# Vibro-acoustic energy flow models implemented by finite elements

**I. Moens*, D. Vandepitte, P. Sas**
Department of Mechanical Engineering, division PMA, K.U.Leuven, Belgium
e-mail : ilse.moens@mech.kuleuven.ac.be
** Research Assistant of the Fund for Scientific Research - Flanders (Belgium)(F.W.O.).*

## Abstract

The use of energy flow as an analytical method is a concept that has received considerable attention for predictions of vibrational behaviour in the mid to high frequency range. This paper describes the use of energy density and energy flow as primary variables in a vibro-acoustic analysis. Basic energy equations are derived for both plates and acoustic cavities individually. Joint parameters describe the vibro-acoustic coupling relationship. This relationship is derived in terms of plate radiation efficiency, which is a function of geometry, frequency and material properties. Similar to the coupling of structural subsystems, the finite element implementation of the energy-based vibro-acoustic coupling equation relates the energy density of the plate and the acoustic subsystem. This joint relationship uses power transmission and power reflection coefficients. This paper gives an overview of the method and an example of a coupled structural-acoustic system.

## 1. Introduction

A lot of tools are available for the study of the dynamic behaviour of mechanical structures. The classical Modal Analysis and the dynamic Finite Element Method (FEM) and Boundary Element Method (BEM) are the most conventional tools and are widely used in the low frequency range to predict the first modes of the structure and frequency response functions. However theoretically correct, these methods have some deficiencies at higher frequencies because of the shorter wavelengths and the higher modal density. In FEM, because of the shorter wavelengths, the number of elements must be increased with frequency. This makes the method costly at high frequencies. Moreover, since results at high frequencies are more sensitive to parameter changes, these deterministic methods are no longer feasible beyond a certain frequency range.

At high frequencies some new tools are developed, which predict the average or smoothed dynamic behaviour in a statistical way. These tools use mechanical energy (sum of kinetic and potential energy) as primary variable in stead of displacements as in the low frequency range. Because the mechanical energy is spatially smoother, this variable is better suited for problems at high frequencies where the wavelength becomes shorter.

Nowadays, Statistical Energy Analysis (SEA) is probably the most commonly used technique for a dynamic analysis in the high frequency range [5,6]. Predictive SEA divides a complex built-up structure into a number of subsystems. Out of an energy balance, based on the energy exchange between subsystems and the energy dissipation within subsystems, an overall vibration response of each subsystem can be obtained. However, no information is available on the spatial variation of the energy within a subsystem.

Energy Flow Analysis (EFA) is a more recent tool for the prediction of the vibrational behaviour of structures in the high frequency range [1-4]. Energy Flow Analysis, like predictive SEA, predicts mechanical energy based on energy equilibrium equations. But EFA also predicts the spatial variation of the mechanical energy in the structure. Energy flow analysis is able to model local effects such as localised power inputs and local damping treatments.

The energy distribution and the energy flow of the different waves are predicted in some basic components like beams, plates, acoustic cavities... An important advantage is that the energy equations in these basic components are conceptually similar to the equations of static heat flow. The energy distribution and energy flow within the basic components can thus easily be computed with existing finite element codes for thermal computations. This is called the Energy Finite Element Method (EFEM). At the coupling of the basic components, transmission coefficients describe the reflection and transmission of waves of different types. Out of these transmission coefficients, the energy flow at the joints can be studied. In the Energy Finite Element Method a special joint element based on these transmission coefficients is included at each joint. Because of the finite element formulation of the method, the EFEM is similar to the database needed for a FEM calculation. So, a low frequency FEM analysis can be easily extended to an analysis in higher frequency bands by EFEM.

In this paper, basic energy equations are derived for both plates and acoustic cavities individually. Joint parameters describe the vibro-acoustic coupling relationship. This relationship is derived in terms of plate radiation efficiency, which is a function of geometry, frequency and material properties. This paper gives an overview and an example of EFEM for a coupled structural-acoustic system.

# 2. Theoretical overview

The basic energy equation for a single subsystem in steady state conditions can be derived by expressing energy equilibrium of a differential subsystem :

$$\Pi_{in} = \nabla\vec{I} + \Pi_{diss} \qquad (1)$$

with $\Pi_{in}$ the applied power , $\vec{I}$ the intensity or energy flow at the borders of the subsystem and $\Pi_{diss}$ the internal dissipated power.

Dissipation of energy can be caused by a great number of mechanisms. In literature, viscous and hysterical damping are most commonly discussed ( Cremer & Heckl [7] ). The structure's ability to dissipate energy is quantified by the material damping loss factor $\eta$. Using the hysteresis damping model, the time averaged dissipated energy density per time unit ( dissipated power ) can be derived :

$$< \Pi_{diss} >= \omega\eta \; <e> \qquad (2)$$

with $\omega$ the pulsation, $e$ the energy density; '< >' denotes a time averaged quantity.

In EFA, mechanical waves transport the energy throughout the structure. In the presented cases in this paper, only a plate subsystem and an acoustic cavity are studied. The energy transport equation in conceptually similar for both cases.

## 2.1 Plate subsystem

In this paper only out-of-plane flexural waves in plates are considered. Out of the solution of the wave equation for bending waves in thin plates, a relationship between the spatially smoothed energy flow or intensity vector and the gradient of the spatially smoothed energy density can be derived :

$$< \underline{\vec{I}} >= -\frac{c_g{}^2}{\eta\omega} \nabla < \underline{e} > \qquad (3)$$

with $c_g$ the group speed of bending waves in plates. '_' denotes a spatially smoothed quantity.

This approximation is only valid in the farfield and for plane waves. The farfield of the plate is any part of the plate that is away (typically half the wavelength) from boundaries and excitation points.

The combination of (1), (2) and (3) yields the general governing energy equation within a plate element :

$$-\frac{c_g{}^2}{\eta\omega} \nabla^2 < \underline{e} > + \eta\omega < \underline{e} > = < \Pi_{in} > . \qquad (4)$$

This is a second order differential equation which governs the smoothed energy density distribution in a vibrating plate. The equation is very similar to a steady-state heat transfer equation, where the temperature is analogous to energy density and the heat flow corresponds to the energy flow or intensity. As a consequence, the traditional finite element code for a steady-state heat transfer problem can be used to solve the energy density and energy flow in a single plate system.

## 2.2 Acoustic subsystem

Out of the Helmholtz wave equation :

$$(\nabla^2 + k_c{}^2) P = Q \qquad (5)$$

with P the pressure, $k_c$ the complex wave number and the Euler equation :

$$U = \frac{j}{\rho_0 . \omega} \nabla P \qquad (6)$$

with U the displacement, $\rho_0$ the fluid mass density and with the approximations of plane waves in a diffuse field and a spatial smoothing over one wave length, one can derive a equation similar to (3) and (4). Note that in this case the intensity and the energy density have to be interpreted for a 3D acoustic element.

### 2.3 Discretisation with finite elements (EFEM)

The energy balance equation as in equation (4) can be solved by a discretisation with finite elements. This is called the Energy Finite Element Method (EFEM). The element matrix can be calculated as follows [2] :

$$[K]\{e\} = F - P \qquad (7)$$

with $K_{ij} = \int_D (\frac{c_g^2}{\eta.\omega} \nabla\Phi_i \nabla\Phi_j + \eta.\omega.\Phi_i\Phi_j) dD$

$F_i = \int_D \Phi_i \Pi_{in} dD$

$P_i = \int_\Gamma \Phi_i (\vec{I}.\vec{n}) d\Gamma$

and shape functions $\Phi_i, \Phi_j$

Different options for the boundary conditions $P_i$ are:

- *intensity boundary conditions*
  In this case, the normal intensity is specified :

$$\vec{I}.\vec{n} = -I_{BC} \qquad (8)$$

  with $I_{BC}$ the applied normal intensity at the boundary.

- *absorption boundary conditions (acoustic elements only)*
  With the Sabine room acoustics model, the absorption boundary condition is expressed as :

$$\vec{I}.\vec{n} = \frac{\alpha}{4}.c_0.e \qquad (9)$$

  with $\alpha$ the absorption coefficient, $c_0$ the wave speed in air
  This boundary condition yields an extra term in the global systemmatrix [K] :

$$K_{ABS} = \int_D (\frac{\alpha}{4}.c_0 \Phi_i \Phi_j) dD. \qquad (10)$$

- *coupling with other types of elements*
  In this paper, the coupling between an acoustic element and a plate element is studied as described in the next paragraph 3.

## 3. Vibro-acoustic coupling

The procedure to handle the coupling of a plate element with an acoustic cavity is as follows :

- add extra nodes at the coupling to predict different energy density values for bending waves in the plate element and the acoustic waves in the acoustic element

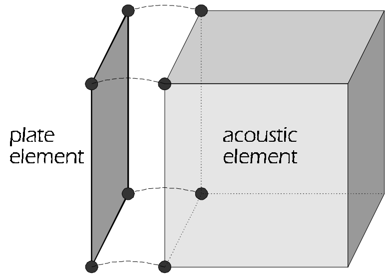


Figure 1: Coupling of an acoustic element with a plate element

- add a coupling matrix to the global system matrix [K]. The coupling matrix is calculated from the transmission coefficients, as described in paragraphs 3.1 and 3.2.

### 3.1 Transmission coefficients

Transmission coefficients are defined as ratios of transmitted to incident power. For the calculation of the vibro-acoustic transmission coefficients a diffuse acoustic field is assumed. The result is a non-symmetric transmission coefficient matrix which is a function of geometry, material properties and the radiation efficiency. A complete derivation of the transmission coefficients can be found in [1].

The structural to acoustic transmission-coefficient is:

$$\tau_{str \to ac} = \frac{\Pi_{transmitted,ac}}{\Pi_{incident,str}} = \frac{2\beta_{21}\sigma_{rad}}{2 + \beta_{21}\sigma_{rad}} \qquad (11)$$

with $\sigma_{rad}$ the radiation efficiency and

$$\beta_{12} = \frac{\rho_0.c_0}{\rho_{str}.c_{str}},$$

where $\rho_0$ is the mass density of air, $c_0$ the wave speed in air, $\rho_{str}$ the mass density of the structural component (plate), $c_{str}$ the phase speed of bending waves in the plate.

Some references for the calculation of the radiation efficiency, which is a function of frequency, geometry and material properties, can be found in literature [5].

The acoustic to structural transmission-coefficient is:

$$\tau_{ac\rightarrow str} = \frac{\Pi_{transmitted,str}}{\Pi_{incident,ac}} = \beta_{21}\frac{c_0^2}{c_{str}^2}\frac{\sigma_{rad}}{fh} \quad (12)$$

with $\beta_{12}$, $c_0$, $c_{str}$ and $\sigma_{rad}$ as in (11); $f$ is the frequency and $h$ the thickness of the plate.

The reflection coefficients can be easily calculated out of the transmission coefficients :

$$\rho_{str\rightarrow str} = 1 - \tau_{str\rightarrow ac} \quad (13)$$

$$\rho_{ac\rightarrow ac} = 1 - \tau_{ac\rightarrow str} \quad (14)$$

The transmission coefficient matrix $\tau$ is :

$$\tau = \begin{bmatrix} \rho_{str\rightarrow str} & \tau_{str\rightarrow ac} \\ \tau_{ac\rightarrow str} & \rho_{ac\rightarrow ac} \end{bmatrix} \quad (15)$$

which is a non-symmetric matrix.

### 3.2 Coupling matrix

The coupling matrix connects the energy density levels with the energy flows at the plate-acoustic joint. The calculation of this joint matrix only requires the knowledge of the different group velocities and the transmission coefficients, as described in [4] :

$$[J] = [I - \tau][I + \tau]^{-1}[c_g] \quad (16)$$

with $I$ the 2x2 unity matrix, $\tau$ the transmission coefficient matrix and $[c_g]$ a 2x2 diagonal matrix with the group velocities of the different wave types along the diagonal.

A straightforward calculation yields :

$$[J] = \frac{1}{2 - \tau_{str\rightarrow ac} - \tau_{ac\rightarrow str}}\begin{bmatrix} \tau_{ac\rightarrow str}.c_{str} & -\tau_{str\rightarrow ac}.c_{ac} \\ -\tau_{ac\rightarrow str}.c_{str} & \tau_{str\rightarrow ac}.c_{ac} \end{bmatrix} \quad (17)$$

In order to predict the different energy density values at the nodes at the vibro-acoustic coupling, this expression is discretised over the 8 nodes at the coupling.

## 4. Example : 1D acoustic tube

In practise, the procedure for a EFEM calculation is as follows :

1. *pre-processing*
   create a mesh (e.g. with a commercial FE package) and add loads, local dampers, boundary conditions, frequency
2. detect *joints* of different types (beam-beam, plate-plate, plate-acoustic) and add extra nodes at the joint
3. *global assembly of the system matrix equation*
   calculate element matrices [K] (beam-plate-acoustic), joint (coupling) matrices [J], add local dampers and assemble the global system matrix and the load vector
4. *solve* the system matrix equation
5. *post-processing*
   visualisation and interpretation of the results

In this paragraph, a simple one dimensional vibro-acoustic example is studied as shown in figure 2. An acoustic tube of 2m length is excited by a plate of 0,02m x 0,02m at one end (left). EFEM results are compared to a reference case : a classic vibro-acoustic FE calculation with a very fine mesh. Two cases are studied. In a first case, only damping in the air is considered. In a second case some absorption material is added at the other (right) end of the tube.

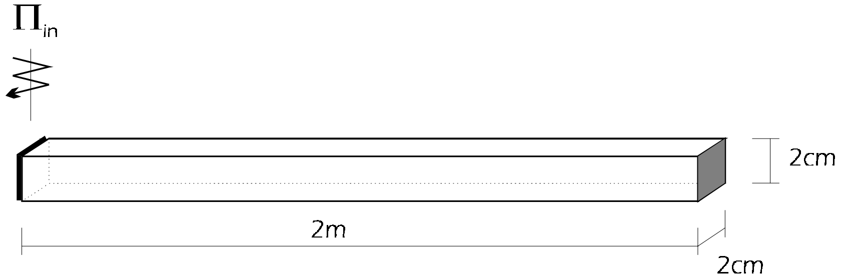


Figure 2 : One dimensional acoustic tube, excited by a vibrating plate.

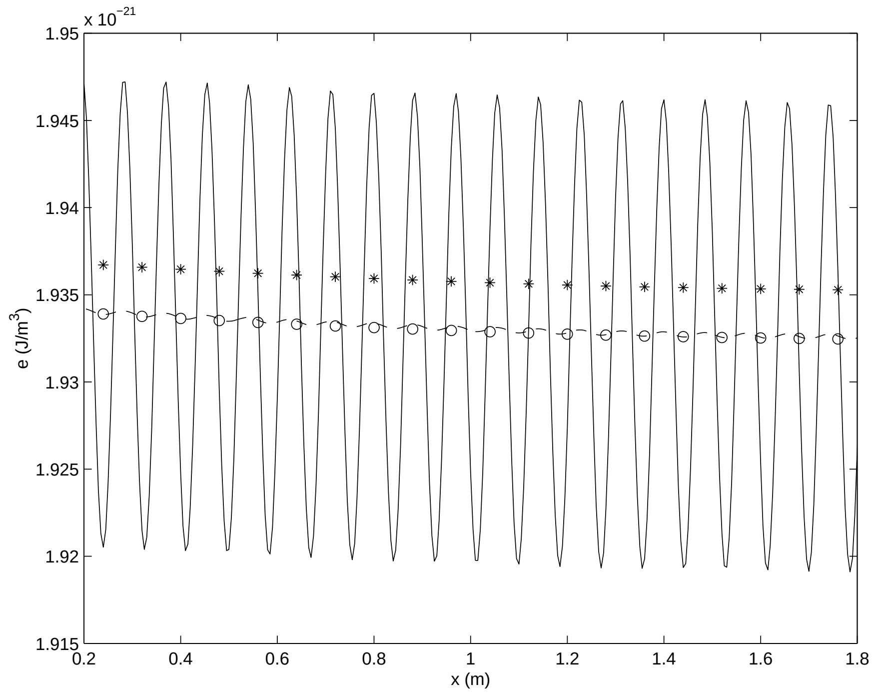


Figure 3: 1D acoustic tube with only fluid (air) damping

| | |
|---|---|
| ----------- | reference FE calculation |
| -o- -o- - | smoothed reference FE |
| * * * * | EFEM result |

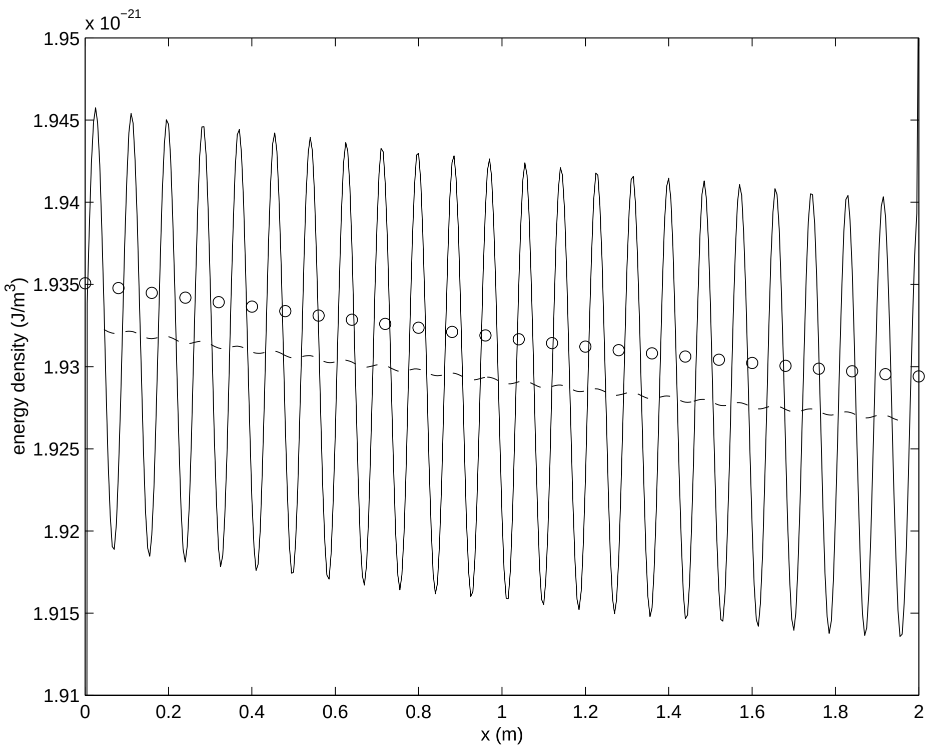


Figure 4: 1D acoustic tube with fluid (air) damping and absorption boundary conditions (a = 0.4)

| | |
|---|---|
| ----------- | reference FE calculation |
| - - - - - | smoothed reference FE |
| o o o | EFEM result |

The EFEM results are verified by a classic vibro-acoustic FE model with 400 acoustic elements along the length of the acoustic tube. The figures 3 and 4 also show the spatially smoothed result of the reference calculation. The EFEM model contains only 20 element. The number of elements in the EFME can still be reduced.

Some general properties of the model are summarised in table 1.

| | ***Symbol*** | ***Value*** |
|---|---|---|
| *wave speed in air* | $c_0$ | 343 m/s |
| *fluid mass density* | $\rho_0$ | 1.21 kg/m$^3$ |
| *fluid damping* | $\eta$ | 6.10$^{-4}$ |

Table 1: properties of the acoustic tube

Figure 3 shows the results when only fluid damping in the air is applied. The EFEM predictions correspond very well to the spatially smoothed results of the FE reference case.

In figure 4, the results are shown when some absorption material is applied to one end of the acoustic tube. The absorption coefficient $\alpha$ is set to 0.4 in this case. Also in this case, the EFEM results are close to the spatially smoothed FE reference case.

## 5. Conclusion

This paper gives an overview of the Energy Finite Element Method EFEM for vibro-acoustic problems. The coupling relationships between a vibrating plate and an acoustic cavity are derived. The outline of the EFEM procedure is explained and applied to a simple 1D example with an acoustic tube excited by a plate. The EFEM results are verified by a reference classic Finite Element calculation with a very fine mesh. The EFEM results calculated with a more coarse mesh, show a good agreement with the FE reference case.

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