# Identification of Rigid Body Properties of 3-D Frame Structure by MCK Identification Method


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## Abstract

This paper presents the result of using a spatial matrix identification method to estimate the rigid body properties of a frame structure. Frequency response functions in three directions at 10 points of the structure are measured by hammer testing. The boundary condition of the structure is free-free. Using the single-input-multiple-output FRFs, the identification method can estimate the rigid body properties of the structure with practical accuracy. The rigid body properties are estimated in three different conditions. The first estimation is done using FRFs up to a frequency between the third and the fourth resonant frequency. The second estimation is carried out using FRFs up to a frequency between the second and the third resonant frequency. The last estimation is carried out using FRFs up to a frequency between the first and the second resonant frequency. All results are practically accurate.


## 1. Introduction

Rigid body properties are defined by the mass quantity, the center of gravity, the principal inertia of moments and their associated principal axes. Rigid body properties are indispensable as well as dynamic properties for various kinds of analysis, synthesis and control design with respect to both structural and mechanical dynamics. For example, one of the core roles of CAE is to predict the dynamic behavior of complex mechanical systems composed of many mechanical and structural components accurately and reliably. Such predictions are carried out synthesizing both the static and dynamic properties of components generally. Simulations cannot be successful without rigid body properties of components. In practical situations, both kinds of properties of all components should be stored in a database. Then, they are downloaded into various simulation methods

The theoretical definition of the rigid body properties is straightforward and well known. However, it is often difficult in practice to identify the rigid body properties of actual mechanical systems and structures accurately due to their complexity. Theoretical approaches, such as the finite element method, need to precise theoretical model. Constructing such accurate models, however, is often difficult due to the geometrical complexity of the structure. Therefore, experimental approaches often play an important role in the identification of rigid body properties in practical situations.

Among the experimental approaches, pendulum testing is used as a primitive method. Another method uses acceleration frequency response functions in the low frequency range in which the inertia is dominant_[1]. This method requires a priori knowledge of one principal axis of the test structure. Using a spatial matrix identification method_[3], Butsuen and Okuma presented a method[2] for identifying the rigid body properties of structures, such as a softly mounted engine. The method was developed further in [4]. These methods assume that the test structure is acting as a rigid body. In order to identify the rigid body properties of flexible structures, a method using the theory of the modal analysis was presented in [5]. It requires the measurement of FRFs of a test structure under the excitation of at least three different locations. That is, at least three sets of single-input-multiple-output FRFs (SIMO FRFs) are required. However, it will be sometimes hard to carry out the measurement of multiple-input- multiple-output FRFs (MIMO FRFs) in practical situations due to the cost of experiments.

One of the authors of this paper has been developing an experimental method for identifying spatial matrices of flexible structures using only SIMO FRFs [6-8]. During his research, it has been found that the method is capable to identify the rigid body properties of flexible structures with practical accuracy. The authors know no other method that can estimate the full set of the rigid body properties of a three dimensional flexible structure using only SIMO FRFs. At this time, a collaborative research work is carried out for investigating the capability using an actual frame structure. This paper reports the result.

## 2. Identification Method

The identification algorithm was already presented in the papers listed as references [6] and [7]. Check these papers for more details. Only the outline of the algorithm is mentioned in this section.

Frequency response functions are measured on the structure to be identified under the free-free boundary condition. Frequency response functions and the coordinates of measurement points are essential for the identification. In addition, the measurement of the coherence functions is recommended. After vibration testing, modal parameters in the frequency range of interest are estimated by a modal parameter estimation method. The parameters to be identified are: the residue of inertia term representing the influence of inertia on the FRFs in the frequency range of interest, the natural frequencies, damping ratios, mode shapes located within the frequency range, and the residual parameters representing the influence of residual natural modes.

After vibration testing, one defines the connectivity between the measurement points, using the coordinates of the measurement points. The identification method automatically determines the location of zero elements in the spatial matrices. In addition, the method creates a set of constraint equations about the relation among non-zero elements in spatial matrices. The set of constraint equations regarding the mass matrix is created based on the physical principle mentioned below. That is, the mass matrix of any system having multi degrees of freedom must keep the relation expressed by

$$[\Psi]^T[M][\Psi]=[M_{rigid}] \,, \tag{1}$$

where [ƒμ] is the matrix of mutually independent rigid motion modes, the matrix [M] is the mass matrix to be identified, and $[M_{rigid}]$ is a rigid body mass matrix. The matrix [ƒμ] is easily formulated with the coordinates of the measurement points. The rigid body mass matrix has the proper formation as described by Eq.(2), provided [ƒμ] is composed of the translational modes in the direction of x-axis, y-axis and z-axis in the first, the second and the third column respectively, and of the rotational modes about x-axis, y-axis and z-axis in the fourth, the fifth and the sixth column respectively.

$$[M_{rigid}]=\begin{bmatrix} m & 0 & 0 & 0 & C & -B \\ 0 & m & 0 & -C & 0 & A \\ 0 & 0 & m & B & -A & 0 \\ 0 & -C & B & I_{xx} & I_{xy} & I_{xz} \\ C & 0 & -A & I_{yx} & I_{yy} & I_{yz} \\ -B & A & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \tag{2}$$

where m is the mass of the structure; $I_{xx}$, $I_{yy}$, $I_{zz}$ are inertia moments around x-axis, y-axis and z-axis respectively; $I_{yz}$, $I_{zx}$, $I_{zy}$ are products of the moments of inertia; A, B, C are parameters based on the relations: $A=mx_g$, $B=my_g$, $C=mz_g$. ( $x_g$, $y_g$, $z_g$ ) is the coordinate of the center of gravity. Eventually, even without knowing the rigid body properties of the structure, several constraint equations among the elements of the mass matrix [M] can be created. Namely, some non-zero elements of the mass matrix [M] are expressed as dependent variables by the linear combination of the other non-zero elements, which are dealt with as independent variables. Furthermore, if some of the rigid body properties are already known, it is possible to use the known values to create the constraint equations.

The constraint equations regarding the stiffness matrix and the damping matrix are created as follows. Eq.(3) can be formulated according to the principle that no stress is generated at any point of the structure for any feasible rigid body motion:

$$[K][\Psi]=[0] \,, \tag{3}$$

where [K] is the stiffness matrix to be identified, and [0] is a zero element matrix. Constraint equations regarding the stiffness matrix can be

created using Eq.(3). The constraint equations of the damping matrix are identical to those of the stiffness matrix.
A set of the initial matrices has to be created to begin the identification method because of the iterative nature of the method. The initial values of the mass matrix and the stiffness matrix are set up by substituting random numbers into the independent variables of the constraint equations. The substitution of random numbers is fast and simple, and no better way to set up the initial matrices has been found yet. After the set up of the initial matrices, the mass matrix and the stiffness matrix are improved to become a positive definite matrix and a positive semi-definite matrix respectively by a sensitivity based optimization method with respect to negative eigen-values. Then, [K] and [M] are fine tuned such that some lower natural frequencies, computed from the eigen-value problem expressed by Eq.(4), correspond well to the experimentally observed natural frequencies.

$$([K]-\lambda_i[M])\{\phi_i\}=\{0\} \qquad (4)$$

In addition to the control of eigen-values, also the correlation between the corresponding model and measured mode shapes is gradually improved. Once a high degree of correspondence of the natural frequencies and modes within the frequency range of the identification is achieved, the spatial matrices are further improved in order to make the magnitude of the model FRFs fit the experimental FRFs.
Since the objective of this research is to estimate the rigid body properties of flexible structures, it is unnecessary to determine the damping matrix. The rigid body properties can be derived from the mass matrix using Eq.(1) and (2). However, it is noted here that the experimental spatial matrix identification method is not only for the estimation of rigid body properties but mainly for the estimation of a set of spatial matrices that can represent the dynamic characteristics as well as rigid body properties.

# 3. Identifications of 3-D frame structure

## 3.1 The Test Structure and Measurement

Fig.1 shows the test structure to be identified. This structure is built of iron pipe with a square cross-section. Fig.2 shows its schematic view and the location of 10 measurement points.

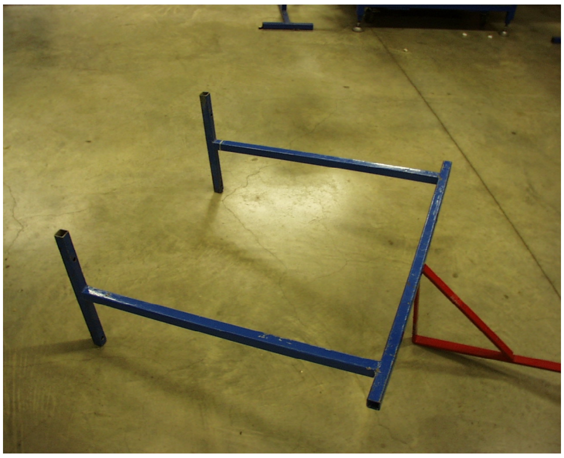

Fig.1 The Test Structure

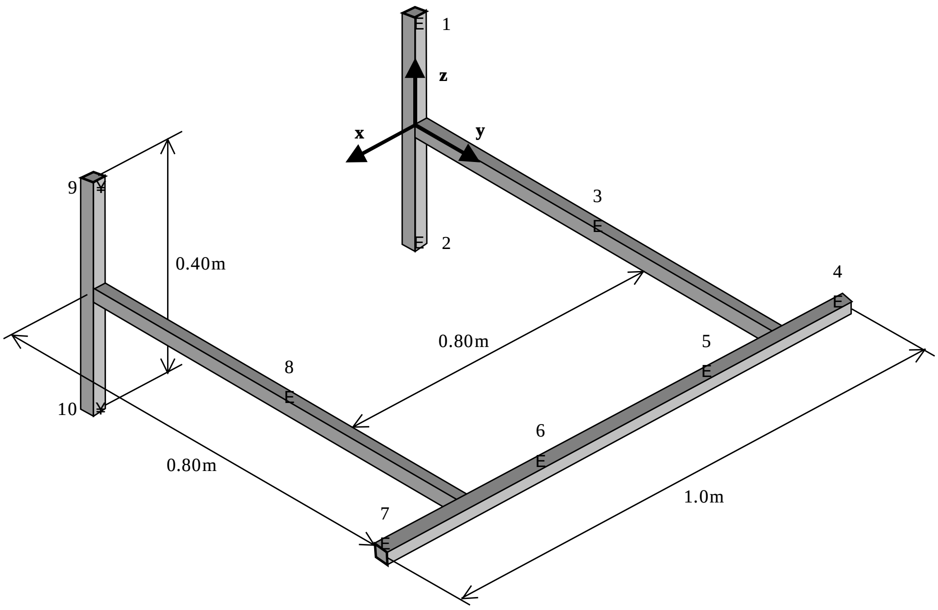


Fig.2 Schematic View of Location of Measurement Points

Three directional accelerometers measured the responses to hammer excitation. Consequently, the number of degrees of freedom of the spatial matrices to be identified is 30. The coordinate system is also shown in Fig.2. The origin is set at the center of the measurement points No.1 and No.2. Hammer input is applied only in the x-direction at measurement point No.1. Fig.3 shows an example of an experimental FRF. It is the direct FRF at measurement point No.1.

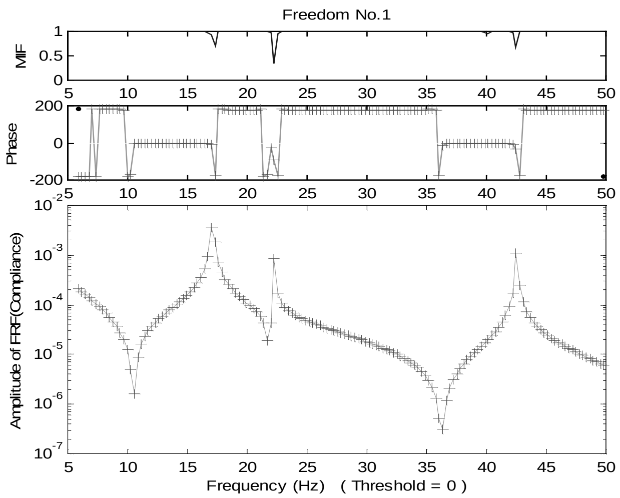


Fig.3 Experimental FRFs of the excited freedom( Point No.1, X-direction)

### 3.2 Identification using FRFs up to a frequency between the third and the fourth resonance

This section presents the results of the rigid body properties identification using FRFs from 6 Hz up to 50 Hz. As shown in Fig.3, the first, second and third resonances are located in the frequency range. Using the truncated FRFs, the identification method determines a set of spatial matrices having 30 degrees of freedom. The identified set of spatial matrices can represent the input experimental FRFs accurately as shown in Fig.4 and Fig.5. The identified spatial matrices are not shown explicitly in this paper due to their size.

The identification method is programmed on MatLab[9] using only the standard Matlab package. The method is an iterative algorithm_[6,7]. Therefore, initial values of spatial matrices are required to start the identification method. The initial values are made using the uniform random number generation function of MatLab. In order to investigate the influence of the initial values, the identifications are carried out ten times using different initial spatial. Table 1 shows the identified rigid body properties. In the table, the column labeled as “Mean” describes the mean values of the results of ten identifications. The column labeled as “S.D.” describes the standard deviations of these results. The values listed in the column labeled as “(Min., Max)” are the minimum and the maximum output values of the ten identifications. The values described in the column labeled as “Ref.” are obtained by manual calculation using the geometry and material data. It is found that the initial values influence the results of the rigid body properties. However, the influence is so slight that the results can be acceptable in practice.

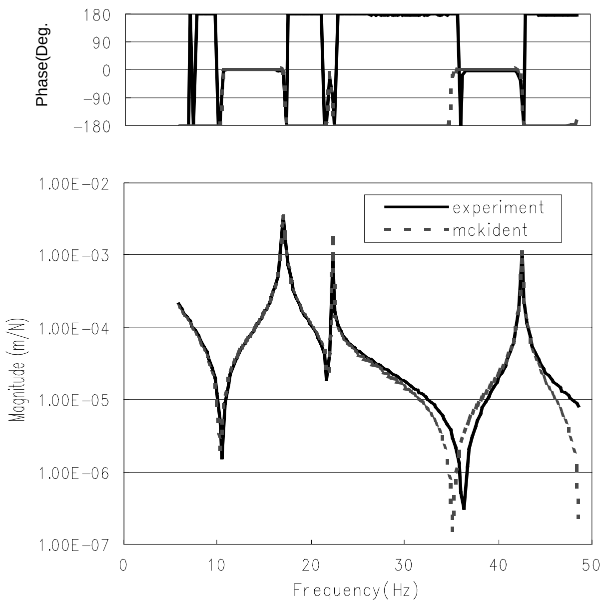


Fig.4 Fitting of FRFs between Experiment and Identification (X-direction of Measurement Point No.1)

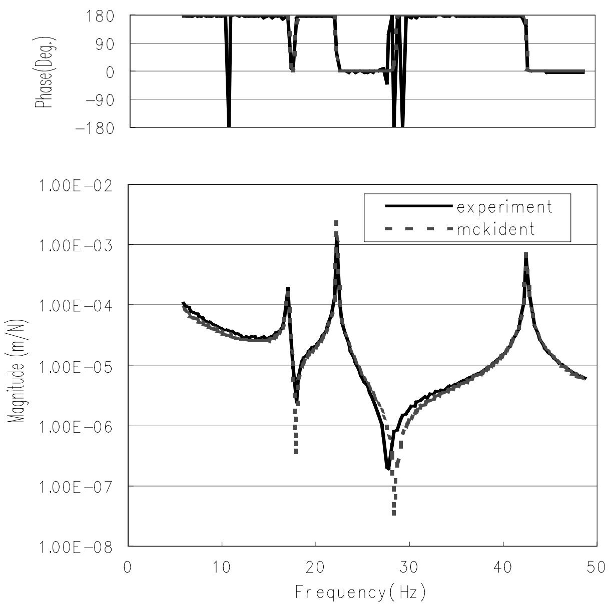


Fig.5 Fitting of FRFs between Experiment and Identification (Y-direction of Measurement Point No.10)

### 3.3 Identification using FRFs up to a frequency between the second and the third resonance

This section presents the results of the identification using FRFs from 6 Hz up to 26 Hz. The resonant peaks located in the frequency range are only the first and second natural frequency as shown in Fig.3. Table 2 lists the result. These values are the results of ten identifications. The results are not different from the ones in Table 1. The identified values can be also practically acceptable.

**Table 1. Rigid Body Properties identified using FRFs from 6 Hz to 50 Hz**

| Propertie \ Methods | | Identification using 6Hz-50Hz | | | Ref. |
|---|---|---|---|---|---|
| | | Mean | S.D. | (Min., Max.) | |
| Mass (kg) | | 5.2 | 0.21 | (5.0, 5.7) | 5.7 |
| Center of Gravity (m) | X coordinate | 0.4 | 0.0071 | (0.389, 0.410) | 0.4 |
| | Y coordinate | 0.41 | 0.0043 | (0.396, 0.410) | 0.439 |
| | Z coordinate | 0.0029 | 0.013 | (-0.035, 0.014) | 0 |
| Principal Inertia of Moment (kgm^2) | I11 | 0.78 | 0.03 | (0.750, 0.841) | 0.697 |
| | I22 | 0.89 | 0.028 | (0.853, 0.956) | 0.684 |
| | I33 | 1.53 | 0.065 | (1.43, 1.67) | 1.48 |
| Principal Axes of Inertia of Moments (Directional Cosine) | V1: x-coord. | -0.98 | 0.0052 | (-.972,-0.999) | 1 |
| | V1: y-coord. | 0.0079 | 0.19 | (-0.181, 0.220) | 0 |
| | V1: z-coord. | 0.0095 | 0.055 | (-0.0623, 0.069) | 0 |
| | V2: x-coord. | 0.079 | 0.19 | (-0.179, 0.217) | 0 |
| | V2: y-coord. | 0.98 | 0.049 | (0.971, 0.997) | 1 |
| | V2: z-coord. | -0.045 | 0.073 | (-0.141, 0.0906) | 0 |
| | V3: x-coord. | -0.0036 | 0.068 | (-0.0808, 0.0708) | 0 |
| | V3: y-coord. | 0.0382 | 0.079 | (-0.074, 0.139) | 0 |
| | V3: z-coord. | 0.995 | 0.0036 | (0.990, 0.999) | 1 |

**Table 2. Rigid Body Properties identified using FRFs from 6 Hz to 26 Hz**

| Propertie \ Methods | | Identification using 6Hz-26Hz | | | Ref. |
|---|---|---|---|---|---|
| | | Mean | S.D. | (Min., Max.) | |
| Mass (kg) | | 5.2 | 0.16 | (5.0, 5.5) | 5.7 |
| Center of Gravity (m) | X coordinate | 0.4 | 0.0063 | (0.387, 0.408) | 0.4 |
| | Y coordinate | 0.41 | 0.0029 | (0.406, 0.416) | 0.439 |
| | Z coordinate | 0.00035 | 0.012 | (-0.0185, 0.024) | 0 |
| Principal Inertia of Moment ( kgm^2) | I11 | 0.81 | 0.057 | (0.773, 0.826) | 0.697 |
| | I22 | 0.87 | 0.025 | (0.908, 0.829) | 0.684 |
| | I33 | 1.51 | 0.064 | (1.42, 1.63) | 1.48 |
| Principal Axes of Inertia of Moments (Directional Cosine) | V1: x-coord. | -0.99 | 0.0014 | (-.999,-0.958) | 1 |
| | V1: y-coord. | 0.0011 | 0.14 | (-0.286, 0.227) | 0 |
| | V1: z-coord. | -0.0014 | 0.055 | (-0.103, 0.125) | 0 |
| | V2: x-coord. | 0.0019 | 0.14 | (-0.287, 0.217) | 0 |
| | V2: y-coord. | 0.99 | 0.013 | (0.956, 0.999) | 1 |
| | V2: z-coord. | -0.034 | 0.057 | (-0.137, 0.0654) | 0 |
| | V3: x-coord. | -0.0026 | 0.058 | (-0.0958, 0.0142) | 0 |
| | V3: y-coord. | 0.019 | 0.042 | (-0.066, 0.0627) | 0 |
| | V3: z-coord. | 0.99 | 0.005 | (0.985, 0.999) | 1 |

**Table 3. Rigid Body Properties identified using FRFs from 6 Hz to 20 Hz**

| Propertie \ Methods | | Identification using 6Hz-20Hz | | | Ref. |
|---|---|---|---|---|---|
| | | Mean | S.D. | (Min., Max.) | |
| Mass (kg) | | 5.3 | 0.09 | (5.1, 5.4) | 5.7 |
| Center of Gravity (m) | X coordinate | 0.4 | 0.0009 | (0.399, 0.401) | 0.4 |
| | Y coordinate | 0.41 | 0.003 | (0.407, 0.416) | 0.439 |
| | Z coordinate | 0.0073 | 0.014 | (-0.0304, 0.00322) | 0 |
| Principal Inertia of Moment ( kgm^2) | I11 | 0.79 | 0.024 | (0.75, 0.816) | 0.697 |
| | I22 | 0.89 | 0.033 | (0.838, 0.910) | 0.684 |
| | I33 | 1.48 | 0.022 | (1.44, 1.51) | 1.48 |
| Principal Axes of Inertia of Moments (Directional Cosine) | V1: x-coord. | 0.97 | 0.0021 | (0.939, 0.990) | 1 |
| | V1: y-coord. | -0.14 | 0.19 | (-0.297, 0.200) | 0 |
| | V1: z-coord. | 0.0168 | 0.032 | (0.0141, 0.0785) | 0 |
| | V2: x-coord. | 0.14 | 0.19 | (-0.199, 0.339) | 0 |
| | V2: y-coord. | 0.97 | 0.021 | (0.939, 0.990) | 1 |
| | V2: z-coord. | -0.0098 | 0.029 | (-0.0614, 0.0249) | 0 |
| | V3: x-coord. | -0.01 | 0.025 | (-0.0532, 0.0176) | 0 |
| | V3: y-coord. | 0.016 | 0.036 | (-0.0168, 0.084) | 0 |
| | V3: z-coord. | 0.99 | 0.0017 | (0.995, 0.999) | 1 |

### 3.4 Identification using FRFs up to a frequency between the first and the second resonance

In this section, it is investigated how accurately the full set of the rigid body properties can be estimated using FRFs in the frequency range including only the first resonance. The frequency range is from 6 Hz up to 20Hz. Only the first resonant peak is located in the frequency range as shown in Fig.3. Table 3 lists the results. The values are the results of six identifications with different starting values. Since extra identifications runs yield the same results, no more identifications are carried out for this case. The accuracy of the estimated values is also practically acceptable. It becomes clear that the rigid body properties can be estimated with practical accuracy even using SIMO FRFs in a frequency range including only the first resonant peak by the experimental spatial matrix identification method. All results show that the principal inertia moments are always overestimated. Research is going on in order to remove the bias errors theoretically.

## 4. Conclusions

This paper presented the results of estimating the rigid body properties of a flexible frame structure using its FRFs measured by a hammer test. For the experimental identification, only SIMO FRFs are necessary. Using the FRFs in the frequency ranges including only a few resonant frequencies, the rigid body properties can be estimated accurately in practice. As a typical condition, it has been found that the identification of rigid body properties is possible even using FRFs in a narrow frequency range including only the first resonant peak. The initial random values of spatial matrices influence the identified rigid body properties only a little. It can be ignored in practice.

## Acknowledgements

This research work was carried out with the support of Venture Business Laboratory Funds of Tokyo Institute of Technology. The authors are grateful for the support.

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