Proper excitation for the derivation of the best related linear dynamic system to describe sprayer boom dynamics

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Abstract

This paper discusses the derivation of the best related linear dynamic system approximation to a tractor-sprayer combination and a parametric model for the related linear dynamic system is identified. It will be of interest to check the degree of non-linearity in order to consider the validity of linear dynamic models. A simple method has been utilised based on the kernel idea of well-chosen periodic excitations where only some of the considered frequency components are excited. The non-excited frequency lines will be used to detect, qualify and quantify the non-linear distortions.

To compare, the tractor-sprayer combination has been excited with a normal multisine and an odd-odd multisine excitation. Next, a parametric transfer function model has been identified in the frequency domain for both data records respectively. The experiment affirms that odd multisines can give positive results in the presence of non-linear distortions. Indeed, derived models even better describe the system than those models obtained from experiments with a multisine excitation.

1. Introduction

The purpose of linear system theory consists in the derivation of a linear model structure to describe approximately the dynamics of a physical system. In this application, an agricultural boom sprayer represents the system of interest. Field sprayers are deployed in agriculture to spray crops with liquid pesticides so that cultivars can grow up in optimal conditions with a minimum pressure from weeds, diseases and insects. The efficiency of any chemical crop protection is mostly controlled by the homogeneity of the droplet deposition and the uniform spray covering on the canopy [1]. Non-uniformity of application across the swath has been shown to have a significant effect on weed and disease control [2].

In practice, uneven doses are mainly the result of wind effects and unwanted sprayer boom vibrations. Simulations demonstrate variations in spray deposit between 0 % and 1000 % for the vertical boom vibrations and between 20 % and 600 % for the horizontal ones [3].

Within the scope of a general study to investigate the effects of sprayer boom vibrations on spray liquid deposition, dynamic models of the spray boom are derived to calculate the occuring boom motions under operating conditions. Combination of the vibrating boom amplitudes with hydraulic liquid distribution models provides information about the uniformity of spray deposits [4].

Prior research at the laboratory has shown that sprayer boom dynamics can be accurately approximated with linear parametric input-output models identified in the frequency domain via the maximum likelihood estimater [5]. Since proposed method is an experimental technique, work has been spent on the selection of a proper reference signal for the experimental test. A multisinusoidal signal seems to have superior properties in relation to a periodic noise excitation [5]. This conclusion points out the presence of possible non-linear distortions on the system output.

It will be of interest to check the degree of non-linearity in order to consider the validity of those linear techniques. The goal of this paper is to detect, qualify and quantify the presence of non-linear distortions on the output of the sprayer. Next, a related linear dynamic system (RLDS) approximation to the non-linear system is defined and a parametric model for the RLDS is identified.

2. Mathematical framework

Here, a method for the detection and the qualification of the non-linear distortions on an FRF or transfer function measurement is presented. The kernel idea is to use well-chosen periodic excitations where only some of the considered frequency components are excited. The non-excited frequency lines will be used to detect, qualify and quantify the non-linear distortions.

The objective of this section is to provide insight in the behaviour of non-linear distortions and their impact on FRF measurements. This allows not only a better understanding of the error mechanism involved, this knowledge can also be applied to the experimental design in order to get the best results under the imposed operational conditions. Hence, a brief description of the mathematical framework used to describe the non-linear distortions is provided in this section. A detailed description, stating precisely all underlying assumptions is given in the work of *Schoukens et al.* [6].

The non-linear distortions are described by a Volterra series. An extended introduction to this technique is given in the book of *Schetzen* [7]. Therefore, this technique includes only non-linearities that can be described by Volterra series.
In order to demonstrate the technique, a general structure is used as depicted in figure 1. The measured output $y(t)$ consists of a linear y_L and a non-linear y_{NL} contribution. It is assumed that y_L dominates over y_{NL} for sufficiently small inputs $u(t)$.

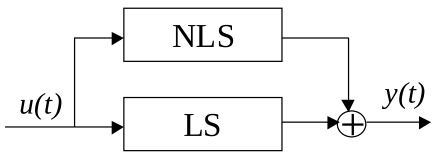

Figure 1: General set-up of considered structure

The idea is to describe the output using multi-dimensional convolutions:

$$y(t)=\int_{-\infty}^{+\infty}h_1(\tau)u(t-\tau)d\tau+\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2+... \quad (1)$$

Two options exist under this assumption: 1) the goal of the measurement is to get the FRF of the underlying linear system (LS), minimising the impact of the non-linear system (NLS) on the measurements; 2) try to find the best linear approximation to the global system, including the NLS. The second choice is preferred if the model will be used to describe the relation between input and output using linear models.

Here, the non-linearity can be linearised around the operation point of the test. The best linear approximation is called the related linear dynamic system (RLDS). It is clear that the best linear model will always be an excitation-dependent approximation of the true non-linear system [6].

The analysis has been carried out for random multisine excitations, which is a periodic random excitation with a user defined choice for the amplitude spectrum and independent uniformly distributed random phases between [0, 2π]:

$$x(t)=\sum_{k=-N}^{N}X(k)e^{j2\pi\frac{f_{max}}{N}kt} \quad (2)$$

with $X(k)=X^*(-k)=|X(k)|e^{j\varphi_k}$,

f_{max} the maximum frequency of the signal
N the number of frequency components
φ_k the phases, random distributed between [0, 2π[.

For periodic excitations with N harmonics of frequencies kf_{max}/N, $k = 1,\ldots, N$, these convolutions simplify to a sum over all possible frequency combinations adding to the output Fourier coefficient $Y(l)$ at frequency lf_{max}/N [8]:

$$Y(l)=\sum_{\alpha=1}^{\infty}Y^{\alpha}(l), \quad (3)$$

with $Y^{\alpha}(l)$ the contribution of degree α:

$$Y^{\alpha}(l)=\sum_{k_1=-N}^{N}...\sum_{k_{\alpha-1}=-N}^{N}H^{\alpha}_{L_k,k_1,...,k_{\alpha-1}}X(k_1)...X(k_{\alpha-1})X(L_k) \quad (4)$$

and $L_k=l-\sum_{i=1}^{\alpha-1}k_i$

H^{α} is the symmetrized frequency domain representation of the Volterra kernel of degree α [7] so that the order of the frequencies $L_k, k_1,\ldots, k_{\alpha-1}$ has no importance.

3. Asymptotic properties of non-parametric FRF

The asymptotic properties of the FRF have been analysed in [6] and it has been demonstrated that the non-linear distortions split in two classes: systematic and stochastic contributions.

There exists a RLDS to which the expected value of the FRF estimate converges. It differs from the underlying linear system by the systematic contributions of the non-linear distortions. This bias (systematic errors) is a deterministic component, independent of the random phase of the excitation that models the systematic contribution of the non-linear distortions to the FRF.

In addition, there are also the so-called stochastic non-linear distortions. The FRF estimate is not smooth as a function of the frequency, even for a very large number of frequencies. It is scattered around its expected value and these deviations do not converge to zero. The stochastic contribution depends on the random phase of the excitation and consequently it is a random component modelling the stochastic contribution of the non-linear distortions to the FRF.

Taking this into account, the measured FRF $G(\omega_k)$ can be written as the sum of four parts:

$$G(\omega_k)=G_{LS}(\omega_k)+G_B(\omega_k)+G_S(\omega_k)+N_G(\omega_k) \quad (5)$$

with $G_{LS}(\omega_k)$ the underlying linear system, $G_B(\omega_k)$ the bias or systematic errors due to the non-linear distortions, $G_S(\omega_k)$ the stochastic non-linear contribution and $N_G(\omega_k)$ the errors due to the output noise. $G_S(\omega_k)$ is called a stochastic contribution since it behaves as uncorrelated noise.

It has been shown that neither of both non-linear contributions is decreasing if the number of frequencies N increases (asymptotic behaviour of the FRF if the number of harmonics $N \to \infty$).

Therefore, the output can be split in two parts: a first part that is linearly related with the input (leading to $G_{RLDS} = G_{LS} + G_B$) and it extends the concept of linear systems, and a second part that is uncorrelated with the input (leading to G_S). Measurements of the best linear approximation (G_{RLDS}) are obtained by eliminating the stochastic non-linear contributions (G_S) and the noise contributions (N_G). Both stochastic contributions G_S and N_G can be reduced through averaging. The RLDS can be considered as the best linear approximation of the non-linear system, but it is clear that this approximation strongly depends on the considered class of excitation signals. It has been shown that the best linear approximation can be obtained with a minimum number of averages using odd multisines with minimised crest factor [6, 9].

4. Detection of non-linear distortions

The aim of this section is to provide a simple test to check the non-linear behaviour of the device under test. The basic idea of this test is to excite the system with an odd-odd multisine, where only the frequencies $4k + 1$, $k = 0, 1,\ldots, kmax$ in Eqn (2) have amplitudes different from zero. Afterwards the output spectrum is calculated with a DFT. From Eqn (3) and (4) it follows that the even non-linearities excite only the even harmonics at the output ($2k$, $k=1, 2,\ldots$), while the odd non-linearities appear only at the odd harmonics ($2k+1$, $k=1, 2,\ldots$). Determined by the choice of the excitation signal, the following possibilities are obtained:

- At lines $4k+1$: the output consists of the linear contribution plus odd non-linear distortions
- At lines $4k+2$: only the even non-linear distortions appear
- At lines $4k+3$: only odd non-linear distortions appear.

This allows to get an idea of the non-linear behaviour of the system. If at least $M \geq 2$ successive periods are measured in one block, the same conclusions still apply respectively at lines $M(4k+1)$, $M(4k+2)$ and $M(4k+3)$. On top of that also the noise level can be characterised by looking at the lines that are no multiple of M since these cannot be excited by a signal with M periods, indicating that only the noise can contribute there.

5.The measurement set-up

The derivation of the best linear approximation of the selected tractor-sprayer combination, a John Deere 2850 tractor with mounted Vicon sprayer, 800-l tank and 16-m boom, forms the fundamental objective of this paper. This means that the considered system should be subjected to a vibrational motion, meanwhile the input disturbance and system's response are recorded.

An electro-hydraulic shaker with one degree of freedom has been selected as excitator, because of its interesting properties as high power generation and possibility of free excitation. The reference signals forwarded to the PID controller are made up of displacements of the shaking unit. Figure 2 shows a picture of the test set-up on which a view is given of the test object, the data-acquisition equipment, the transducers and the excitation mechanism.

Figure 2: Picture of test set-up to perform experimental system identification on Vicon sprayer.

The tractor-sprayer combination is placed with the left rear wheel on the excitation device for shaking it in the vertical direction. Periodic excitation is used, composed of 10 successive measurement periods, each lasting 20.48 seconds. All signals are sampled at 100 Hz sample frequency and aliasing problems are reduced to a minimum by employing analog low-pass Butterworth filters with a cut-off of 20 Hz. The measurements have been started when the system reached the steady state regime.

Two types of measurements have been carried out, according to the measured quantities for respectively input and output (SISO):

- Type I are made up of vertical displacements of the tractor wheel for the input and horizontal velocities on the boom tip for the output.
- Type II data are composed of vertical accelerations of the tractor wheel (input) and vertical accelerations for the boom tip (output).

In the test procedure, the considered system is studied in the frequency band between 0.1-5 Hz, in which most dominant resonance frequencies of the device under test (DUT), as well as the main disturbance frequencies of the field undulations are situated. In addition, higher frequencies do not contribute to large spray boom motions and should therefore not be considered.

6. The related dynamic system

6.1. Non-parametric transfer function

The purpose of this small experiment is to provide insight in the behaviour of non-linear distortions and their impact on FRF measurements. This knowledge can be applied to the experimental design in order to get the best results under the imposed operational conditions. The concept is illustrated for two types of reference signals: a multisine and an odd-odd multisine excitation. Both types have a random phase distribution, but a deterministic amplitude spectrum.

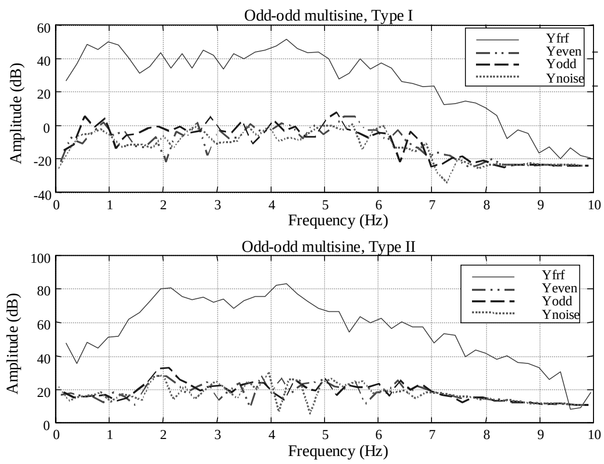

Figure 3: Detection of non-linear distortions at the output of the boom tip for horizontal and vertical direction. Y_{frf}: linear + odd non-linear contributions, Y_{even}: even non-linear contributions, Y_{odd}: odd non-linear contributions, Y_{noise}: noise level.

The tractor-sprayer combination has been excited with an odd-odd multisine (0.1-10 Hz) at the frequencies $f_k = (4k+1)f_0$, with $k = 1, \ldots, 52$ with constant amplitude and random phase. The output spectrum of the left boom tip is calculated with a DFT for respectively the horizontal and vertical direction and the results are shown in figure 3 with $M = 10$ (10 successive periods).

From this simple test one notices even and odd non-linear distortions (the non-linear distortions are much more pronounced for the vertical direction, ± 20 dB), but these are not above the noise level (except in certain frequency ranges).

In case of the horizontal tip motions, the non-linear distortions are very small and far below the linear contribution. The odd non-linear distortions are comparable to the even non-linear distortions at the entire frequency band. Since the even non-linear contributions are rather small, no significant improvement is expected using odd multisines.

Significant even- and odd non-linear distortions are present in the vertical output data. Once again, the odd- and even non-linear distortions are similar at the whole frequency range.

6.2. Parametric transfer function model

Subsequently, the behaviour of the linear transfer function model (parametric) in the presence of non-linear distortions will be studied. This section describes the identification of a transfer function model of the sprayer boom based on the frequency response function measurements. Two steps are considered in this identification procedure: the selection of the model structure (model orders: d-n_h-n_l) followed by an estimation of the 'best' possible model parameters within this model structure.

The model structure selection problem is reduced to the choice of the correct order of the numerator polynomial (n_l, n_h; with n_h and n_l the highest and lowest power of the numerator polynomial, respectively), and the order of the denominator d of the transfer function. This choice is based on some *a priori* information available from the non-parametric transfer function estimates. The model representation can be either in discrete domain as in the Laplace domain (continuous time).

The model parameter estimates are the result of a minimisation of a cost function with respect to the unknown values, *i.e.* the model parameters. This leads to a non-linear equation in the model parameters. Therefore, numerical algorithms are proposed that are based on Gauss-Newton or Levenberg-Marquardt iterative schemes. The initial guess for the parameters is given by the linear least squares estimate. All algorithms to solve the minimisation problem were implemented in MATLAB code.

Various model structures (orders) have been selected and the corresponding model parameters have been identified in the frequency domain (maximum likelihood estimator for multisine data records and non-linear least squares estimator for data obtained with odd-odd multisine excitation).

Information obtained from the frequency response functions was helpful by the model order selection. This information served as starting values for a trial-and-error procedure to obtain finally the best model orders. Table 1 summarises the results for the multisine and odd-odd multisine excitation. The model structure (d-nh-n_l) and the normalised root mean squares value (e_n) are mentioned for only the best model sets, who are a compromise between accuracy and complexity.

Table 1: Best models (continuous time) describing sprayer boom dynamics for multisine and odd-odd multisine excitation with corresponding normalised root mean square errors e_n; model structures given as d-n_h-n_l where d is the power of the denominator polynomial, n_h and n_l is the highest and lowest power of the numerator polynomial, respectively.

Location	Multisine excitation		Odd-odd multisine excitation	
	Model structure	Error (e_n)	Model structure	Error (e_n)
Type I	6-6-0	0.81	6-5-1	0.57
Type II	6-6-0	0.61	6-6-1	0.47

The value of the normalised root mean square error e_n, Eqn (6), reflects the reliability of the predictions and is an easy indicator to compare different model structures for different experiments:

$$e_n = \frac{\sqrt{\frac{1}{N}\sum_{k=1}^{N} E_k^2}}{\sqrt{\frac{1}{N-1}\sum_{k=1}^{N}(y_k - \bar{y})^2}} \qquad (6)$$

where: E_k is the residual; y_k is the variable; $\bar{y}$ is the mean of the variable y; k is an index; and N is the number of data points of vector y.

Table 1 reveals striking differences in e_n-values with a preference for the odd-odd multisine. Hence, these models will be discussed in more detail below. Two models are identified in the frequency domain through the non-linear least squares estimator (NLSE) of the data achieved with an odd-odd multisine excitation, one for the horizontal motions (6-5-1) and another for the vertical direction (6-6-1).

Table 2 gives the coefficients of numerator and denominator polynomials for the longitudinal (type I) and vertical (type II) directions, together with the poles and zeros from the models. Comparisons between the transfer function estimates and the measured frequency response functions are shown in figure 4.

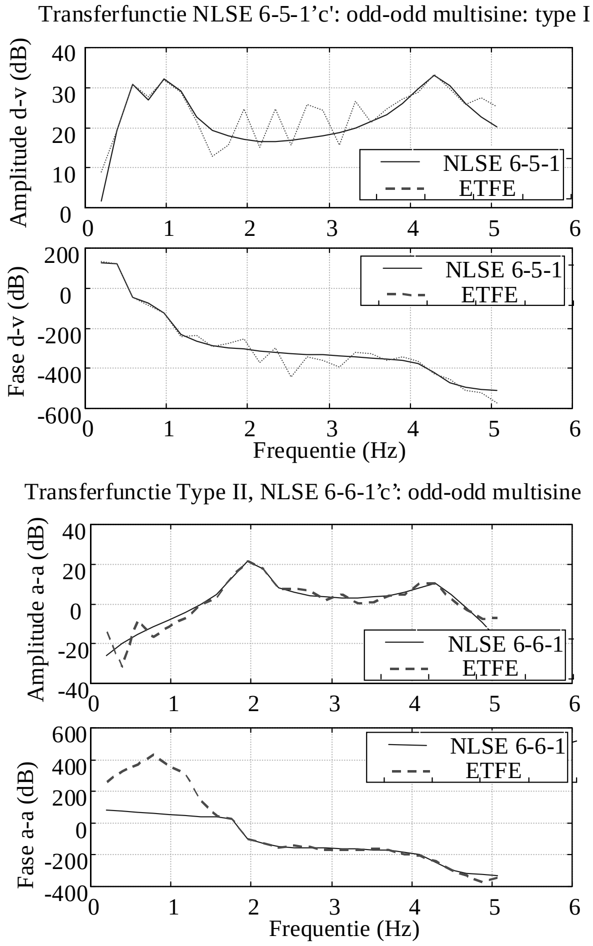

Figure 4: Comparisons between the amplitudes and phases of the transfer function estimates (full line) and the measured frequency response function (dotted line) for longitudinal and vertical direction respectively; excitation with odd-odd multisine.

Table 2: Results of the 6-5-1 model of the horizontal boom tip motions and of the 6-6-1 model describing vertical tip motions, both estimated through the NLSE (odd-odd multisine excitation).

Type I: NLSE 6-5-1 'c' from odd-odd multisine: $e_n = 0.57$	
NLSE = [nd nhi nlo] = [6 5 1] 'continuous time'	
N_{NLSE} = [0 -7.91 2.82 10^3 -2.56 10^4 1.48 10^5]	
D_{NLSE} = [1 3.74 799 1.16 10^3 4.08 10^4 1.75 10^4 3.39 10^5]	
Zeros	Poles
-0.16	-0.019 ± 0.52 I
0.81 ± 0.99 i*	-0.09 ± 1.05 I
55.34*	-0.19 ± 5.27 I
Type II: NLSE 6-6-1 'c' from odd-odd multisine: $e_n = 0.47$	
NLSE = [nd nhi nlo] = [6 6 1] 'continuous time'	
N_{NLSE} = [-0.4 3.59 -508.4 4.59 10^3 -7.65 10^4 7.42 10^5]	
D_{NLSE} = [1 2.93 1.06 10^3 1.38 10^3 2.64 10^5 1.47 10^5 1.9 10^7]	
Zeros	Poles
-0.04 ± 2.13 i	-0.04 ± 1.9 I
0.005 ± 5.27 i*	-0.02 ± 2.14 I
1.5*	-0.17 ± 4.31 I
* non-minimum phase	

6.3. Validation of dynamic models

The accuracy of the models describing the dynamics of the device under test can be evaluated in several ways. A straightforward manner is the inspection of the bodeplots of the model, together with the estimated frequency response function. Good similarity over the whole frequency band corroborates the correct model representation for both directions.

To check the performance of the model to different inputs, a simulation experiment has been carried out. The tractor-sprayer system is excited with an arbitrary signal, representing a standardized track, crossed at a speed of 6 km/h. Simultaneously, input displacements of the shaking platform and velocity (acceleration) responses have been recorded on the boom. The output of the models to these input signals are calculated in the time domain and compared with the measured time data records. Figure 5 shows the results for the horizontal and vertical, respectively, boom tip motions.

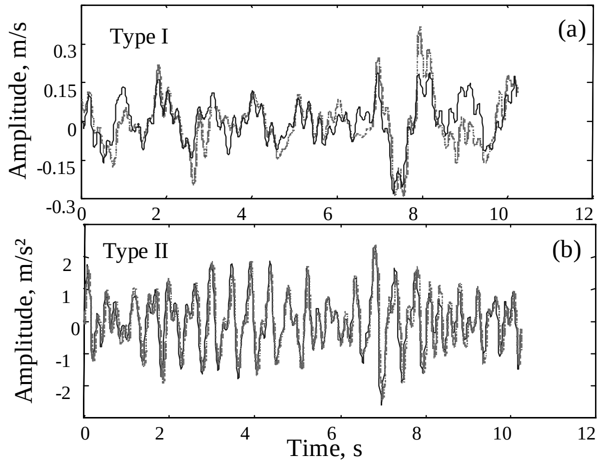

Figure 5: (a) Comparison between the amplitudes of the models outcome (6-5-1) (full line) and the measured horizontal velocity of the boom tip (dotted line); (b) comparison between the amplitudes of the models outcome (6-6-1) (full line) and the measured vertical accelerations of the boom tip (dotted line).

7. Conclusion

Through a simple test, based on special designed excitation signals, possible non-linear distortions can be characterised. Information provided in this way is helpful in experimental design. Analysis confirms the presence of even and odd non-linear distortions on the outputs, however much more pronounced for the vertical direction.

The level of the non-linear distortions can be clearly discerned from the linear system, but they do really influence the results since the latters show conspicuous improvements mainly at the lower frequencies (inspection of transfer function of models) when using special designed odd-odd multisines. This effect is more apparent for the horizontal direction (however not expected from above considerations) and analogous conclusions can be drawn from the simulation results. Major progress is made in the horizontal directions, with a sharp decline of the e_n-value to 0.57 (compared with 0.81, table 1).

By this small illustration, it was shown that the best linear approximation can be obtained with a minimum number of averages using odd multisines. If even non-linear distortions are present, it is better to eliminate them in the tests using odd multisines, because it is known that they do not contribute to the best linear approximation. Indeed, derived models even better describe the system than those models obtained from experiments with a multisine excitation.

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