Evaluation of the FRF Based Substructuring and Modal Synthesis Technique Applied to Vehicle FE Data

K. Cuppens, P. Sas
Department of Mechanical Engineering, Division PMA, K.U.Leuven, Belgium
e-mail : Koen.Cuppens@mech.kuleuven.ac.be
L. Hermans
LMS International, Leuven, Belgium
e-mail : Luc.Hermans@lms.be

Abstract
FRF based substructuring (FBS) predicts the dynamic behaviour of a coupled system on the basis of free-interface FRFs of the uncoupled components. The modal synthesis technique determines the dynamic behaviour of a coupled system on the basis of a normal mode description of the uncoupled components. This paper gives a brief description of the theoretical background of these techniques. When using these techniques it is important to take into account the error due to modal truncation. This error can be significantly reduced by application of compensation methods. This paper discusses two possible methods, namely dynamic FRF compensation and compensation by residual modes. In order to evaluate the FBS and modal synthesis techniques and the compensation methods, they are applied to vehicle FE data, more specifically to the coupling of a vehicle subframe with a body-in-white. In the case of multi-component coupling with high frequency resolution and bandwidth the results are in favour of modal synthesis but in the case of frequency dependent joints and the need for easily obtainable interface forces the results are in favour of FBS.

1 Introduction

Virtual prototyping in the NVH design and refinement process of a vehicle could be an answer to the never ending demand of vehicle development time and cost reduction and to the need to take this process to an earlier stage in the design cycle.

A virtual prototyping environment should be able to (1) translate the subsystem and component behaviour to the system behaviour by means of appropriate substructuring methods, (2) analyse the contribution of components to the system behaviour (contribution analysis), (3) integrate numerically and experimentally modelled components or different types of numerically modelled components (hybrid modelling), (4) translate system target responses to subsystem or component target responses (component target setting) and (5) optimise these target responses with respect to NVH performance.

Concerning the translation of the subsystem and component behaviour to the system behaviour, two different classes of substructuring methods can be distinguished [1]: time-domain based and frequency-domain based. In the time-domain methods each component is described by a (generalised) mass, damping and stiffness matrix. This class is represented by the component mode synthesis (CMS) and modal synthesis method. In the frequency-domain based methods each component is described by frequency-dependent data. This class is represented by the FRF based substructuring (FBS) or dynamic flexibility method and the inverse FBS or dynamic stiffness method. On the one hand, CMS and modal synthesis have been applied mainly and extensively on analytical data for many years [2, 3, 4, 5, 6]. On the other hand, FBS has been used mainly on experimentally obtained data [7, 8] although it is preferable over CMS in the case of frequency-dependent joints in an analytical coupling procedure. This paper gives an evaluation of the FBS and modal synthesis technique for FE data.

The first two sections discuss the FBS technique and the modal synthesis technique. Together with a description of the theoretical principles the calculation procedure is described. The next section treats two compensation methods which can reduce the modal truncation errors occurring in both substructuring techniques. Then, the vehicle FE data is presented and the results of the application of both coupling techniques to this data are evaluated.

2 FRF based substructuring

2.1 Theoretical formulation

The FBS technique predicts the dynamic behaviour of a coupled system on the basis of free-interface FRFs of the uncoupled components and possible coupling stiffnesses. For every component the available degrees of freedom (dofs) are classified into two sets, namely (1) the coupling dofs and (2) the internal dofs. These internal dofs correspond with the excitation and response dofs of the components.

The dynamic description of two components A and B is then

$$\begin{Bmatrix} x_{A_R} \\ x_{A_S} \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{A_{RR}} & \mathbf{H}_{A_{RS}} \\ \mathbf{H}_{A_{SR}} & \mathbf{H}_{A_{SS}} \end{bmatrix} \begin{Bmatrix} F_{A_R} \\ F_{A_S} \end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix} x_{B_T} \\ x_{B_S} \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{B_{TT}} & \mathbf{H}_{B_{TS}} \\ \mathbf{H}_{B_{ST}} & \mathbf{H}_{B_{SS}} \end{bmatrix} \begin{Bmatrix} F_{B_T} \\ F_{B_S} \end{Bmatrix} \quad (2)$$

with $\mathbf{H}_{U_{VW}}$ the FRF matrix of component U between the dof groups with subscripts V and W,
$\{x_{U_V}\}$ the V-set dofs of component U,
$\{F_{U_V}\}$ the forces on the V-set dofs of component U,
S the subscript for the coupling dofs of A and B,
R the subscript for the internal dofs of A,
T the subscript for the internal dofs of B.

The general formula for the FRFs of a rigidly coupled two component system C, assembled by the components A and B, as developed in [8] , is then

$$\begin{bmatrix} \mathbf{H}_{C_{RR}} & \mathbf{H}_{C_{RS}} & \mathbf{H}_{C_{RT}} \\ \mathbf{H}_{C_{SR}} & \mathbf{H}_{C_{SS}} & \mathbf{H}_{C_{ST}} \\ \mathbf{H}_{C_{SR}} & \mathbf{H}_{C_{TS}} & \mathbf{H}_{C_{TT}} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{A_{RR}} & \mathbf{H}_{A_{RS}} & 0 \\ \mathbf{H}_{A_{SR}} & \mathbf{H}_{A_{SS}} & 0 \\ 0 & 0 & \mathbf{H}_{B_{TT}} \end{bmatrix} - \begin{bmatrix} \mathbf{H}_{A_{RS}} \\ \mathbf{H}_{A_{SS}} \\ -\mathbf{H}_{B_{TS}} \end{bmatrix} \left[\mathbf{H}_{A_{SS}} + \mathbf{H}_{B_{SS}}\right]^{-1} \begin{bmatrix} \mathbf{H}_{A_{RS}} \\ \mathbf{H}_{A_{SS}} \\ -\mathbf{H}_{B_{TS}} \end{bmatrix}^T \quad (3)$$

This formula shows that only one inversion has to be calculated, namely the inversion of the sum of the driving point FRF matrices of both components. The quality of the predictions is strongly dependent on this inversion. To improve this inversion numerically a singular value decomposition can be used [7]. The condition number of this decomposition is a measure for the quality of the inversion. This condition number is the ratio between the largest and the smallest from zero differing singular values of the FRF matrices.

When the two components are coupled by means of a flexible joint the kernel matrix $[\mathbf{H}_{A_{SS}} + \mathbf{H}_{B_{SS}}]^{-1}$ in equation (3) has to be replaced by $\left[\mathbf{H}_{A_{SS}} + \mathbf{H}_{B_{SS}} + [K_S]^{-1}\right]^{-1}$ with $[K_S]$ the matrix which represents the coupling stiffnesses between the coupling dofs of both components. Note that $[K_S]$ has frequency-dependent elements in the case of a frequency-dependent flexible joint.

The major limitation of this formulation of FBS is that it can only be applied to couple two (groups of) substructures. In the derivation of equation (3) it is assumed that, before coupling, the two substructures are independent of each other. This method is not applicable to cases in which two coordinates to be coupled are located on the same structure. [9] gives an alternative FBS formulation which circumvents these limitations. Further treatment of this alternative is out of the scope of this paper.

2.2 Procedure

The procedure of an analytical FBS coupling consists of two main steps, namely of (1) the determination of the necessary FRFs from the available FE data and (2) the actual coupling.

2.2.1 Determination of the FRFs

The first step in the analytical FBS procedure is the determination of the FRFs. On the one hand FRFs have to be determined for the desired excitation and response points of the components and on the other hand for the coupling points. Moreover the driving points FRFs in the coupling points are essential for the FBS procedure. Two ways to determine the FRFs are discussed [1].

A first way for the determination of the FRFs is by a direct calculation of the FRFs from the available FE data of the components. The FRF at a frequency ω_k between degrees of freedom i and j of the component is the ijth element of the FRF matrix $\mathbf{H}(\omega_k)$ given by equation (4).

$$\mathbf{H}(\omega_k) = \left[-\omega_k^2[M] + j\omega_k[C] + [K]\right]^{-1} \quad (4)$$

with $[M]$ the mass matrix,
$[C]$ the damping matrix,
$[K]$ the stiffness matrix,
ω_k the kth frequency.

This can be a computationally very intensive calculation in the case of component models with a large number of degrees of freedom and/or a wide excitation frequency range. After all, the dynamic stiffness matrix has to be inverted for every discrete frequency in the frequency range of interest.

A second way to determine the FRFs of a component is by performing an FRF synthesis based on a finite number of mode shapes and natural frequencies of the components. For proportionally damped systems this relationship between the synthesized FRF matrix $\mathbf{H}_{syn}(\omega_k)$ and mode shapes is expressed by

$$\mathbf{H}_{syn}(\omega_k) = \sum_{i=1}^{N} \frac{\{\phi\}_i \{\phi\}_i^T}{(\omega_{n_i}^2 - \omega_k^2) + j2\xi_i \omega_{n_i} \omega_k}, \quad (5)$$

with N the number of calculated modes (usually less than the total number of dofs),
$\{\phi\}_i$ the ith mass normalised mode shape,
ω_{n_i} the ith natural frequency,
ξ_i the ith modal damping ratio.

FRF synthesis does not require a matrix inversion of a potentially large dynamic stiffness matrix. Only one eigenvalue analysis is needed to determine the natural frequencies and mode shapes. Then one only has to synthesize the FRFs in the dofs and frequencies of interest.

The FRF matrix $\mathbf{H}_{syn}(\omega_k)$ in equation (5) is just an approximation of the exact FRF matrix $\mathbf{H}(\omega_k)$ because in general the number of calculated modes N is less than the number of dofs of the component model.

2.2.2 Coupling of the FRFs

The actual coupling consists of applying equation (3) to the calculated FRFs of the components. The frequency range and resolution of the component FRFs determine the frequency range and resolution of the coupled FRFs. The core calculation step in this coupling calculation is the calculation of the kernel matrix $[\mathbf{H}_{A_{SS}} + \mathbf{H}_{B_{SS}}]^{-1}$ for every frequency of interest.

3 Modal synthesis

Component Mode Synthesis (CMS) [2, 3, 4, 5] is a substructuring or coupling technique which predicts the dynamic behaviour (the system modes) of a coupled system by using a generalised modal description of the components. This generalised modal description is in general a reduced component description obtained by applying a transformation $[T_i]$ from the original physical dofs $\{x_i\}$ of component i to a reduced set of generalised coordinates $\{q_i\}$. CMS contains three different phases, namely (1) reduction on component level, (2) coupling of the reduced components and solving the coupled system matrices on assembly level and (3) backtransformation of the assembly results on component level.

Prior to coupling there is always a reduction phase on component level in CMS. In general, the dynamic description of one component i in the physical domain is given by

$$\left[-\omega^2[M_i] + j\omega[C_i] + [K_i]\right]\{x_i\} = \{F_i\}, \quad (6)$$

with $[M_i]$ the mass matrix,
$[C_i]$ the damping matrix,
$[K_i]$ the stiffness matrix,
$\{x_i\}$ the displacement vector,
$\{F_i\}$ the force vector.

After applying the transformation

$$\{x_i\} = [T_i]\{q_i\} \quad (7)$$

and left multiplying equation (6) by $[T_i]^T$ the dynamic description of component i in the generalised modal domain is

$$\left[-\omega^2[m_i] + j\omega[c_i] + [k_i]\right]\{q_i\} = \{f_i\} \quad (8)$$

with $[m_i]$ the generalised mass matrix,
$[c_i]$ the generalised damping matrix,
$[k_i]$ the generalised stiffness matrix,
$\{q_i\}$ the generalised coordinate vector,
$\{f_i\}$ the generalised force vector,

where

$$\begin{aligned}
[m_i] &= [T_i]^T[M_i][T_i] \\
[c_i] &= [T_i]^T[C_i][T_i] \\
[k_i] &= [T_i]^T[K_i][T_i] \\
\{f_i\} &= [T_i]^T\{F_i\}.
\end{aligned}$$

The columns of this transformation matrix $[T_i]$ are the component modes of component i. Typically, these component modes can be constraint modes, attachment modes, normal modes (fixed-interface or free-interface) and inertia-relief modes. The number of component modes (especially normal modes) determines the degree of reduction and thus the frequency content of the component description.

When the transformation matrix $[T_i]$ of component i is composed of the normal modes $\{\phi_i\}$ of component i only, i.e. $[T_i] = [\Phi_i]$, then the generalised coordinates $\{q_i\}$ are the modal coordinates of the component. In this special case of CMS the method is called modal synthesis [10, 6].

After the reduction phase the components have to be coupled on assembly level. In a first step the generalised component matrices $[m_i]$, $[c_i]$ and $[k_i]$ are brought together in generalised uncoupled assembly matrices $[m]$, $[c]$ and $[k]$, with

$$[m] = \begin{bmatrix} [m_1] & 0 & \dots & 0 \\ 0 & [m_2] & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & [m_N] \end{bmatrix}. \tag{9}$$

$[c]$ and $[k]$ are constructed in an analogue way. In a second step the equations of compatibility (equal displacement of the connection dofs) and continuity (interface forces in the connection dofs should cancel) are taken into account. Due to the compatibility constraint the generalised dofs $\{q\}$ of the assembly are not independent anymore. Therefore the generalised dofs are divided into dependent and independent dofs, namely $\{q^d\}$ and $\{q^i\}$. The compatibility constraint is then

$$[S]\{q\} = \Big[[S_{dd}][S_{di}]\Big] \begin{Bmatrix} q^d \\ q^i \end{Bmatrix} = \{0\}, \tag{10}$$

which leads to an extra transformation matrix on assembly level, namely

$$\begin{Bmatrix} q^d \\ q^i \end{Bmatrix} = \begin{bmatrix} -[S_{dd}]^{-1}[S_{di}] \\ [I] \end{bmatrix} \left\{q^i\right\} = [R]\left\{q^i\right\}. \tag{11}$$

The dynamic description of the coupled system on assembly level for free-free boundary conditions is then

$$\Big[-\omega^2[m'] + j\omega[c'] + [k']\Big]\{q^i\} = \{0\} \tag{12}$$

where

$$\begin{aligned} [m'] &= [R]^T[m][R] \\ [c'] &= [R]^T[c][R] \\ [k'] &= [R]^T[k][R]. \end{aligned}$$

The assembly eigenvectors and natural frequencies are determined by performing an eigenvalue analysis on equation (12).

The backtransformation phase consists of the expansion of the obtained assembly eigenvectors to the physical dofs of interest by using equations (11) and (7). These backtransformed eigenvectors are the normal mode shapes of the coupled system.

4 Compensation methods

Compensation methods reduce modal truncation errors. These errors occur due to the limited number of modes which are taken into account in FRF synthesis (FBS) and in the modal description of the components (CMS or modal synthesis). The errors consist in the loss of the contribution of higher order modes to the dynamic behaviour in the whole frequency range (even at 0Hz!). In this paper two different compensation methods are treated, namely the dynamic FRF compensation [1] and the compensation by residual modes [11].

4.1 Dynamic FRF compensation

This compensation method which can only be used in FBS, adds a first order approximation of the residual FRF matrix $\mathbf{H}_{res}(\omega_k)$, i.e. the difference between the exact and the synthesized FRF matrix, to the synthesized FRF matrix $\mathbf{H}_{syn}(\omega_k)$. In the case that n is the number of dofs of the component the residual FRF matrix is expressed by

$$\begin{aligned} \mathbf{H}_{res}(\omega_k) &= \sum_{i=N+1}^{n} \frac{\{\phi\}_i\{\phi\}_i^T}{(\omega_{n_i}^2 - \omega_k^2) + j2\xi_i\omega_{n_i}\omega_k} && (13) \\ &\approx [A] + [B]\omega_k^2. && (14) \end{aligned}$$

This approximation is derived under the assumption that the FRF matrix consists of the accelerances between the dofs. The static and dynamic compensation matrices $[A]$ and $[B]$ are determined by calculating the residual FRF matrix $\mathbf{H}_{res}(\omega_k)$ at two distinct frequencies ω_{low} and ω_{high}. If it is possible to obtain the residual FRF matrix for more than two frequencies the compensation matrices can be determined by means of a least squares fit. The choice of ω_{low} and ω_{high} will influence the result. As a rule of thumb [6] one can select ω_{low} below the first flexible mode and ω_{high} at about 75% of the highest frequency of interest.

This compensation method tries to approximate the contribution of all the higher order modes to the FRF matrix in the whole frequency range of interest.

4.2 Compensation by residual modes

This compensation method adds a number of residual mode shapes $\{\psi_i^{res}\}$ to the normal modes $\{\phi_i\}$ in the modal synthesis reduction step of component i. These additional mode shapes are obtained in a two step procedure.

The first step consists of the calculation of static solutions for the component resulting from applied unit loads in the coupling and/or excitation points. These static solutions are the attachment modes $\{\psi_i^a\}$ of component i as defined in [3].

In the second step these attachment modes are orthogonalised with respect to the component normal modes. If these attachment modes $\{\psi_i^a\}$ are not linear combinations of the normal modes $\{\phi_i\}$, they are added to the normal modes in the transformation matrix $[T_i]$ of equation (7) which leads to a new transformation matrix $[T_i^a] = [\Phi_i \mid \Psi_i^a]$. An intermediate set of generalized dofs $\{q_i^a\}$ is then generated by the coordinate transformation

$$\{x_i\} = [T_i^a]\{q_i^a\}. \tag{15}$$

The dynamic description of the component in the newly generalised modal domain is then (cf. equation (8))

$$\left[-\omega^2[m_i^a] + j\omega[c_i^a] + [k_i^a]\right]\{q_i^a\} = \{f_i^a\} \tag{16}$$

where

$$\begin{aligned} [m_i^a] &= [T_i^a]^T[M_i][T_i^a] \\ [c_i^a] &= [T_i^a]^T[C_i][T_i^a] \\ [k_i^a] &= [T_i^a]^T[K_i][T_i^a] \\ \{f_i^a\} &= [T_i^a]^T\{F_i\}. \end{aligned}$$

The eigenvectors and natural frequencies of the component in the intermediate generalised domain are obtained by performing an eigenvalue analysis on equation (16). After backtransformation of the resulting eigenvectors by means of equation (15) the resulting mode shapes in the physical domain are the normal modes $\{\phi_i\}$ and the additional residual modes $\{\psi_i^{res}\}$. The number of residual modes is equal to the applied number of static loads. The natural frequencies of these residual modes are higher than the highest natural frequency of the normal modes if these normal modes are calculated from 0Hz on.

The reduction step in the modal synthesis technique can now be obtained by the new transformation matrix $[T_i^{res}] = [\Phi_i \mid \Psi_i^{res}]$.

This compensation method tries to approximate the static contribution of all the higher order modes by means of some additional higher frequency residual modes where FBS tries to approximate the contribution in the whole frequency range of interest. Both FBS and modal synthesis can use this method since the residual modes can be included in the FRF synthesis.

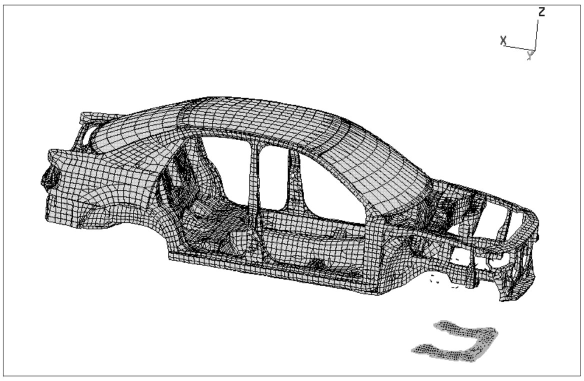

Figure 1: FE model of a vehicle body-in-white and subframe in an exploded view

5 Application to vehicle FE data

The aim of this part of the paper is to evaluate the FBS technique and the modal synthesis technique for the coupling between a vehicle subframe and body-in-white based on FE data in the frequency range from 0 to 200Hz. The directly calculated FRFs from 0 to 200Hz of the full FE model (body and subframe in one FE model) serve as a reference solution.

5.1 Model description

The considered vehicle FE data consist of two components, namely a subframe and a body-in-white as shown in figure 1. The full FE model of the coupled structure is used to obtain the reference solution.

5.1.1 Subframe

The FE model of the vehicle subframe contains approximately 12000 dofs. The subframe has three different excitation points, namely two points which are connected to the front suspension of the vehicle and one point central on the subframe which is a suspension point for the engine. There are six coupling points for the coupling between the subframe and the vehicle body-in-white.

The first five modes of the subframe are located in the frequency range up to 300Hz. The first mode (at about 160Hz) is located in the frequency range of interest of the coupling procedure. The structural damping in the subframe is represented by 1% of modal damping.

5.1.2 Body-in-white

The FE model of the vehicle body-in-white contains approximately 200000 dofs. The body-in-white has two excitation points, namely the two engine suspension points, and six coupling points for the coupling to the subframe. The response points are located on the roof, in the front wheel pits, near the foot ends of the front seats, at the end of the steering wheel and at the foot ends of the rear seats.

In the frequency range up to 200Hz the FE model of the body has up to 200 normal modes. The first mode has a natural frequency of about 30Hz. The structural damping in the body-in-white is represented by 1% of modal damping.

5.1.3 Full FE model

The full FE model of the vehicle subframe-body assembly consists of approximately 212000 dofs. The excitation and response points of the full model coincide with those of the subframe and body-in-white.

5.2 Coupling specification

For the FBS calculation the FRFs of the subframe and the body-in-white have been synthesized with a frequency resolution of 0.1Hz from 0 to 200Hz. On the one hand, in the FRF synthesis of the subframe the rigid body modes and the first five normal modes (up to 300Hz) have been taken into account. On the other hand, in the FRF synthesis of the body-in-white the rigid body modes and the normal modes up to 200Hz have been taken into account. For the dynamic FRF compensation the directly calculated FRFs have been obtained for 10 and 150Hz. For the compensation by residual modes static loads have been applied on the six dofs of the six coupling points of both subframe and body-in-white. The residual modes have been orthogonalised with respect to the modes up to 300Hz for the subframe and up to 200Hz for the body-in-white.

For the modal synthesis calculation the subframe on the one hand has been reduced using the rigid body modes and the first five normal modes (up to 300Hz). On the other hand, the body-in-white has been reduced using the rigid body modes and the normal modes up to 200Hz. The compensation by residual modes is the same as for the FBS calculation. The resulting FRFs of the coupled assembly have been synthesized with a frequency resolution of 0.1Hz from 0 to 200Hz using the rigid body modes and the normal modes up to 200Hz of the coupled assembly.

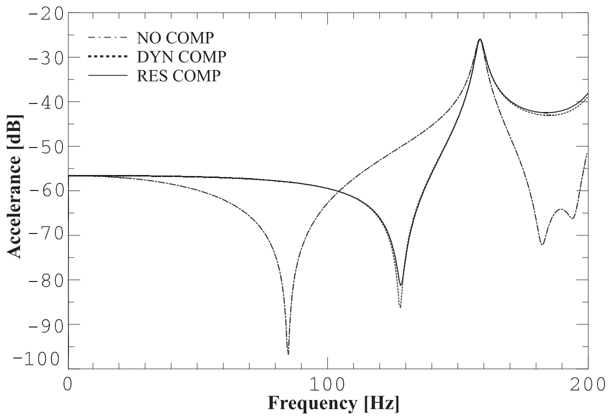

Figure 2: Synthesized FRFs of the subframe with compensation (dynamic FRF compensation and compensation by residual modes) and without compensation (rotational excitation and rotational response dof)

5.3 Coupling evaluation

Both coupling techniques are evaluated on several aspects, namely influence of compensation methods, accuracy of the coupling, calculation time, ease of use and necessary amount of data.

For both coupling techniques the modal truncation error is considerably reduced by applying the compensation methods. The effect on the synthesized FRFs of the components is stronger for FRFs containing rotational dofs. Figure 2 shows the compensation effect for an FRF between a rotational excitation dof and a rotational response dof of the subframe. The compensated FRFs approximate the exact FRF very well, although this is not shown on the figure. Figure 3 shows the difference for an FBS-coupled FRF with and without dynamic FRF compensation. For FBS it is possible to use both compensation methods. In general the compensation by residual modes is preferred because (1) the calculation of these residual modes is a simple and fast extension of the normal modes calculation and (2) there is no need for additional directly calculated FRFs in two or more frequencies. Moreover, the compensation effect of both methods is similar.

For both coupling techniques the results of the coupled FRFs (the components have been compensated by residual modes) show good correspondence in the frequency range of interest, except for a small region around 170Hz. This result is shown in figure 4.

When both techniques are compared with the ref-

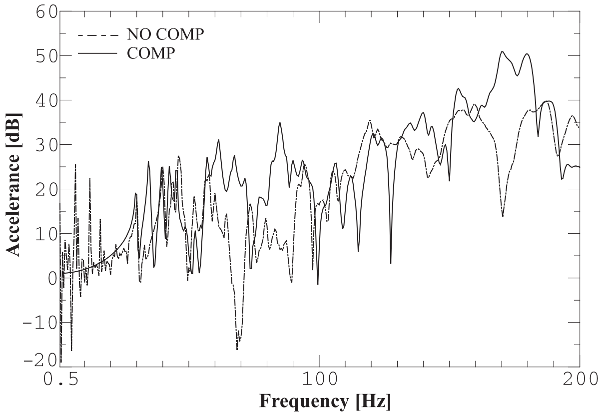

Figure 3: Coupled FRFs obtained by FBS using synthesized FRFs with and without dynamic FRF compensation (vertical excitation on the subframe and vertical response on the body roof)

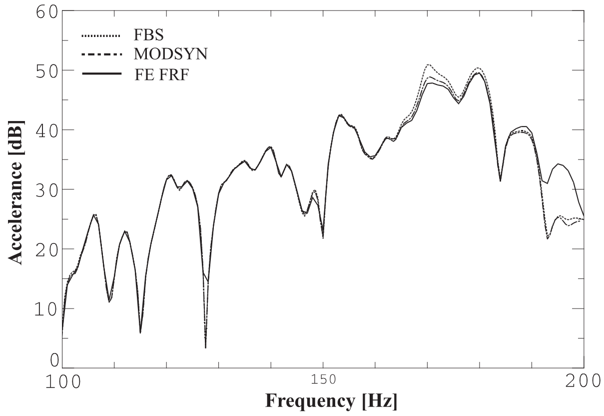

Figure 5: Reference solution and coupled FRFs obtained by FBS and modal synthesis using compensation by residual modes (vertical excitation on the subframe and vertical response on the body roof)

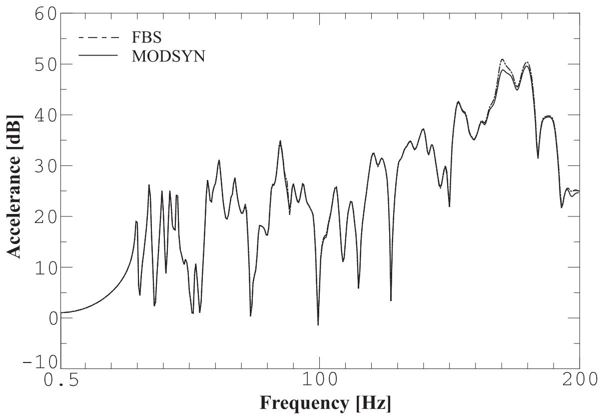

Figure 4: Coupled FRFs obtained by FBS and modal synthesis for normal modes with compensation by residual modes (vertical excitation on the subframe and vertical response on the body roof)

erence solution (see figure 5) also a good correspondence is noticed, except for this small region around 170Hz and above 190Hz. The difference between the reference solution and the coupled FRF in the neighbourhood of 170Hz is more significant for FBS. The condition number for the inversion of the kernel matrix over the frequency range also shows a significant peak around 170Hz which could explain the loss in accuracy. The difference between the reference solution and both coupled FRFs above 190Hz is probably caused by the fact that only the component normal modes up to 200Hz of the body-in-white are taken into account and that the residual modes compensate insufficiently for the high frequency truncation error.

Both coupling techniques are considerably faster than the direct FRF calculation. When both techniques use compensation by residual modes they can be up to four times faster than the direct FRF calculation. On the one hand modal synthesis will be faster than FBS in the case of multi-component coupling since FBS can only couple two components in one step. Multi-component FBS can circumvent this drawback. On the other hand, when one is mainly interested in the interface forces between the components, FBS will be faster than modal synthesis because these forces are easily obtainable out of the calculated FRF matrix of the coupled assembly.

The modal synthesis technique is easier to use in the case of multi-component coupling and in the case where several coupled solutions have to be calculated for different versions of one or a few components. The FBS technique is easier to use when frequency dependent joints are considered in the coupling between the components. Moreover, in the case where one or more components can only be represented by FRF data, FBS has to be used.

Concerning the necessary amount of data, the modal synthesis technique has got again a considerable advantage. A modal formulation is in general more compact than an FRF description, especially for larger bandwidths and high frequency resolution.

6 Conclusion

Two substructuring or coupling techniques are discussed in this paper, namely the FRF based substructuring (FBS) technique and the modal synthesis technique. The modal truncation error is explained with respect to these substructuring techniques and two possible compensation methods are discussed: the dynamic FRF compensation and the compensation by residual modes. Both coupling methods (with and without compensation) are applied to vehicle FE data, namely the coupling between a subframe and a body-in-white, and compared with a reference solution. The evaluation of these results has been done keeping several aspects in mind, namely the influence of the compensation method, the accuracy of the coupling, the calculation time, the ease of use and the necessary amount of data. On the one hand, in the case of multi-component coupling, a wide frequency range of interest, high frequency resolution and a modal component description it is advantageous to use the modal synthesis technique. On the other hand, in the case where a frequency dependent joint is considered in the coupling between components and one is mainly interested in the interface forces between the components it is advantageous to use the FBS technique.

Acknowledgements

The presented research work was carried out in the framework of the Flemish research project 'IWT-VPR', supported by the Institute for the Promotion of Innovation by Science and Technology in Flanders (IWT). The support of the Flemish government is gratefully acknowledged.

References

[1] W. Heylen, S. Lammens, and P. Sas, Modal Analysis Theory and Testing, K. U. Leuven, PMA, Leuven, Belgium (1997).

[2] R. R. Craig Jr., *Coupling of substructures for dynamic analysis*, AIAA Journal, Vol. 6(7), pp. 1313–1319 (July 1968).

[3] R. R. Craig Jr. and C. Chang, *On the use of attachment modes in substructure coupling for dynamic analysis*, Tech. Rep. 77-405, AIAA (1977), presented at AIAA/ASME 18th Structures, Structural Dynamics and Materials Conference, San Diego, CA, March 21-23, 1977, pp. 89-99.

[4] R. R. Craig Jr., *A review of time-domain and fequency-domain component mode synthesis methods*, International Journal of Analytical and Experimental Modal Analysis, Vol. 2(2), pp. 59–72 (April 1987).

[5] P. De Fonseca, *The use of component mode synthesis techniques in the design and the optimisation of mechatronic systems*, Internal Report 98R06, K.U.Leuven, PMA (February 1998).

[6] L. Hermans, P. Mas, W. Leurs, and N. Boucart, *Estimation and use of residual modes in modal coupling calculations: A case study*, Proceedings of the 18th International Modal Analysis Conference (IMAC), San Antonio, Texas, U.S.A. (2000).

[7] D. Otte, J. Leuridan, H. Grangier, and R. Aquilana, *The use of SVD-based data reduction techniques for coupling of structures using measured FRF data*, Proceedings of the 15th International Seminar on Modal Analysis (ISMA), Leuven, Belgium, pp. 99–113 (1990).

[8] A. Klosterman, On the Experimental Determination and Use of Modal Representations of Dynamic Characteristics, Ph.D. thesis, Dept. of Mech. and Ind. Eng., University of Cincinnati (1971).

[9] Y. Ren and C. F. Beards, *On substructure synthesis with FRF data*, Journal of Sound and Vibration, Vol. 185(5), pp. 854–866 (1995).

[10] L. Hermans, M. Brughmans, and N. Tzannetakis, *Use of modal coupling and modification methods for the vibro-acoustic optimization of a car body*, VDI-Berichte 1550: Experimentelle und rechnerische Modalanalyse sowie Identifikation dynamischer Systeme (VDI-Schwingungstagung 2000), pp. 543–557 (2000).

[11] T. Rose, *Using residual vectors in MSC/NASTRAN dynamic analysis to improve accuracy* (1991), presented at the 1991 MSC World Users' Conference.