a rotation tensor for the local part of the global model.

2. A direct frequency response (DFR) analysis is performed on the global model with a harmonic excitation, in which the internal force FRF's and the acceleration FRF's are computed.

3. The boundary force FRF's for the local model are extracted from the internal force FRF's.

4. The acceleration FRF's are interpolated over the nodes of the local model and the interpolated acceleration FRF's are multiplied by the lumped masses to obtain the inertia force FRF's. Both the extraction process and the interpolation process employ the results of the alignment process and the model data of both the FE models.

5. Two LS analyses are performed on the local model. One analysis is performed with the unit boundary forces $F^B_{j,unit}$ and one with the unit inertia forces $F^I_{j,unit}$ resulting in respectively the boundary stresses $\sigma^B_{j,unit}$ and the inertia stresses $\sigma^I_{j,unit}$ (see equation (3)).

6. The linear combination of the boundary force FRF's with the boundary stresses as coefficients and the linear combination of the inertia force FRF's with the inertia stresses as coefficients leads to respectively the boundary stress FRF and the inertia stress FRF.

7. The final stress FRF is obtained by the summation of the boundary stress FRF and the inertia stress FRF.

3. Internal force distribution

An important step in global/local model concept is the internal force distribution over the global/local interface. The distribution is discussed for beam elements with rectangular cross sections only. The internal forces which occur are the two shear forces V_1 and V_2, the two bending moments M_1 and M_2, the axial force F and the torsional moment T as shown in figure 6. The distributed forces are determined by the classical rules of mechanics as stated in the previous section. That is, the shape of the force distribution is the same as the shape of the stress distribution in beam theory (see [2] and [3]) and the total force contribution is equal to the internal force. For example in case of shear force V_1 the shape is parabolic similar to the shear stress

$$F_i = A \cdot (1 - 4(\frac{y_i}{h})^2) \tag{4}$$

where F_i represents the force in y-direction of node i, A the scale factor, y_i the y-coordinate of node i and h the beam height. The scale factor is determined by the equality between the total force contribution and the internal force V_1. The distributed internal forces are summarised in table 1. The index i represents the node number, N the number of local nodes in the cross section, b and h respectively the beam width and height and (y_i, z_i) the nodal coordinate in the cross section.

The formulas in table 1 can not be generalised easily for beam elements with arbitrary cross sections. The distribution of the torsional moment T is more difficult to determine. It is not possible to obtain the

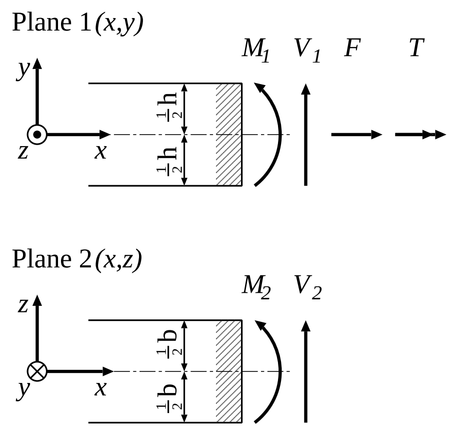

Figure 6: internal forces of beam element

F_{int}	distributed internal force magnitude	dir
M_1	$-M_1 \cdot \frac{y_i}{\sum_{i=1}^{N} {y_i}^2}$	x
V_1	$V_1 \cdot \frac{1-4(\frac{y_i}{h})^2}{\sum_{i=1}^{N}(1-4(\frac{y_i}{h})^2)}$	y
M_2	$-M_2 \cdot \frac{z_i}{\sum_{i=1}^{N} {z_i}^2}$	x
V_2	$V_2 \cdot \frac{1-4(\frac{z_i}{b})^2}{\sum_{i=1}^{N}(1-4(\frac{z_i}{b})^2)}$	z
F	$F \cdot \frac{1}{N}$	x
T	$\frac{T \cdot ({y_i}^2 - \frac{1}{4}h^2) z_i}{\sum_{i=1}^{N} (\, ({y_i}^2 - \frac{1}{4}h^2){z_i}^2 + ({z_i}^2 - \frac{1}{4}b^2){y_i}^2 \,)}$	y
	$\frac{-T \cdot ({z_i}^2 - \frac{1}{4}b^2) y_i}{\sum_{i=1}^{N} (\, ({y_i}^2 - \frac{1}{4}h^2){z_i}^2 + ({z_i}^2 - \frac{1}{4}b^2){y_i}^2 \,)}$	z

Table 1: Distributed internal forces

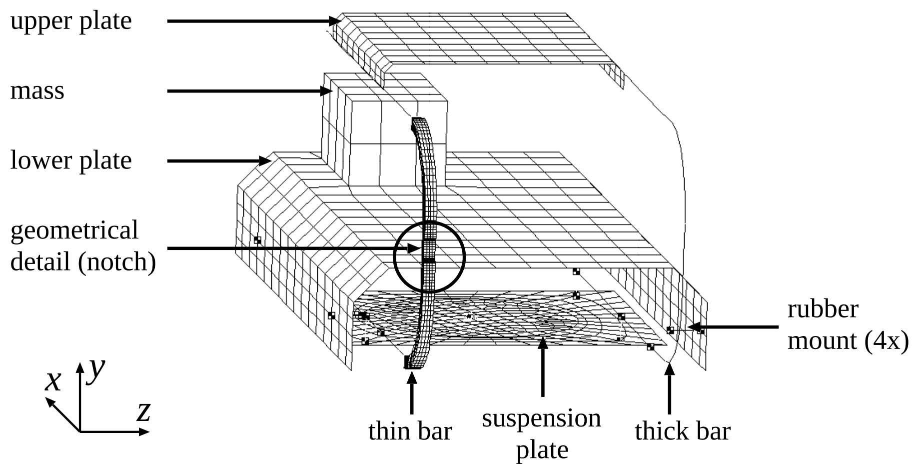

Figure 7: Reference FE model of assembled structure

shear stress distribution of a beam with an arbitrary cross section under torsional load analytically (see [3]).

4. Numerical experiments

In [1] the global/local model concept is illustrated with the 2D academic example of a beam structure. The influence of both the coarseness of the global FE model and the size of the local FE model are examined. De Langhe *et al.* have concluded that the global model should be fine enough to represent all eigenmodes in the frequency range of interest and that the extension of the local model does not influence the stress results substantially.

Here the result of the global/local model method is presented for one test case. An assembled structure (see figure 7) is considered which is excited by a prescribed acceleration of the suspension plate. The total dimensions are 474 mm in length (x-direction), 241 mm in width (z-direction) and 179 mm in height (y-direction). The assembled structure consists of a 8 mm thick aluminium suspension plate, a thick steel bar (7×9) bent in an U-shape, a thin steel bar (7×7) bent in an U-shape and provided with a geometrical detail, rubber mounts, an upper and a lower plate manufactured from a 1 mm thick steel plate, a steel mass of 2 kg and some connection elements. The suspension plate is mounted rigidly on top of the electrodynamic vibration exciter. A global FE mesh is generated using beam elements, plate elements, solid elements, rigid elements and lumped masses only (see figure 8). The frequency range of interest for the dynamic analysis is $0 - 30$ Hz. There are 4 eigenmodes within this frequency range for which the eigenfrequencies are given in table 2. The eigenmodes are presented in figure 10.

A geometrical detail is created in the vertical part of the thin bar (see figure 7). A notch is situated over the total width of the bar in the face with its normal in the negative x-direction of the global FE model. A fine FE model consisting of solid elements only is generated of the vertical part of the thin bar for the LS analyses (see figure 9).

In the dynamic analysis the structure is excited in the y-direction by a base acceleration with a prescribed level

$$a(f,t) = 1 \cdot e^{2\pi j f t} \; \forall \, f \in [0, 30] \tag{5}$$

where $a(f,t)$ represents the prescribed acceleration, f the frequency, t the time and where $j = \sqrt{-1}$. The stress FRF is computed for element 297 in the local model situated at the narrowed part of the thin bar (see figure 9). The result of the global/local model method is validated with a stress FRF computed by a dynamic analysis applied to the reference FE model shown in figure 7. The reference model does not consist of solid elements only since the size of FE models is restricted. Only the bent part of the thin part is modelled using solid elements.

The stress FRF to the critical part is shown in figure 12. There is a good correspondence between the reference stress FRF and the stress FRF computed by the global/local model method except for the third eigenfrequency. The third eigenmode is the first local bending mode of the lower plate as illustrated in figure 10. The deformation of the vertical part of the thin

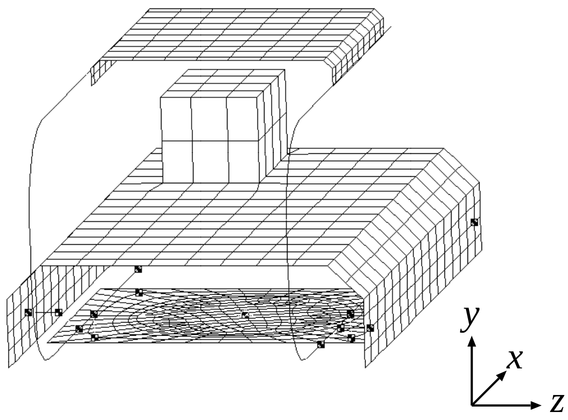

Figure 8: Global FE model of assembled structure

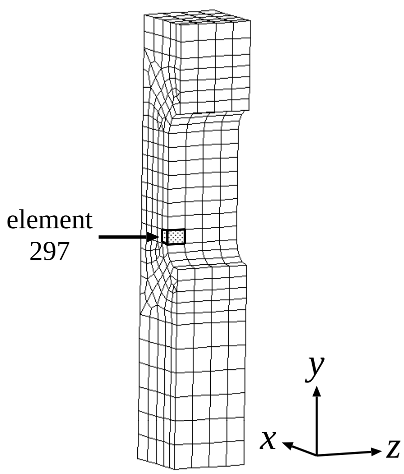

Figure 9: Local FE model of region around geometrical detail

mode	frequency [Hz]
1	17.5
2	18.2
3	21.7
4	25.2

Table 2: Eigenfrequencies of assembled structure

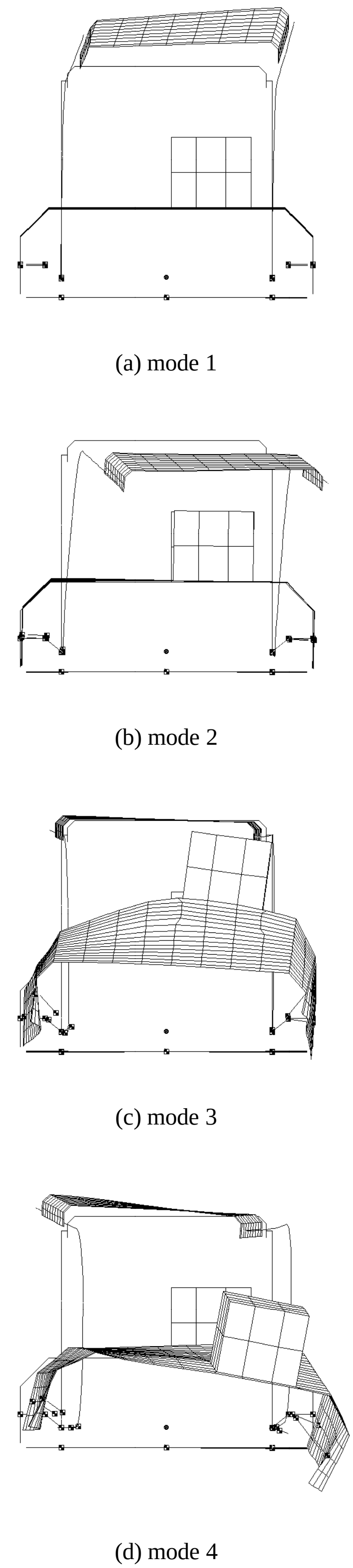

Figure 10: Eigenmodes

bar is small both in case of the global FE model (see figure 10(c)) and in case of the reference FE model (see figure 11). Therefore the stress levels are expected to be small.

5. Conclusions

For the practical application of the global/local model method it is necessary that the method is automated.

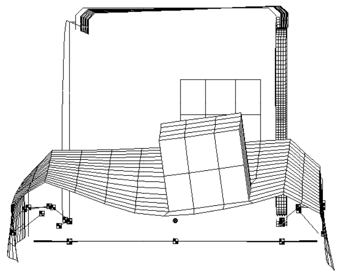

Figure 11: Mode 3 of reference FE model

The important processes besides the finite element calculation are the internal force distribution and the acceleration interpolation. Section 3 shows that the force distribution can be derived for simple beam elements only. The distribution of the torsional moment T depends on the shape of the beam cross section. Finally in section 4 the global/local model is illustrated for stress FRF computation of an assembled structure. The result is validated with the reference stress FRF determined in a direct frequency response analysis performed on the reference FE model. There is a good correspondence between the stress result obtained by the global/local model method and the computed reference.

Acknowledgements

The work presented in this paper is part of the Brite Euram Project BE-96-3035 FATYNAMICS, concurrent engineering approach for durability optimisation of trucks applying an integrated fatigue and dynamics methodology.

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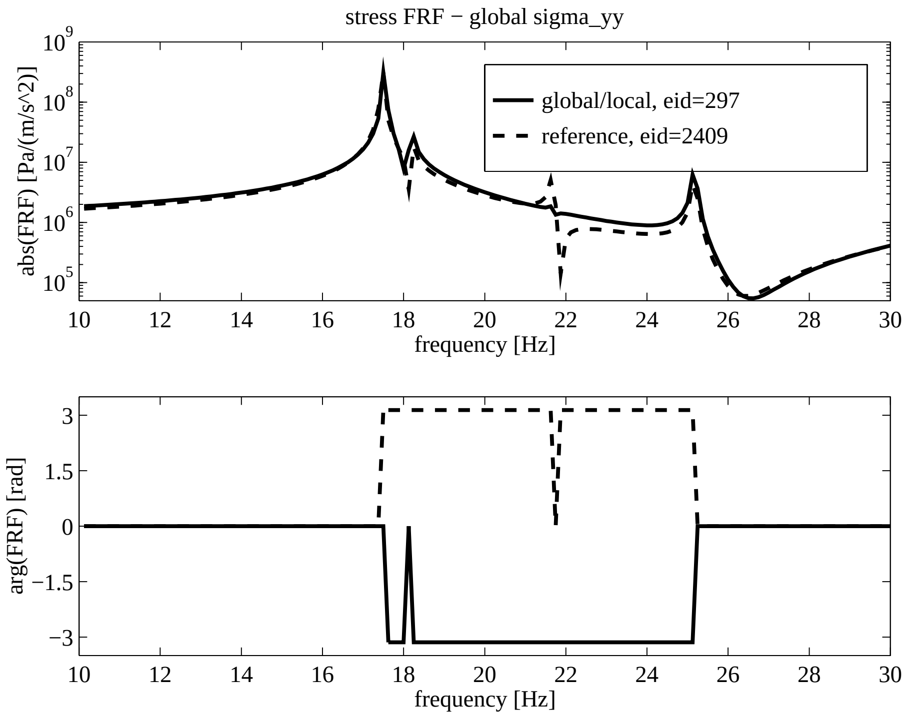

Figure 12: Stress FRF of assembled structure